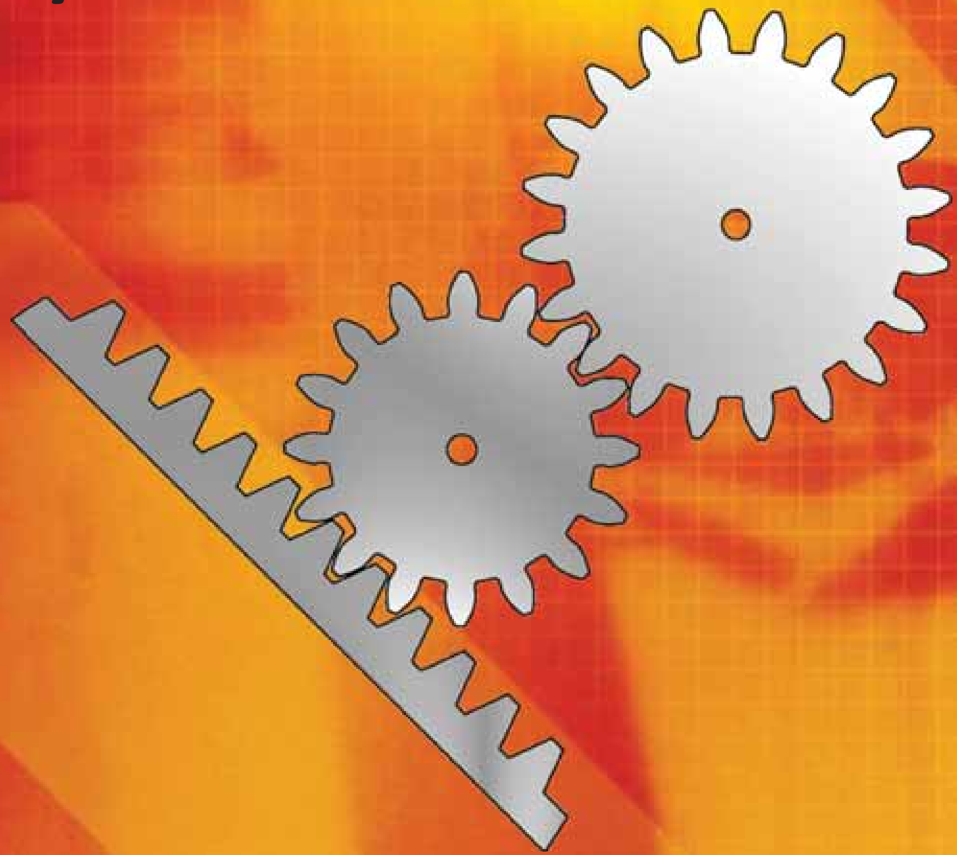
Dynamic Mechanisms



TECHNOLOGY
SUBJECTS
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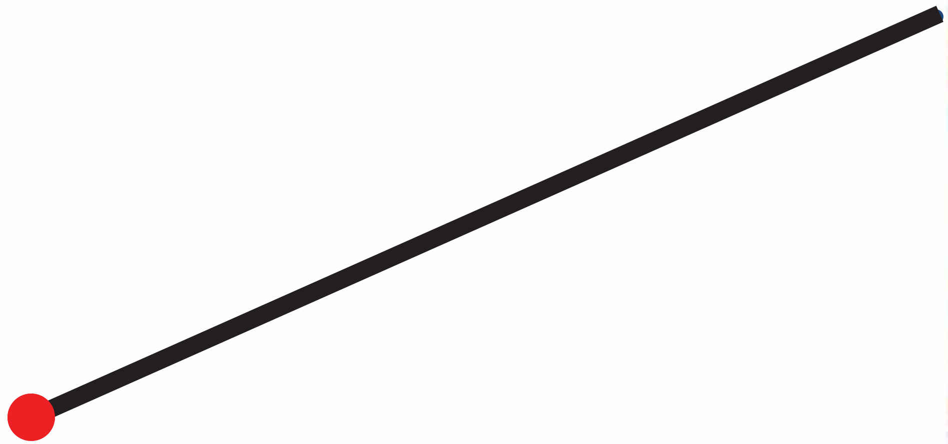
BASICS OF LOCI

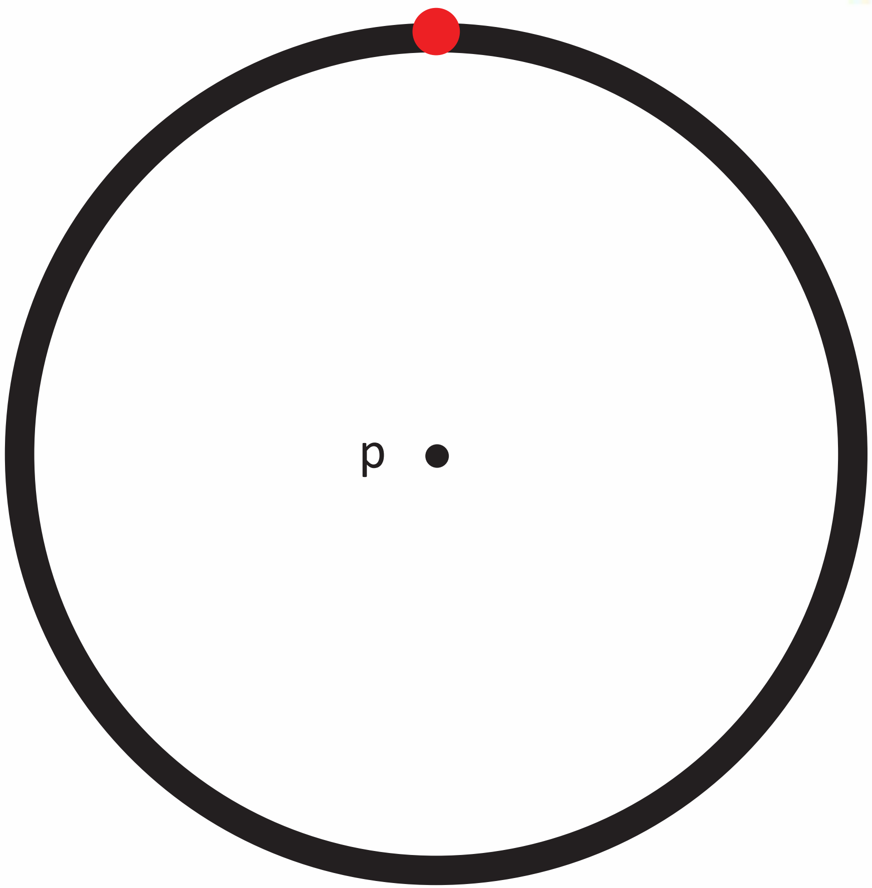
What is a Locus?

- A **locus** is the movement of a point as it follows certain conditions
- For example, a line is the locus of a point as it moves in a straight path



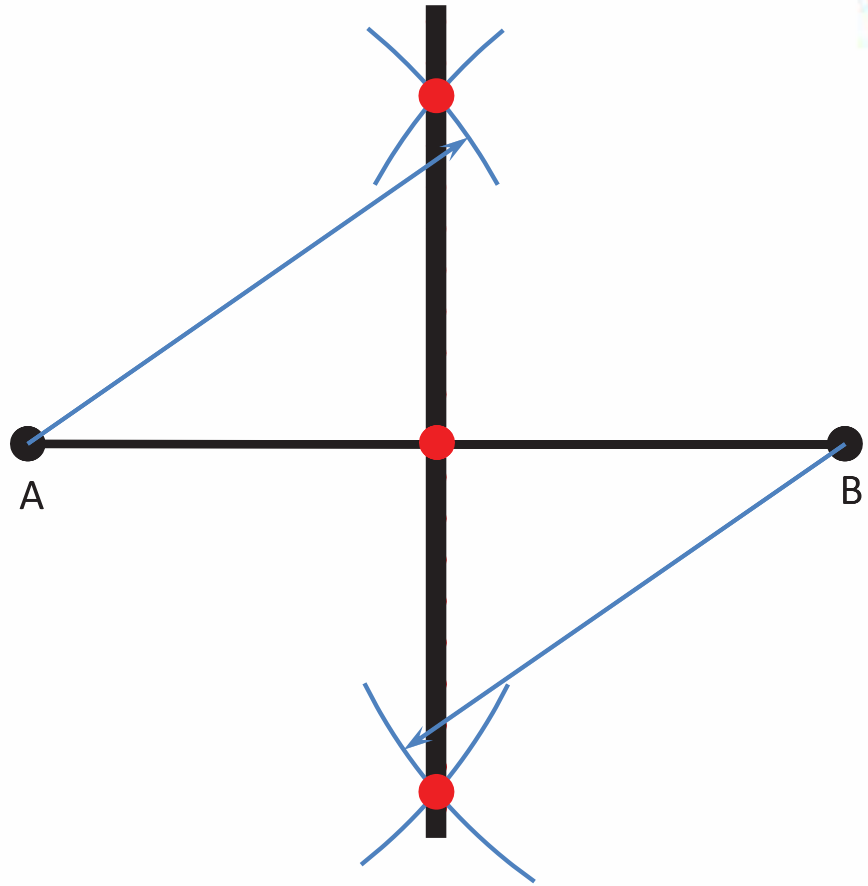
The Circle as a Locus

- A circle is the locus of a point which moves so it remains a constant distance from a fixed point “p”
- The fixed point is called the **centre** and the distance the **radius**



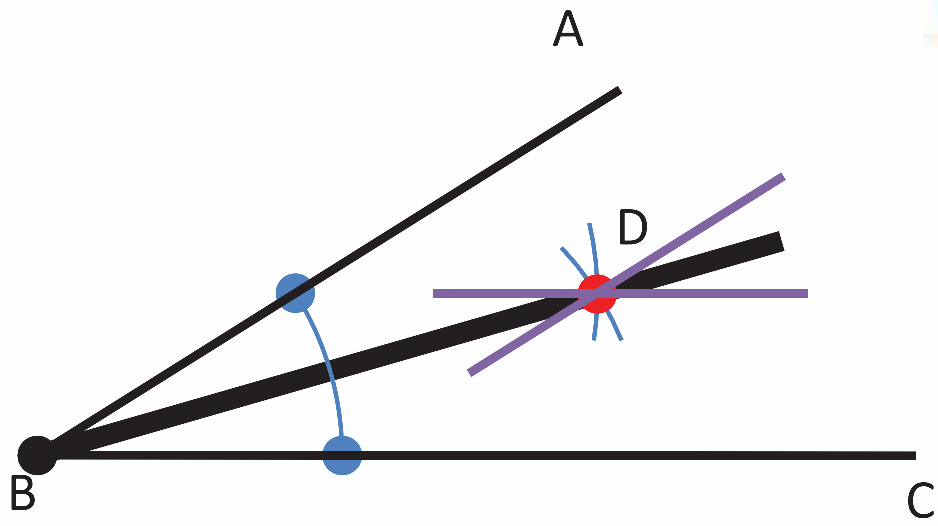
Constructions that use Loci

- We use loci to find the midpoint of a line
- The locus of a point which moves so that it is equidistant from two fixed points is called the **perpendicular bisector** of the line
- The intersection of the perpendicular bisector and the line segment yields the midpoint



Constructions that use Loci

- We can used loci to solve simple problems such as finding the bisector of an angle
- The bisector of an angle can be defined as:
- The locus of a point that moves so that it remains equidistant from 2 fixed lines



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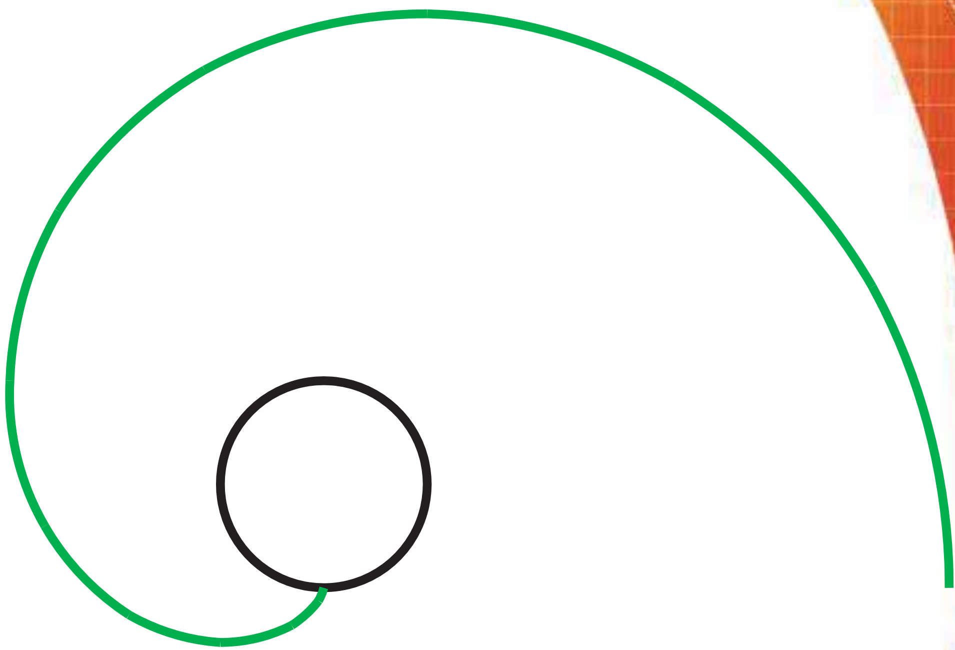
INVOLUTES

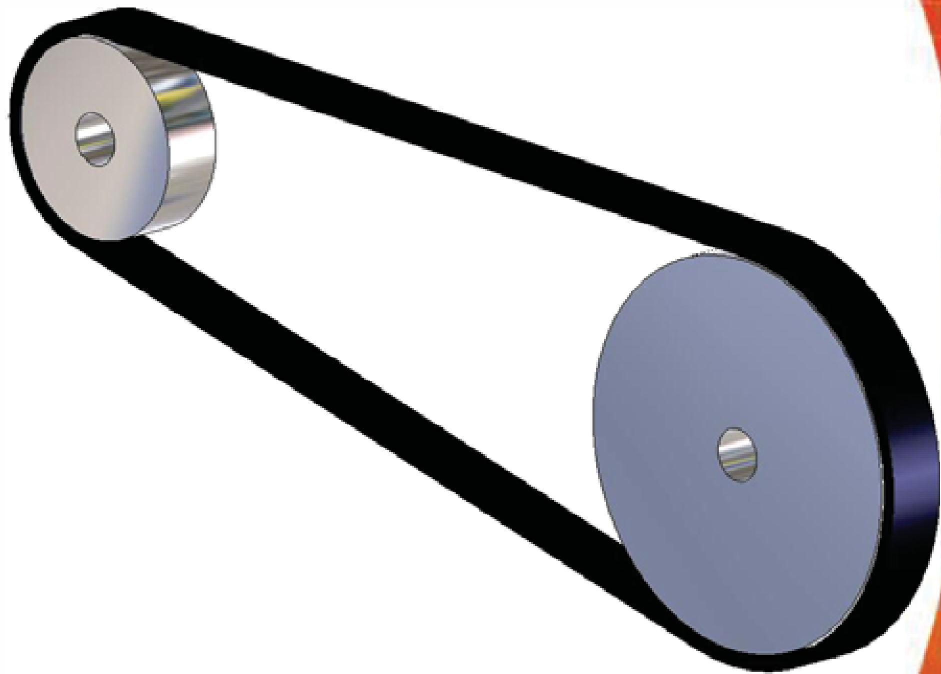
Involutes

- An involute is the locus of a point on a line as the line rolls along a shape
- It can also be thought of as the locus of the end of a piece of string as the string is wound/unwound around the circumference of a plane figure

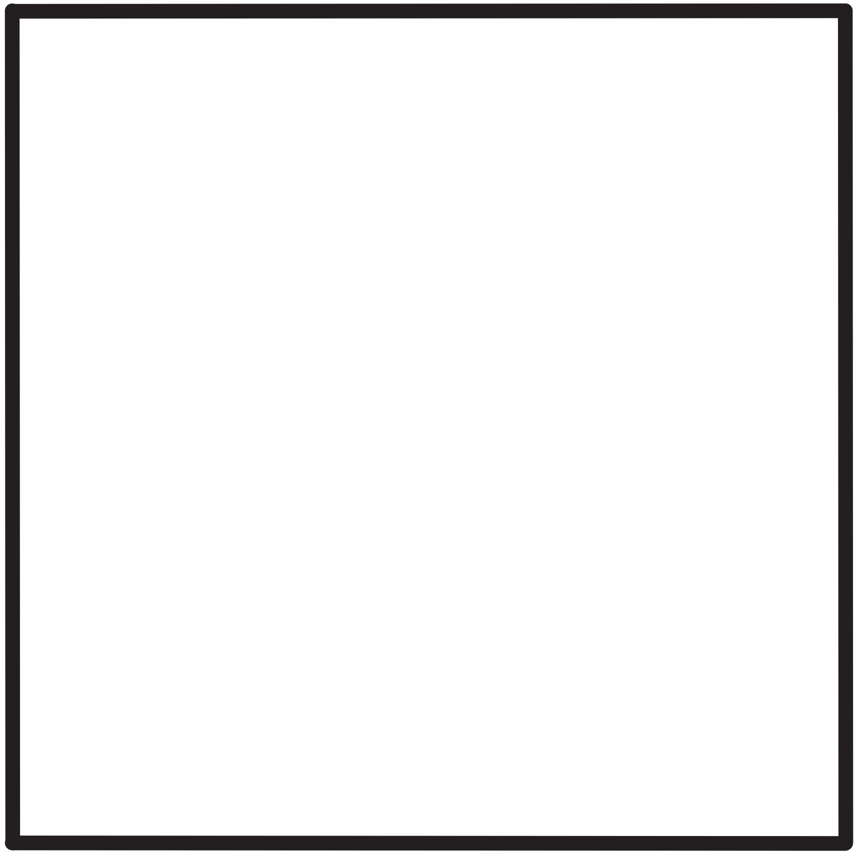


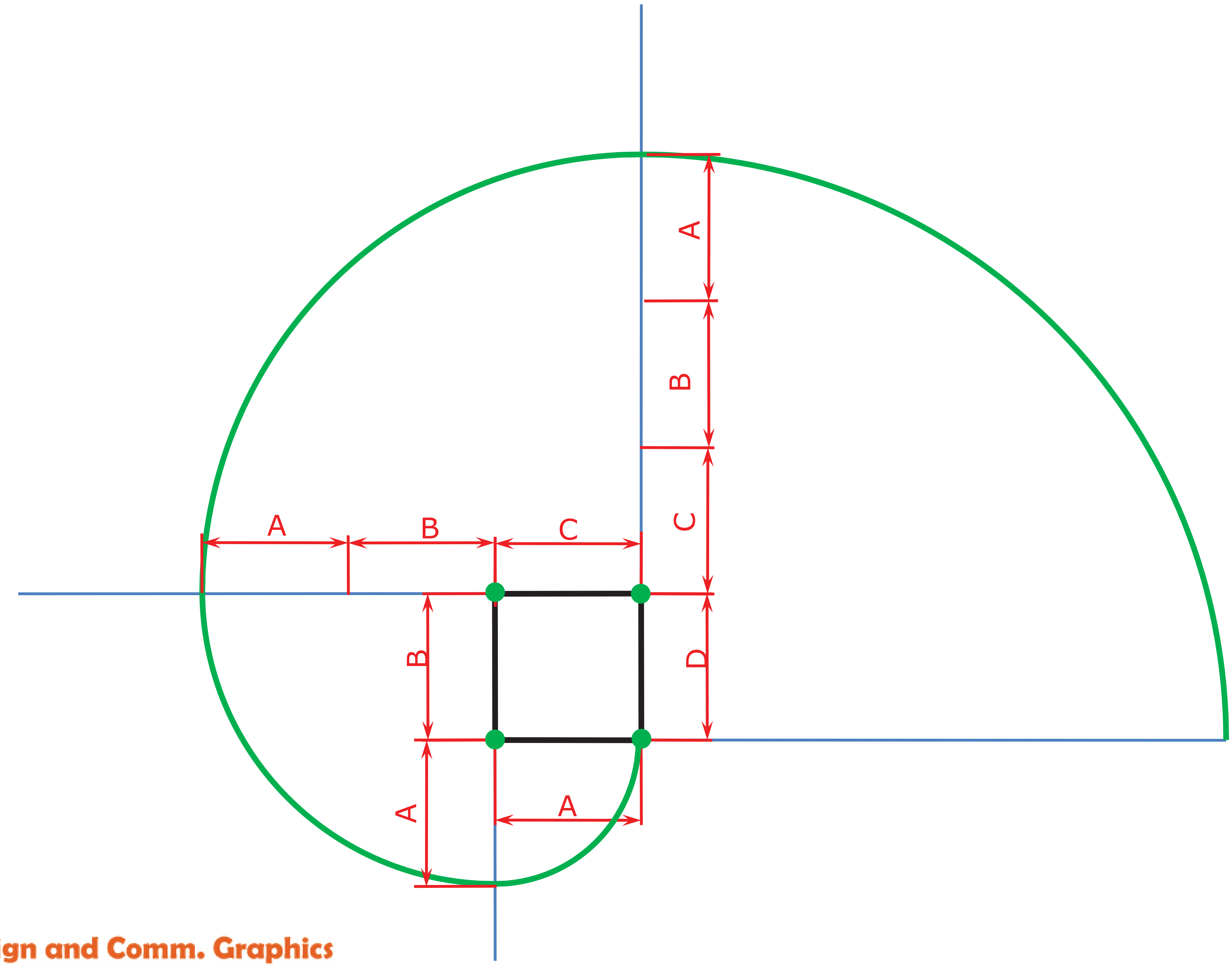
Applications of Involutes

- Involutes are used to determine the length of belts used in pullies and other machines
- Involutes are also used to calculate the amount of material required to create tyres and wheels



Involute of a Square

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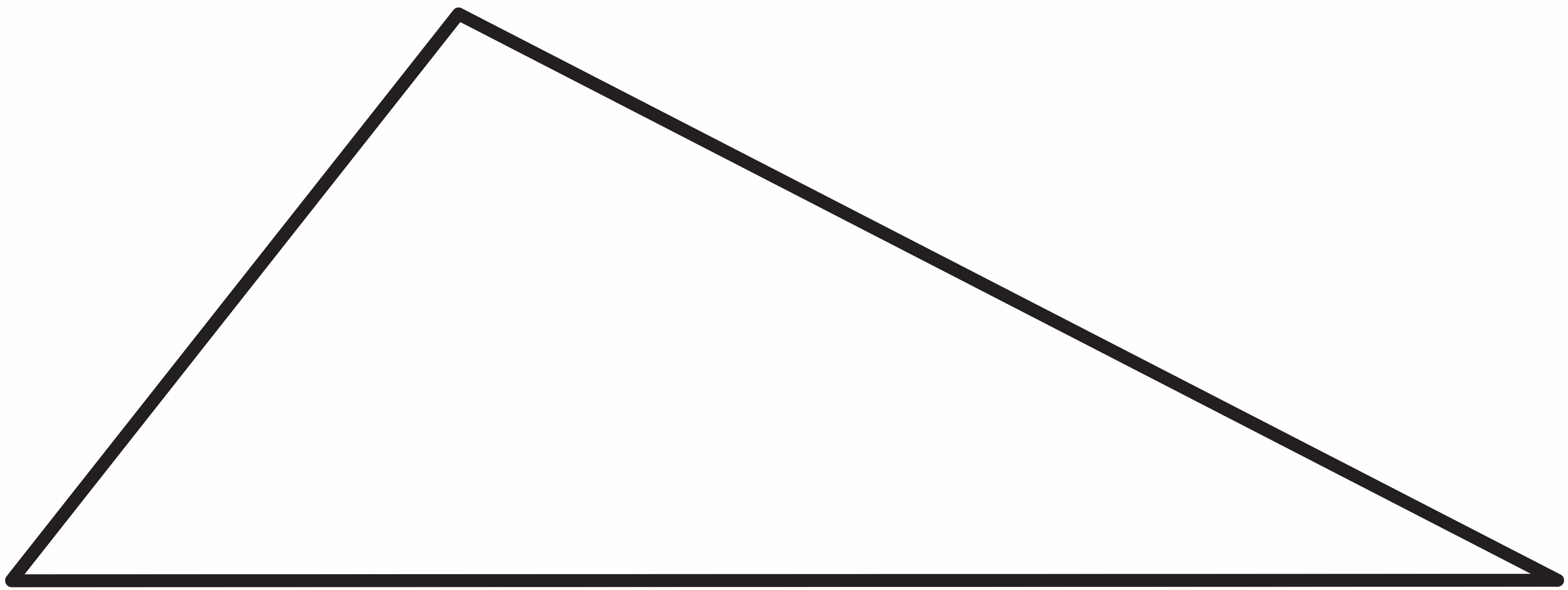
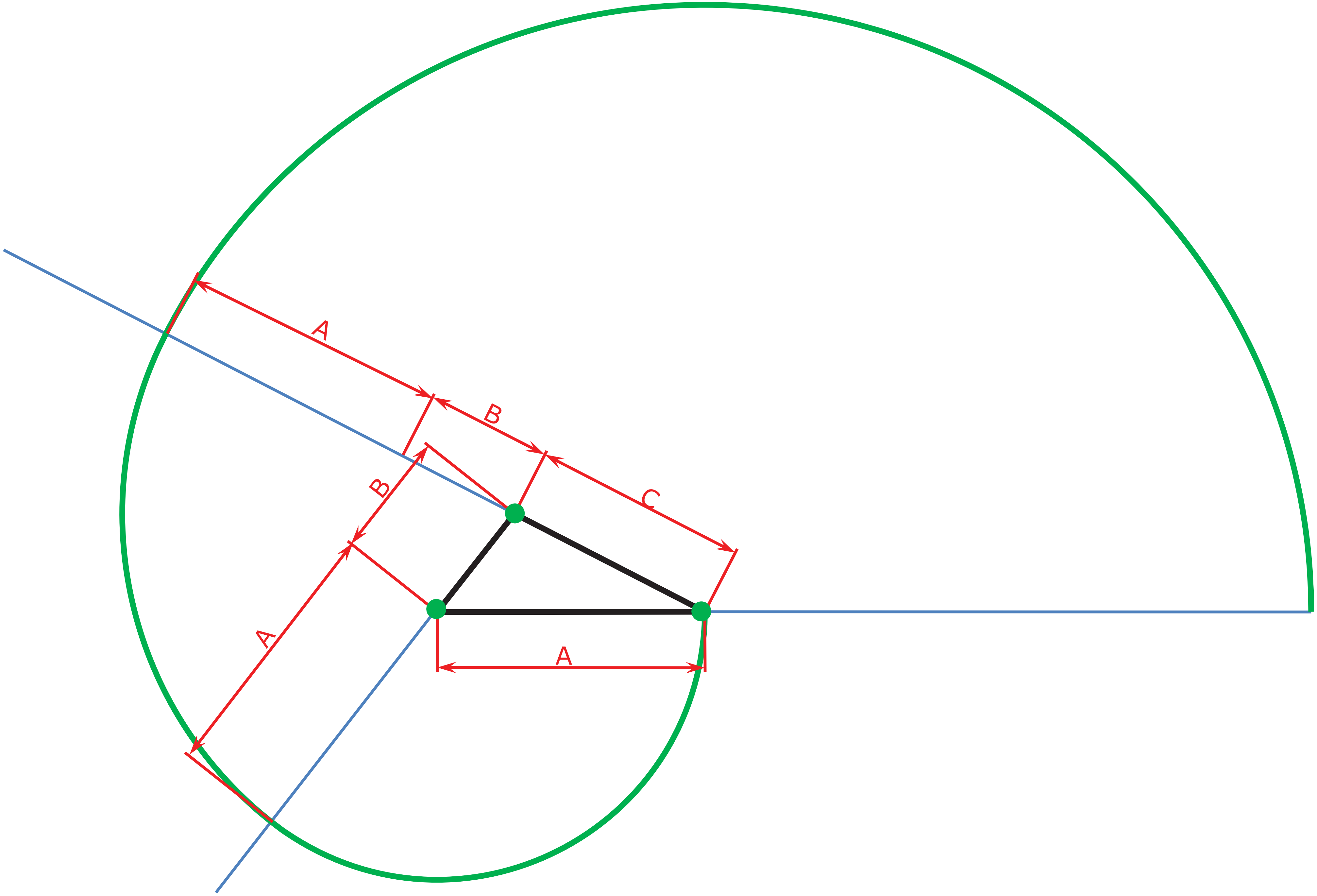
Involute of a Triangle

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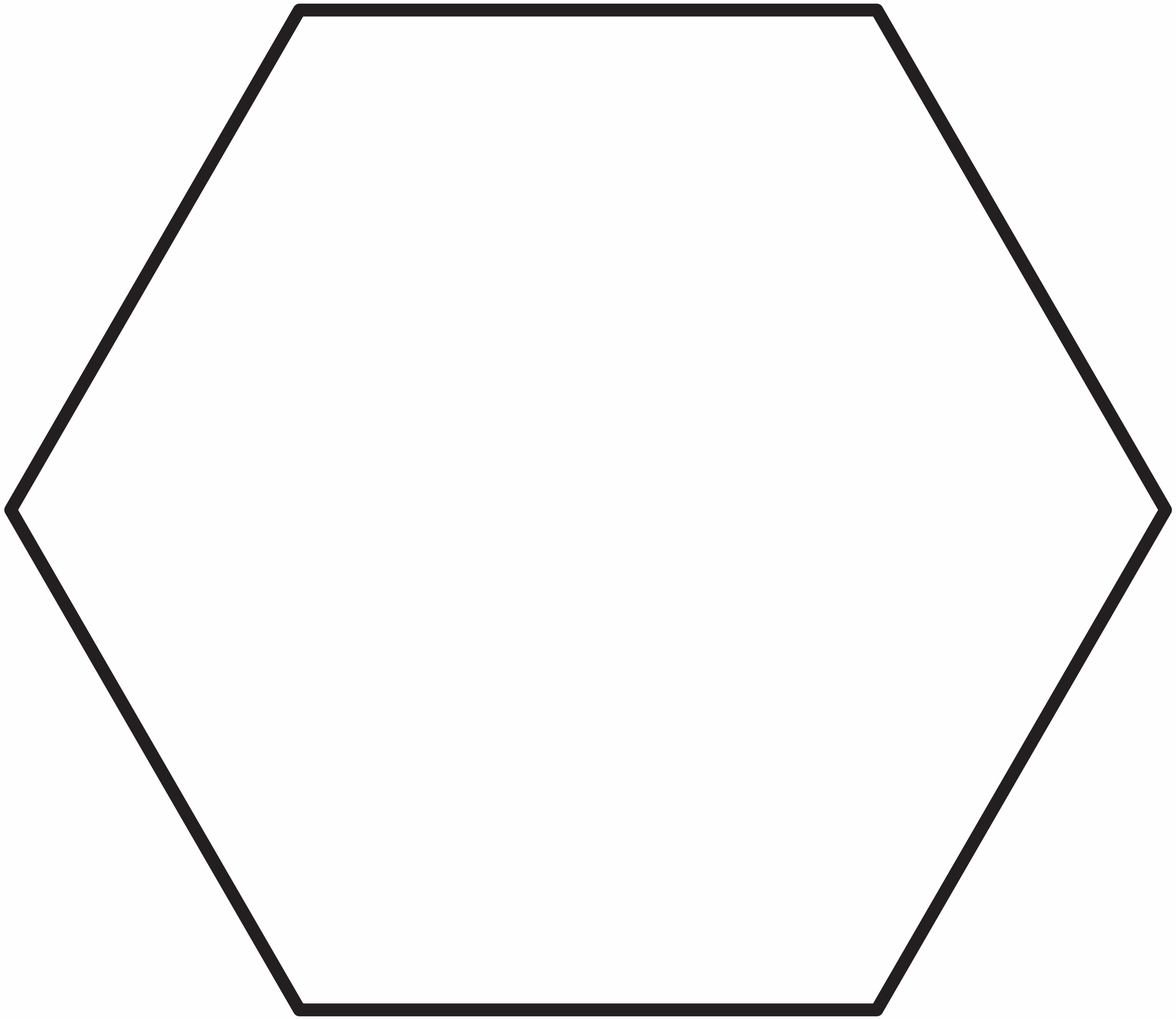
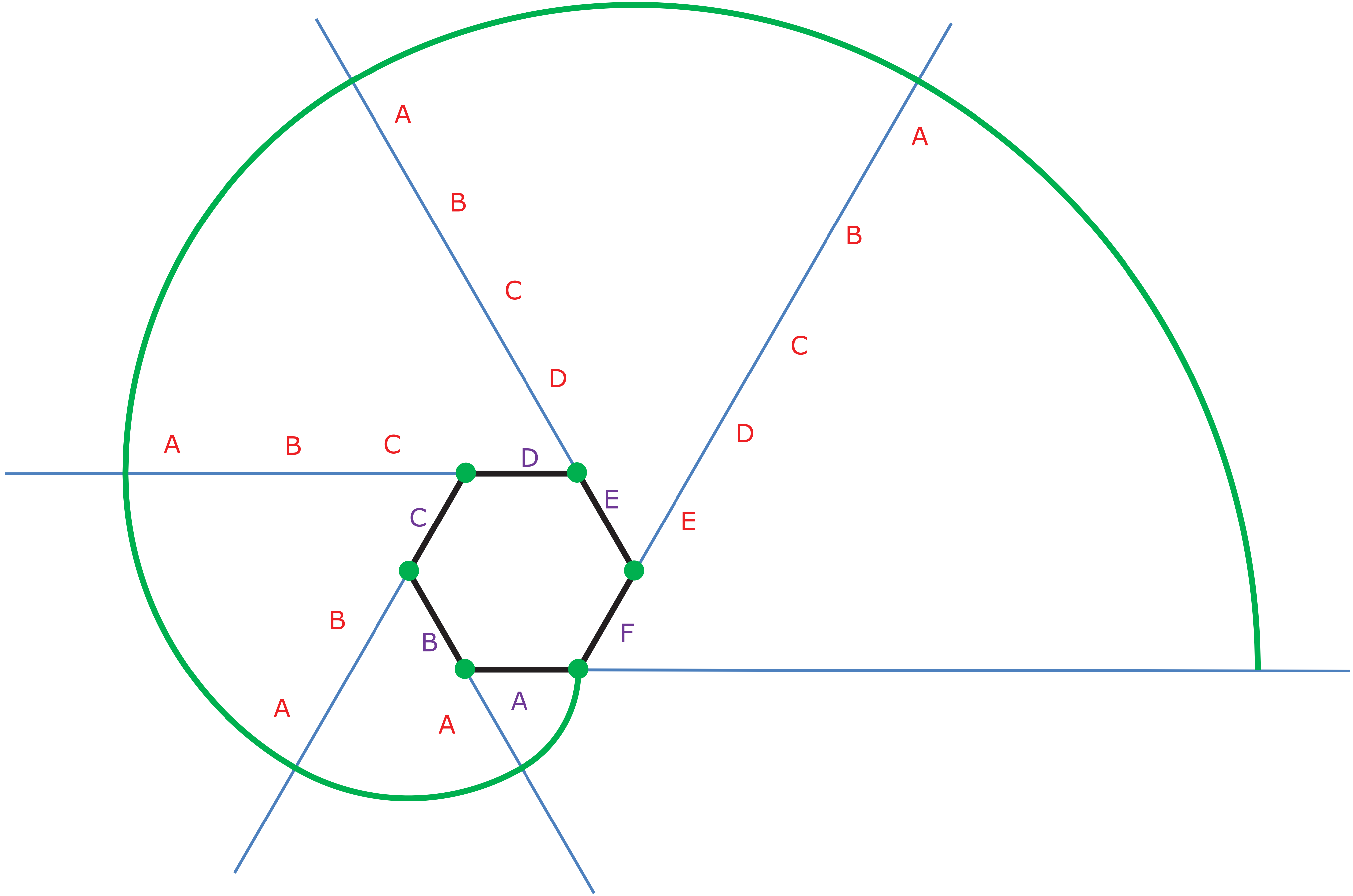
Involute of a Hexagon

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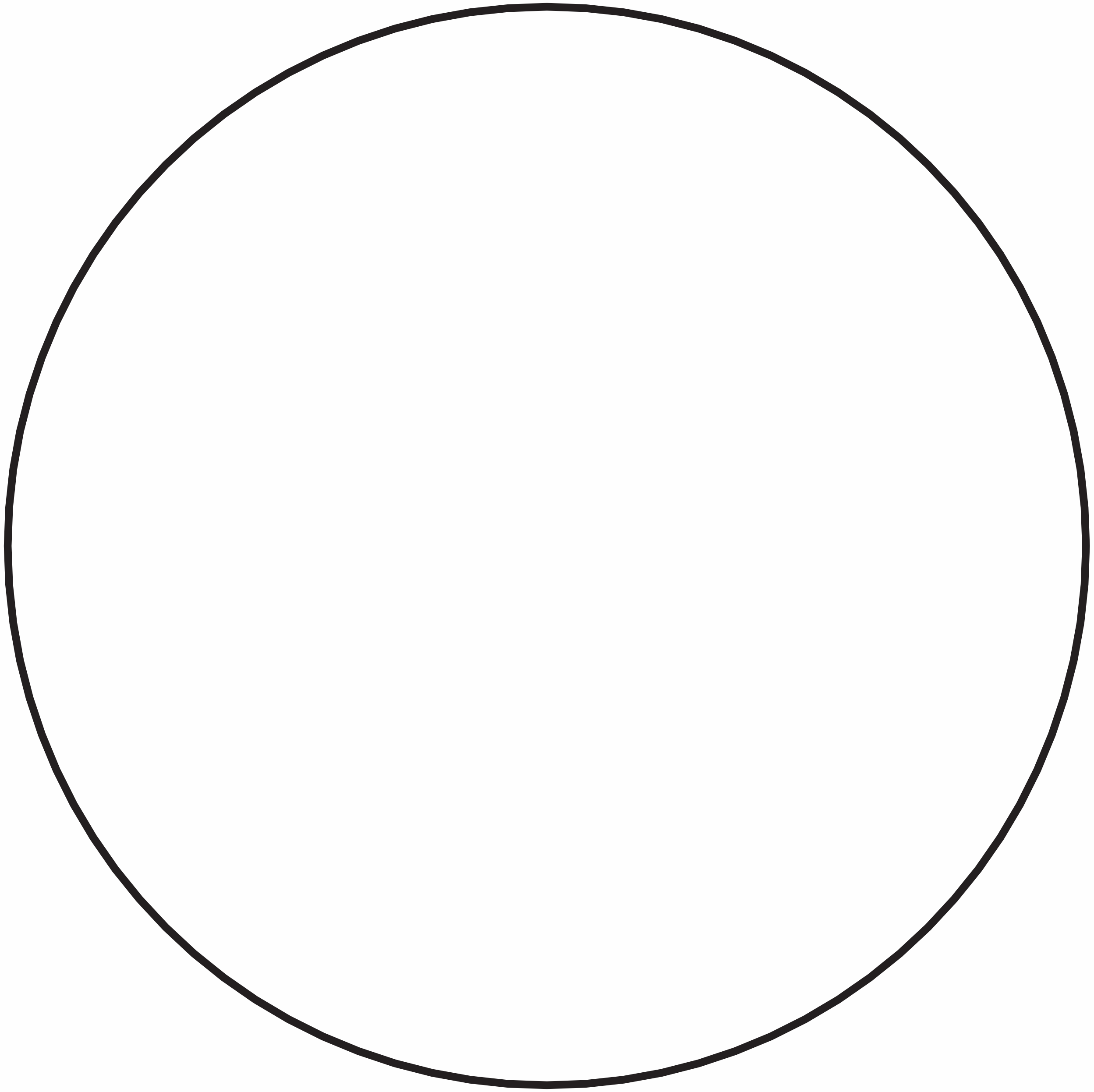
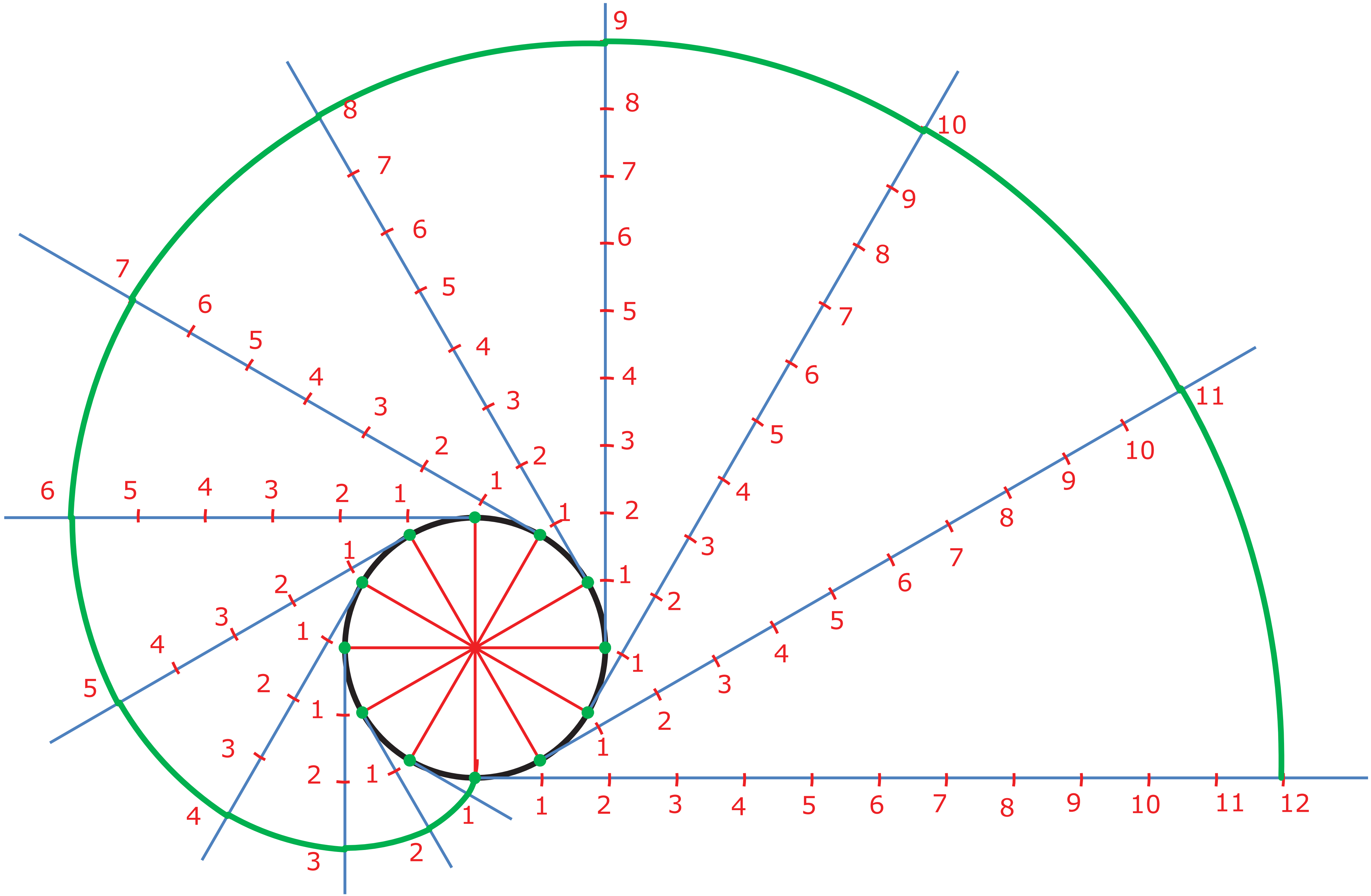
Involute of a Circle





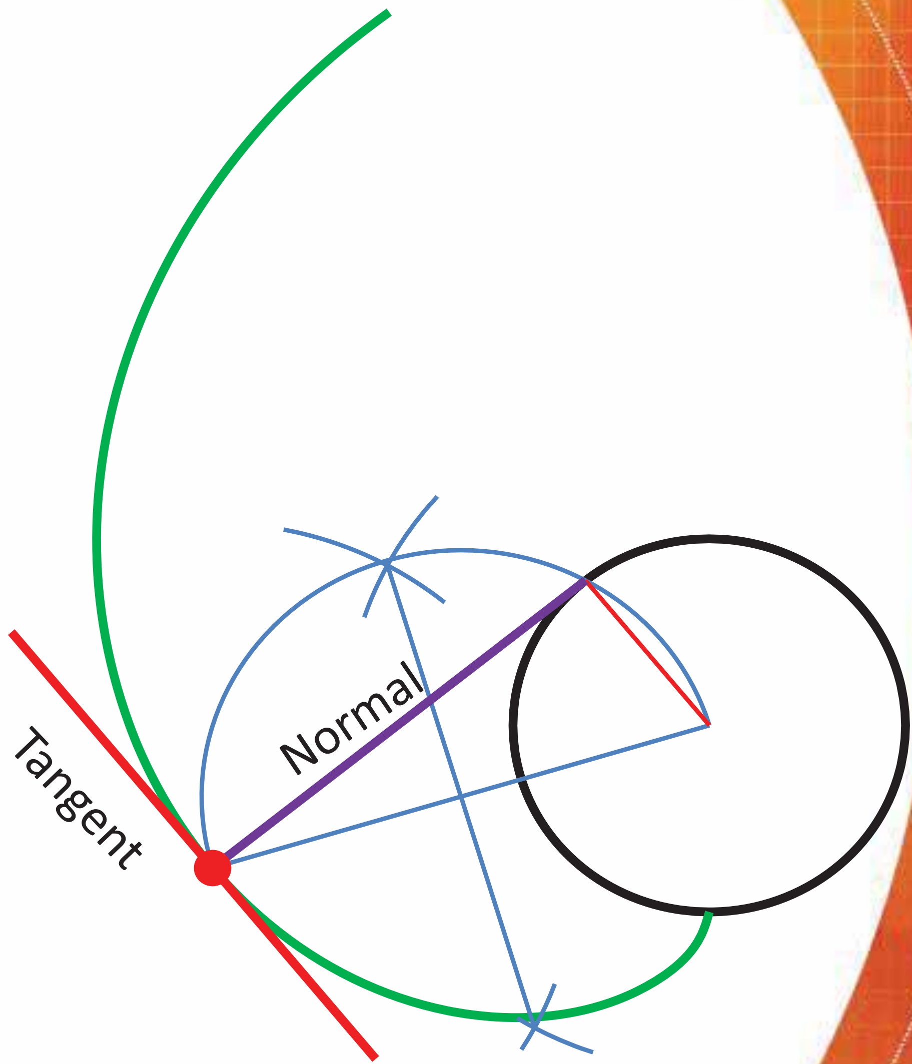
Making Involute models

- http://www.math.nmsu.edu/~breakingaway/Lessons/involute1/involute.html

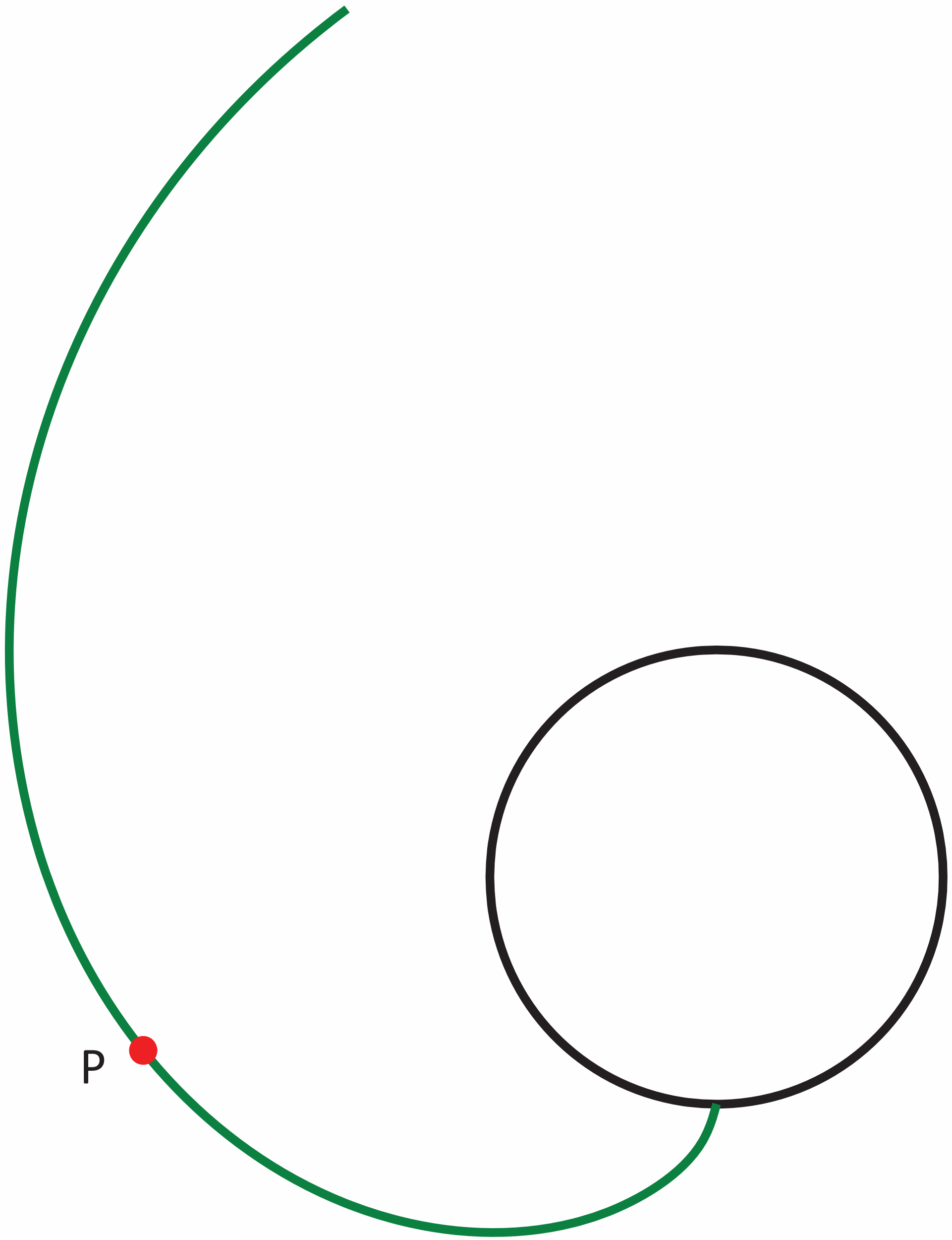
TANGENTS TO INVOLUTES

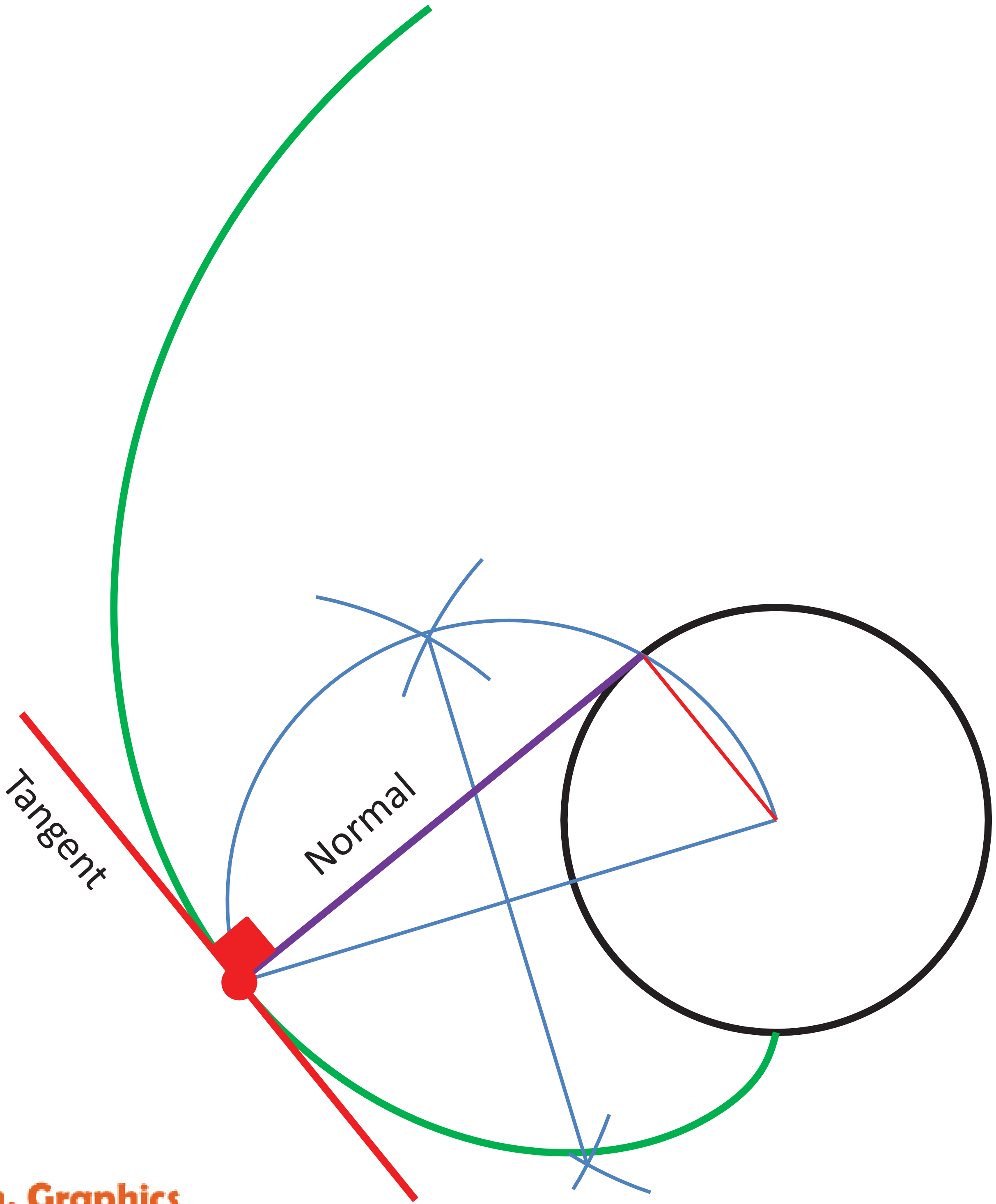
Tangents to Involutes

- Involutes are curves, and as with all curves a tangent can be drawn to the involute

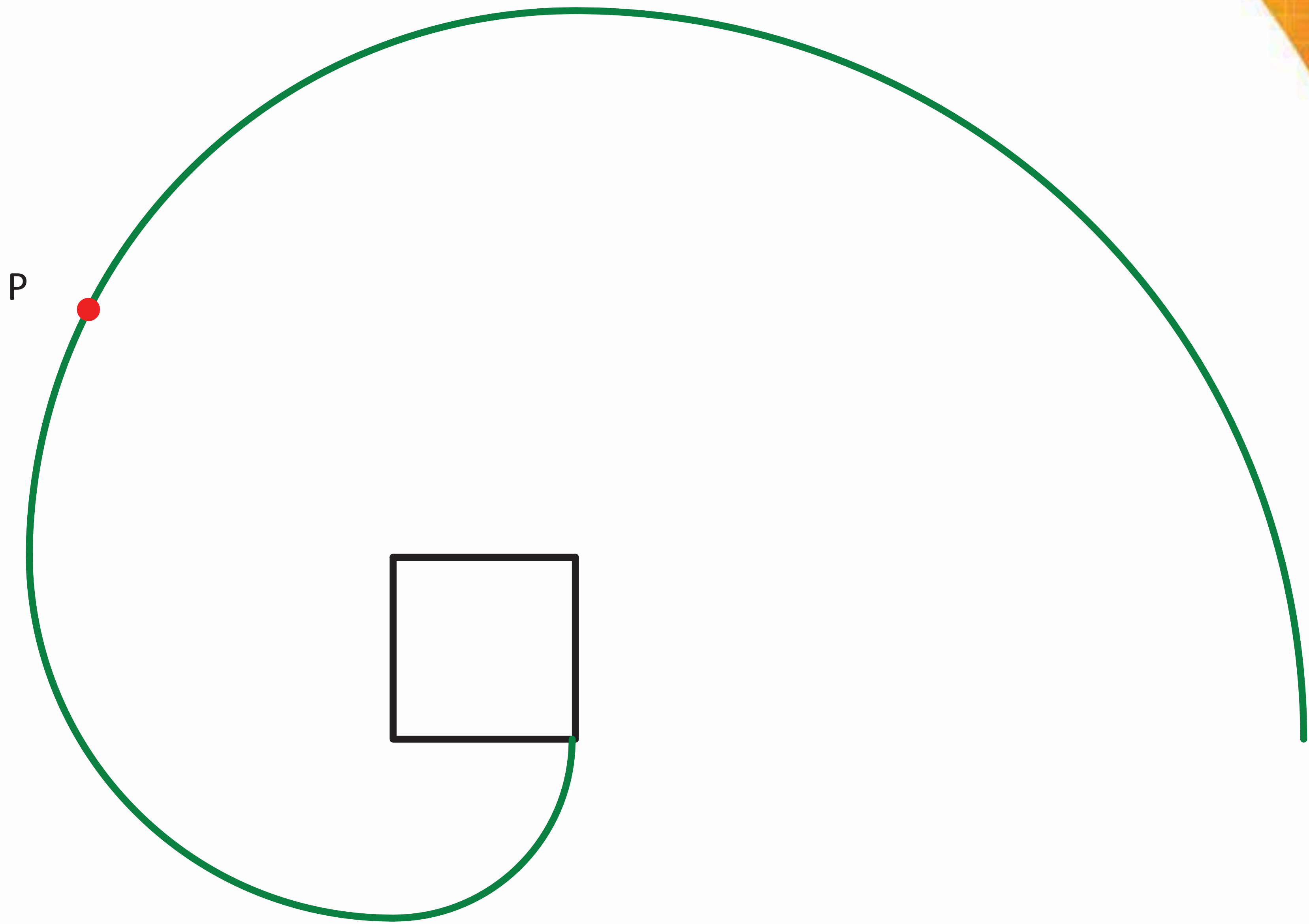


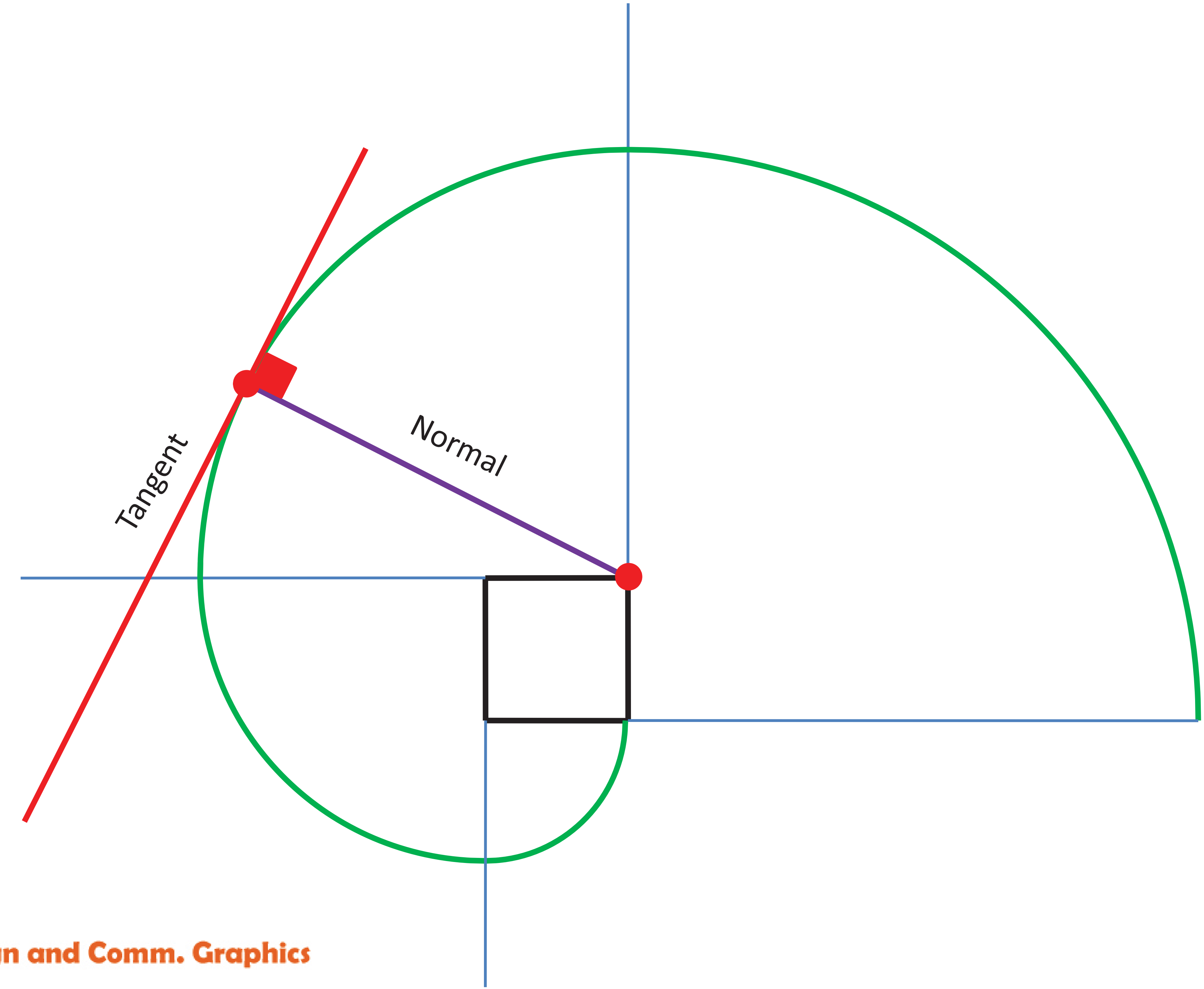
Tangent to an involute at point P




Tangent
Normal

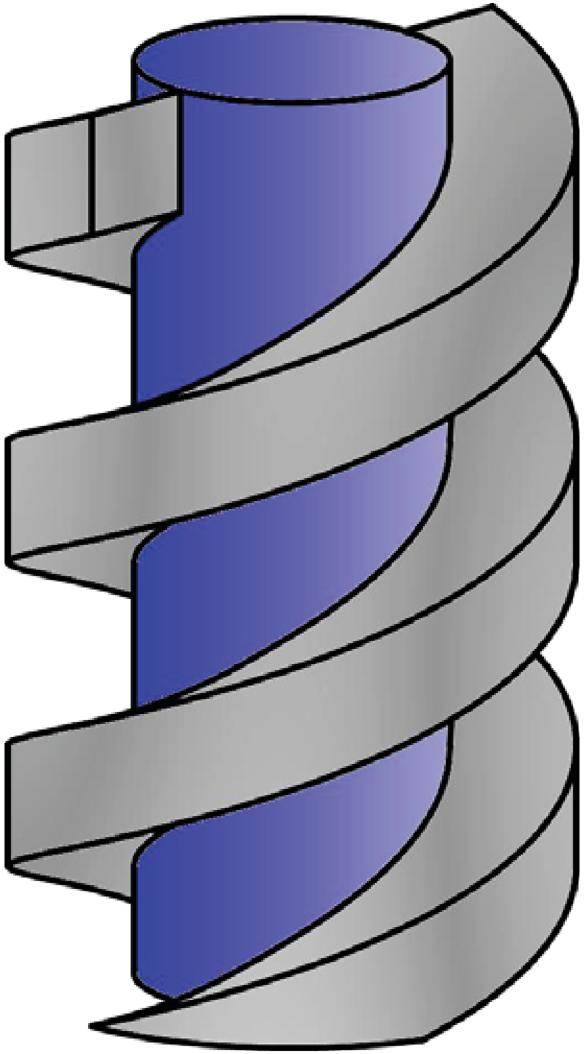
Tangent to an involute at point P




Tangent
Normal

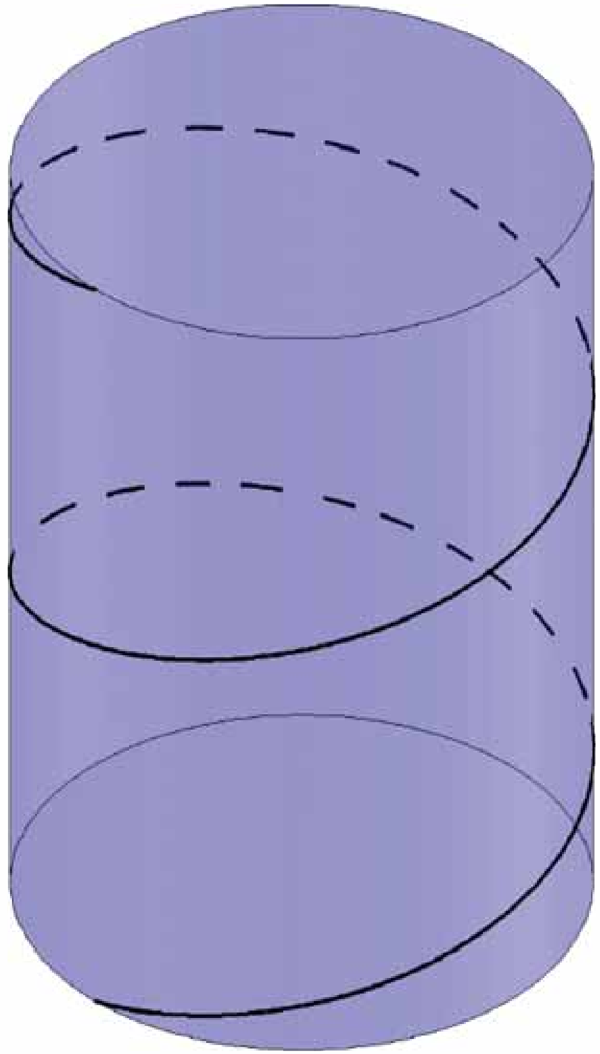
TECHNOLOGY SUBJECTS SUPPORT SERVICE

THE HELIX



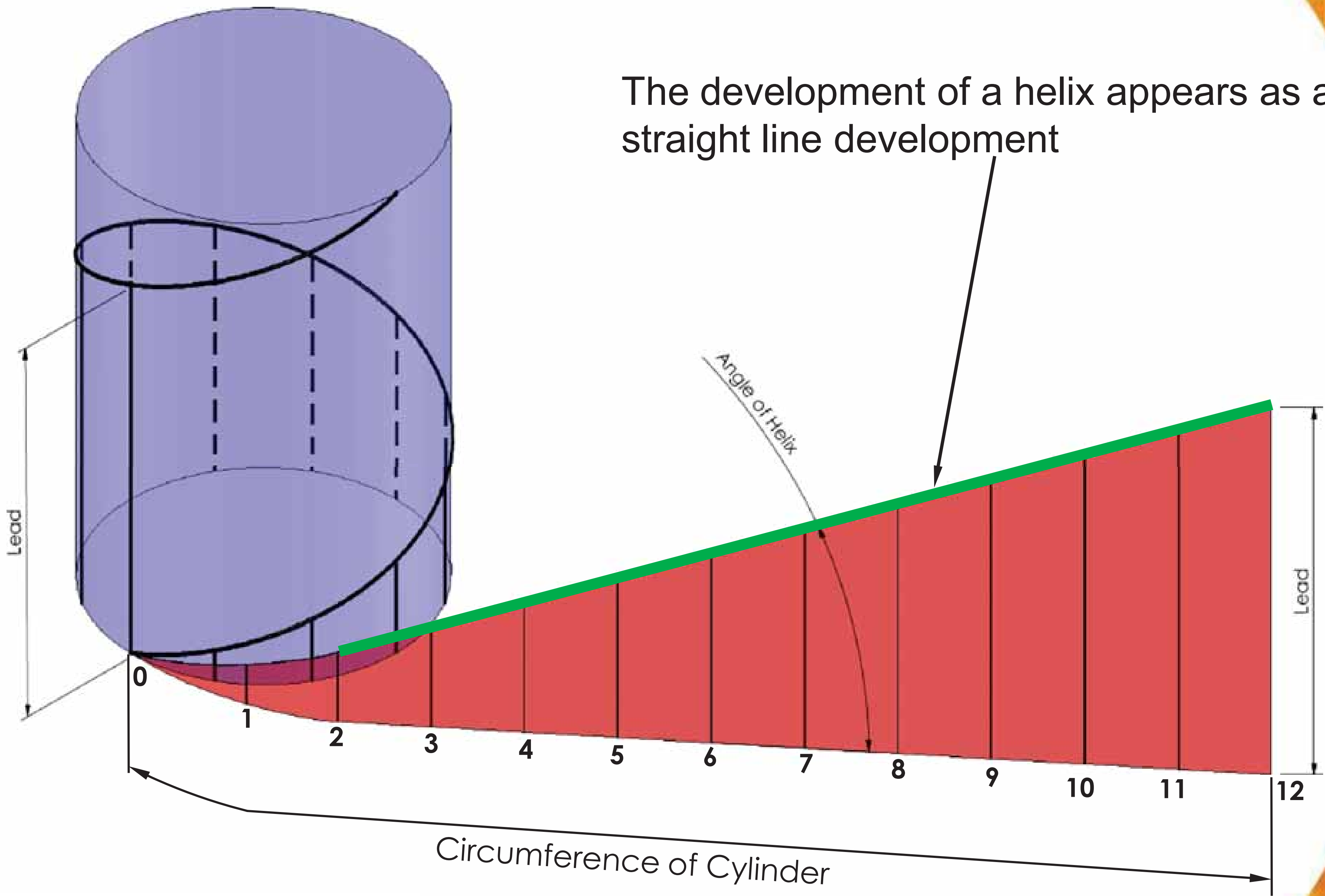
The Helix

- A helix is the locus of a point as it moves on the surface of a cylinder so that it rotates at a constant rate around the surface of the cylinder, while also progressing in the direction of the axis at a constant rate



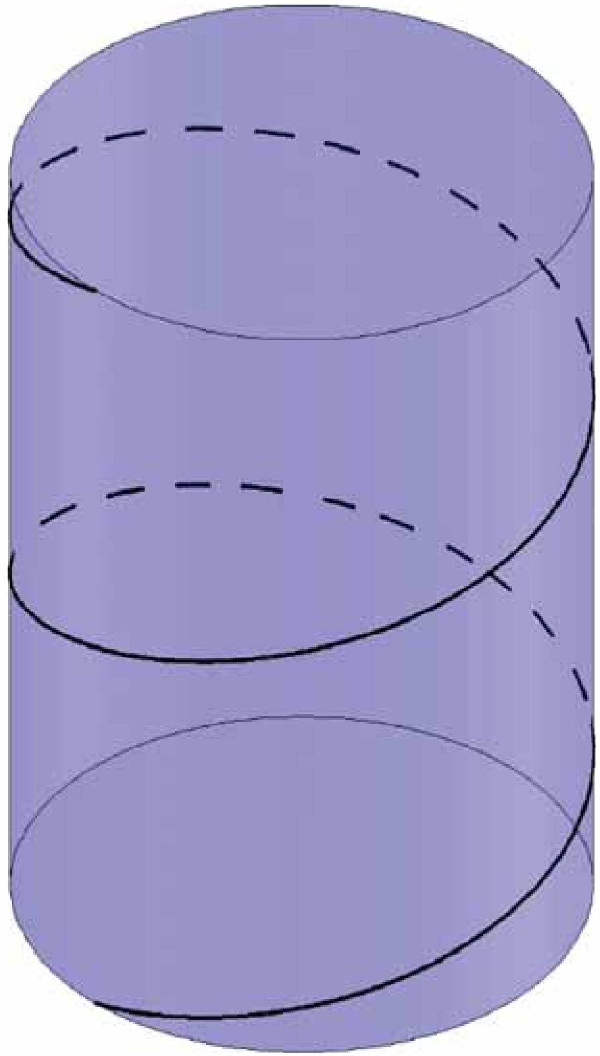
Helix Anatomy

The development of a helix appears as a straight line development



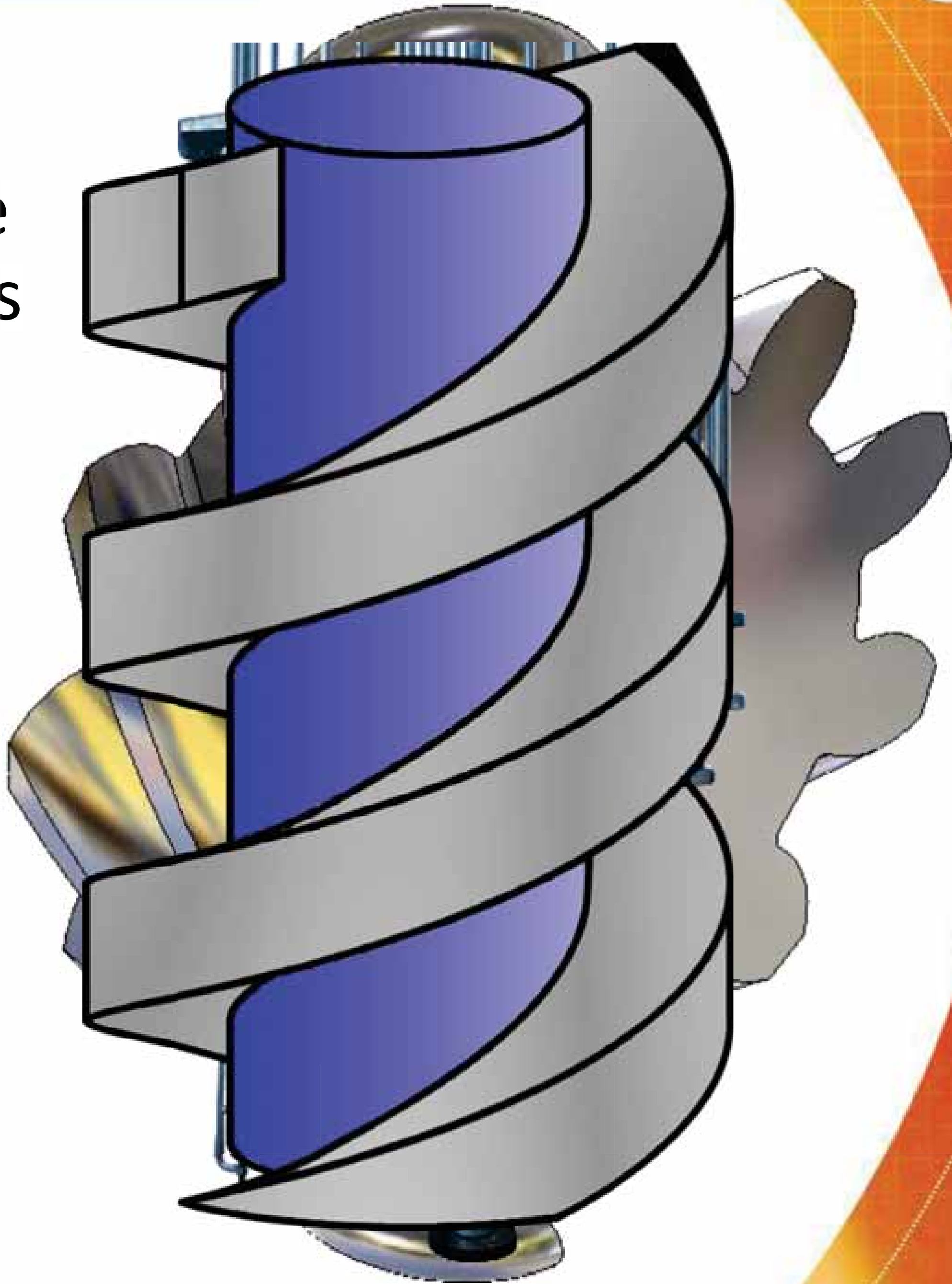
Helix as a Geodesic

- A geodesic is the shortest distance between two points on a surface
- The geodesic of a cylinder may be:
  - A circle
  - A linear element
  - A helix



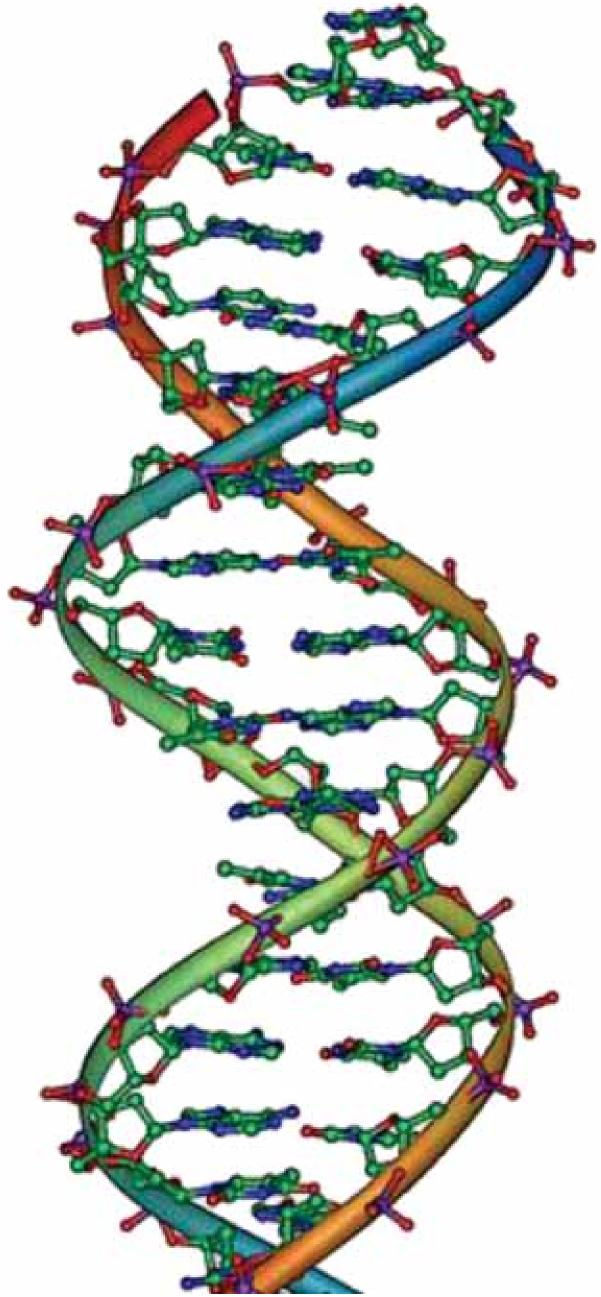
Applications of the Helix

- The helix is used for the thread of bolts, reamers and drill bits
- Springs are derived from the helix
- Helical gears are derived from the helix
- Winding staircases are also derived from the helix



Applications of the Helix

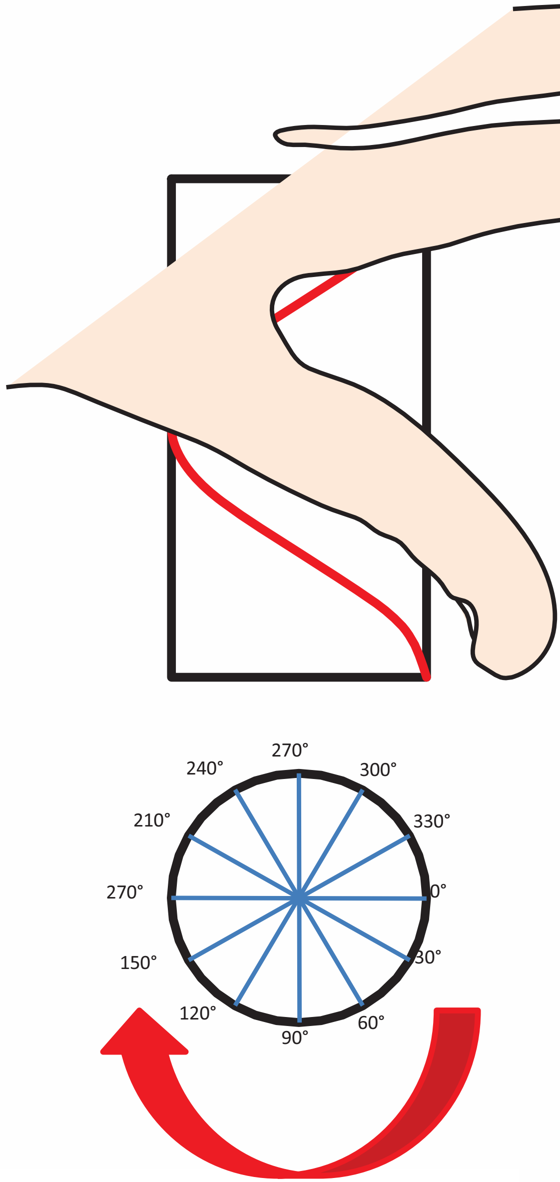
- The helix may be the most important shape in the universe as the human gene code is structured around a helix



Left and Right Hand Helix Rule

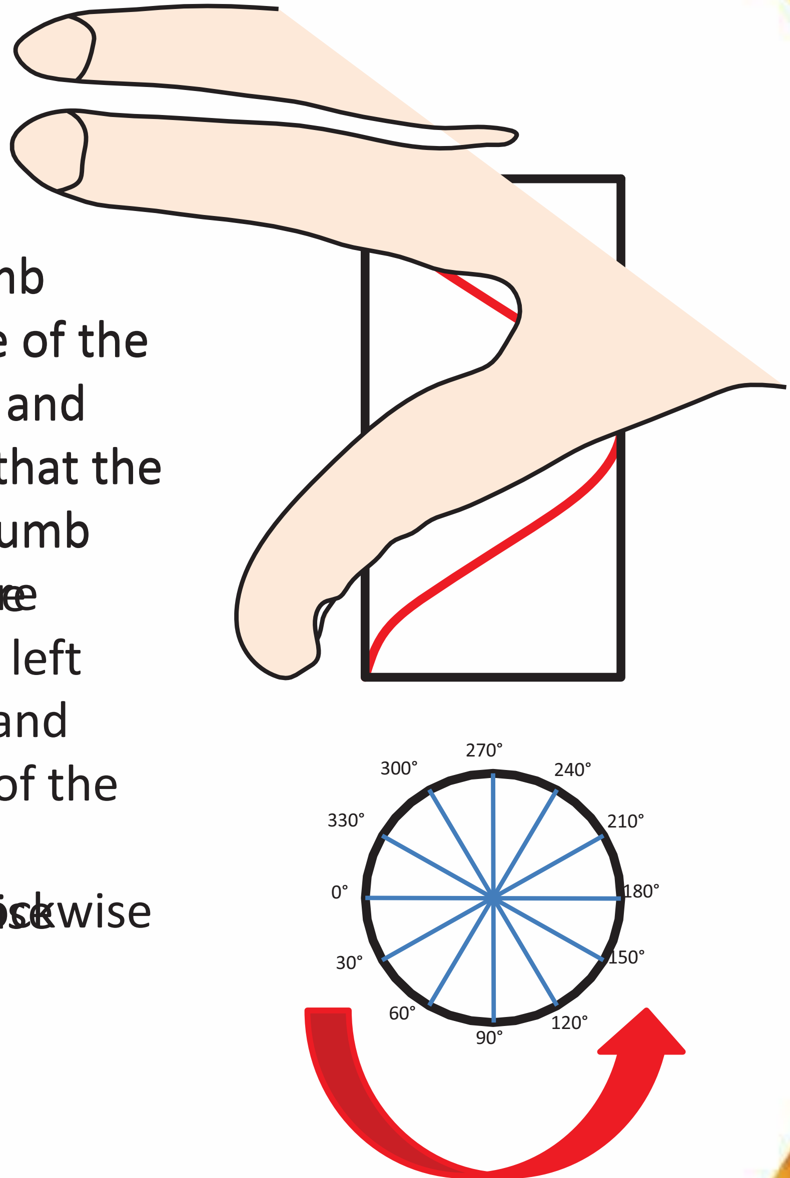
Left Hand Helix

Right Hand Helix

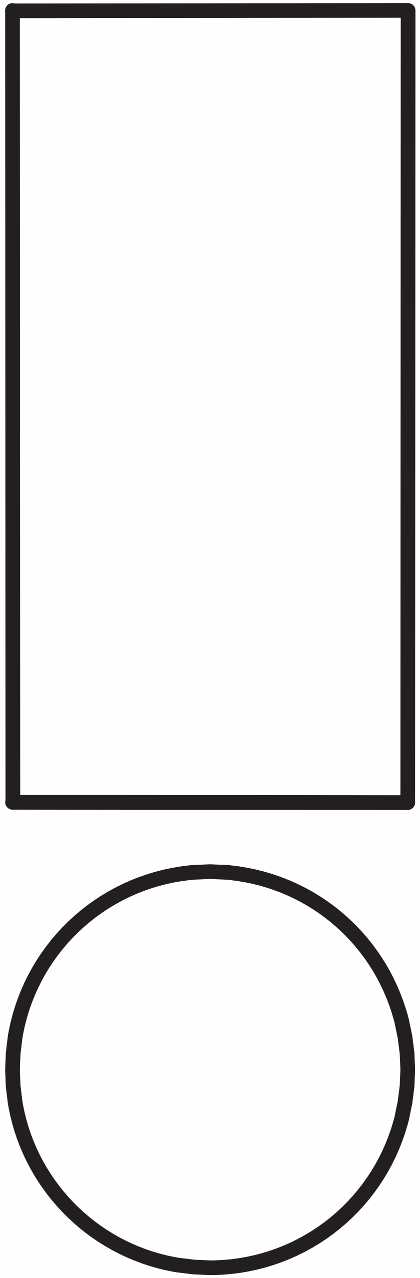


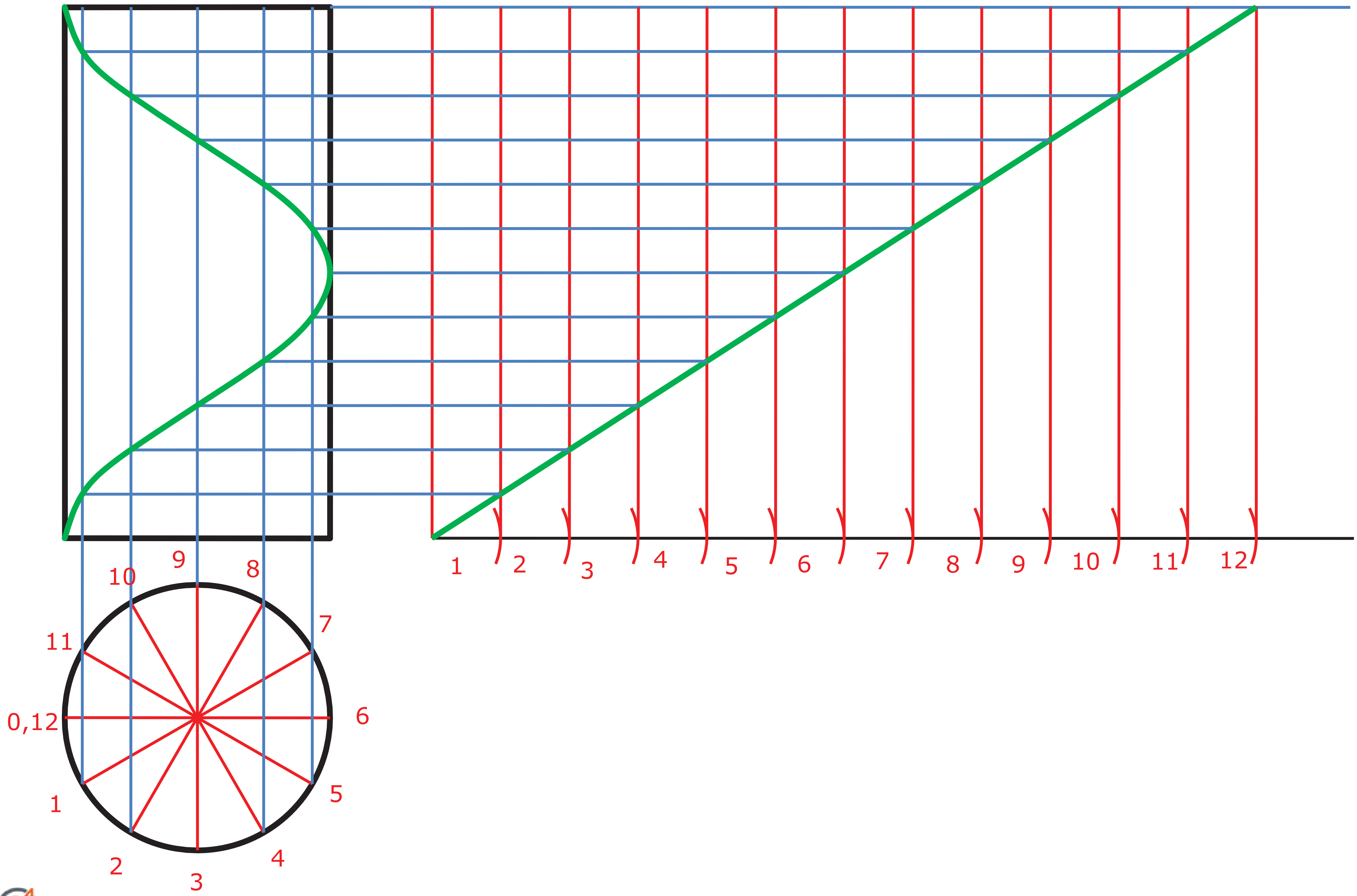
Place your thumb along the shape of the helix as shown, and you will notice that the shape of the thumb and the helix are similar. This is a left hand helix. The plan of the cylinder will be indexed anticlockwise

This is a right hand helix. The plan of the cylinder will be indexed clockwise



A right hand helix of one revolution





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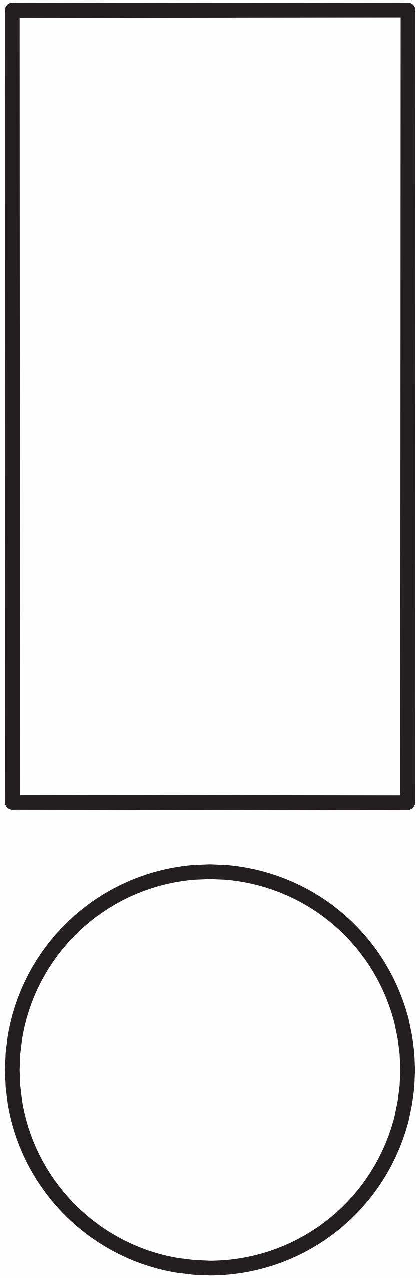
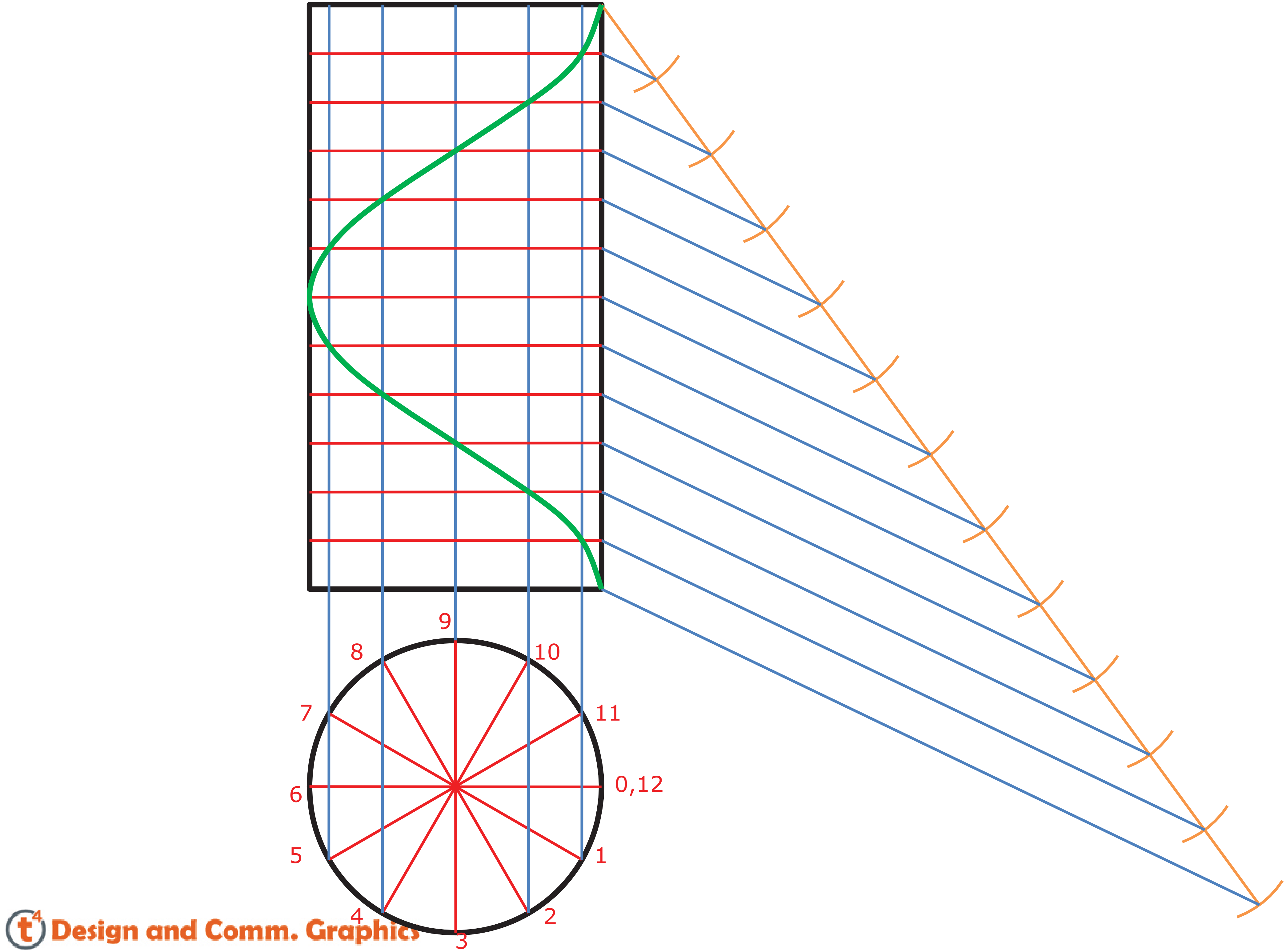
A left hand helix of one revolution





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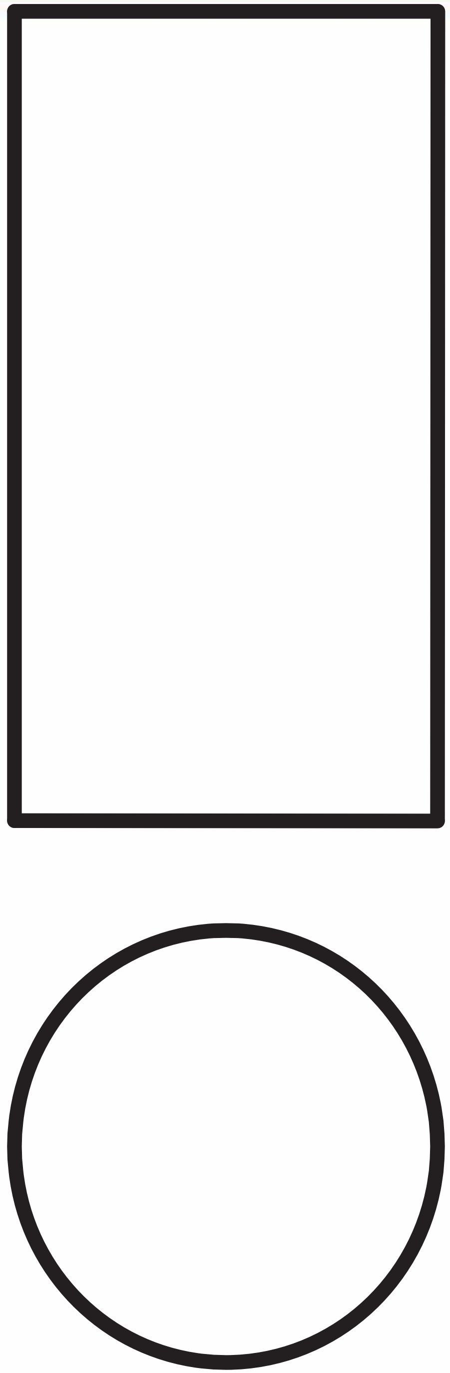
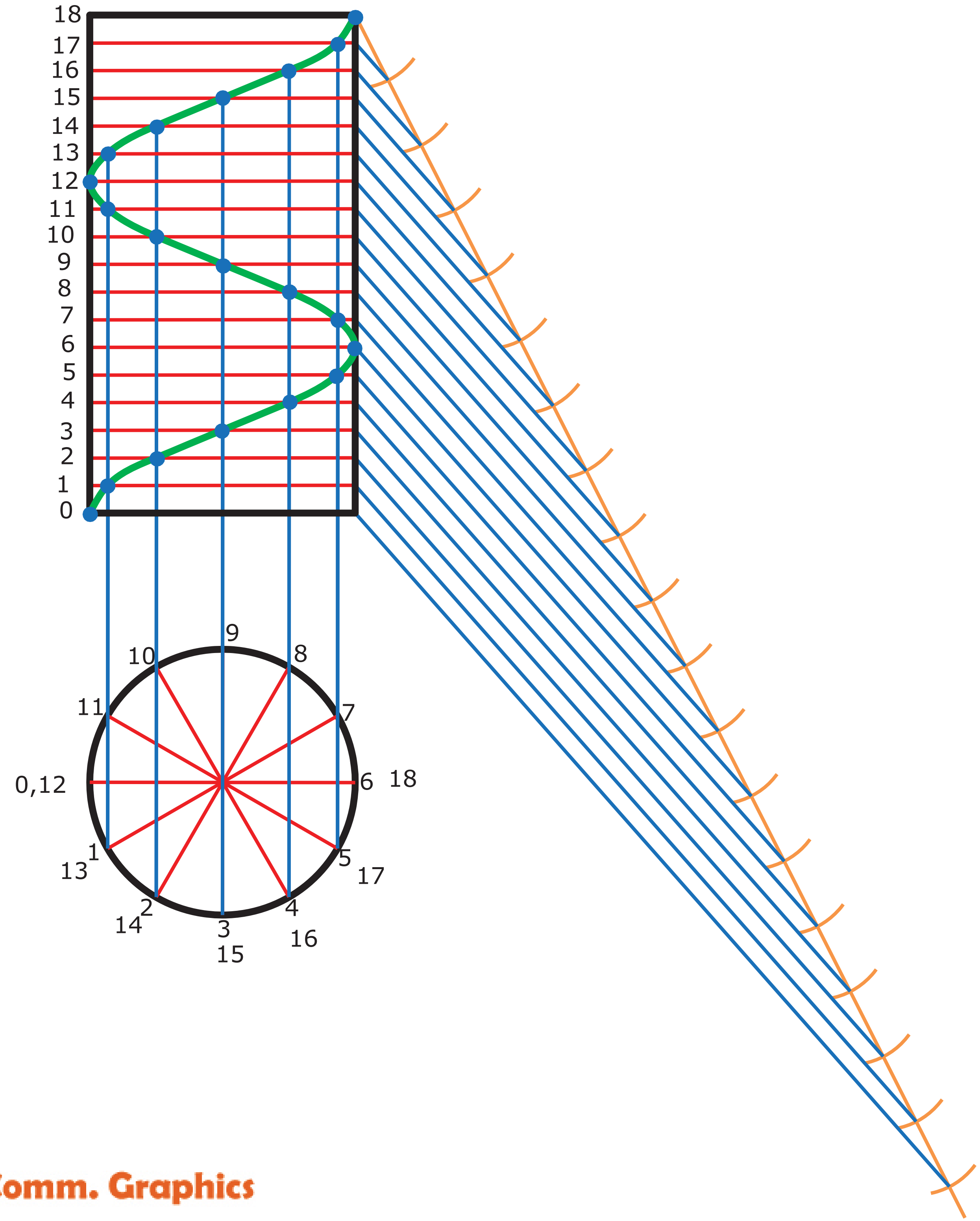
Design and Comm. Graphics

A right handed helix of 1 ½ revolutions

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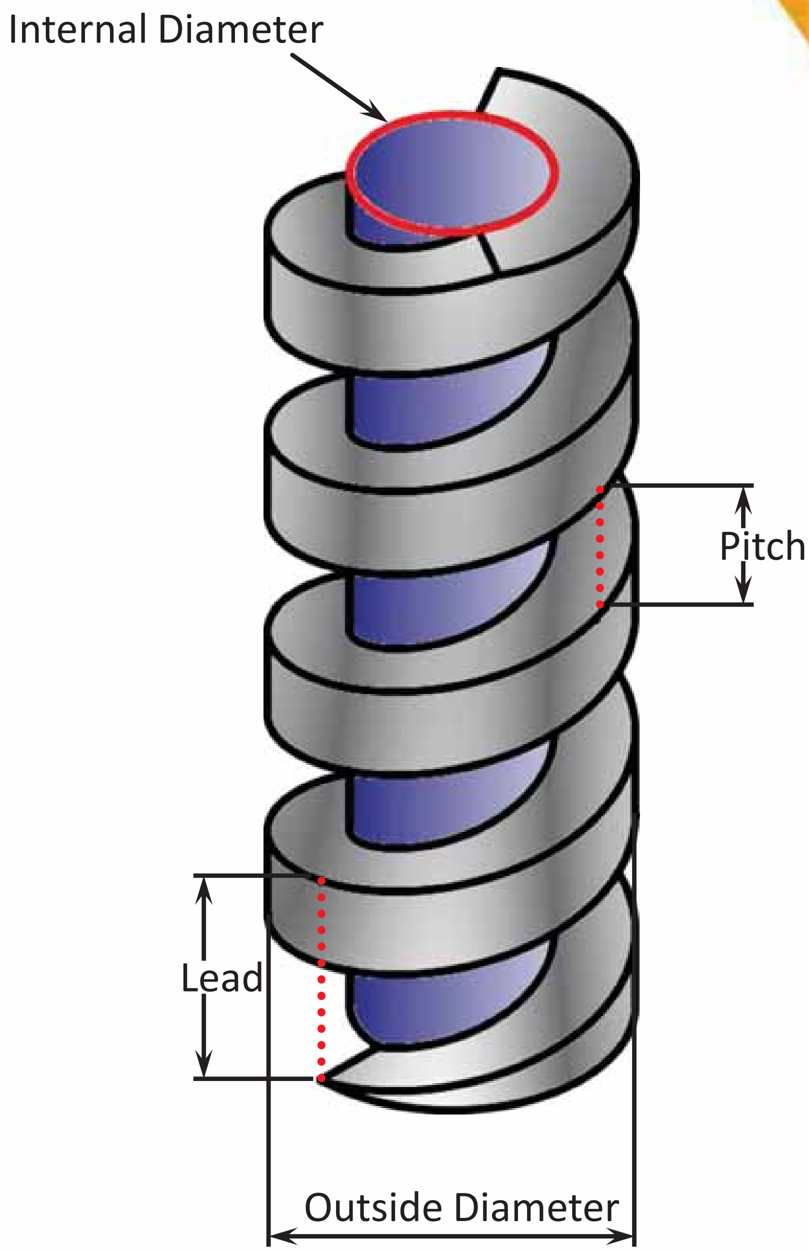
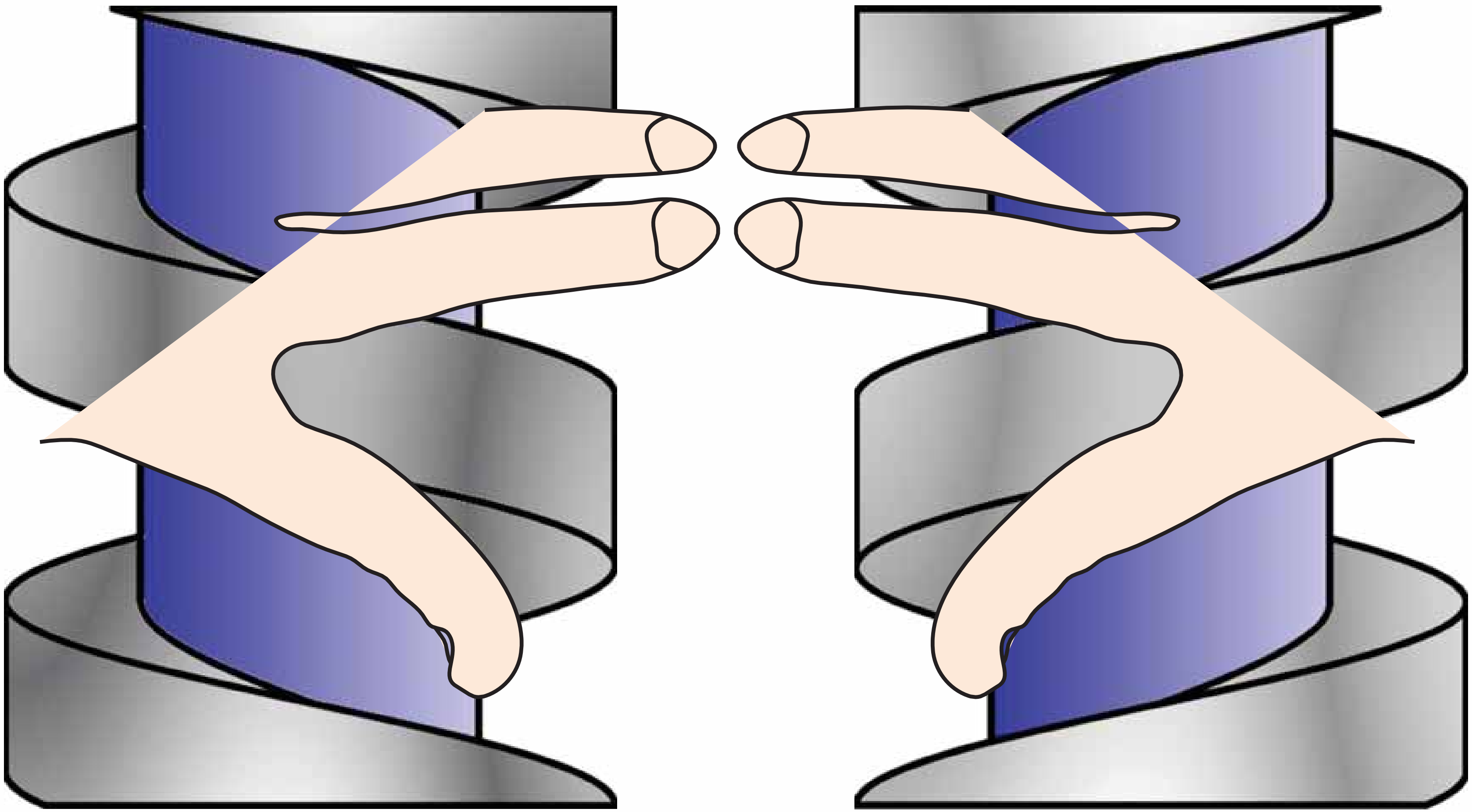
Helical Screw Thread: Terminology

- Internal Diameter: Diameter of Shaft/ internal helical curve
- Outside Diameter: Outermost diameter of the thread/helical curve
- Lead: amount of axial advance during one complete revolution of the helix
- Pitch: is the distance from a point on the helix to a corresponding point on the next revolution measured parallel to the axis



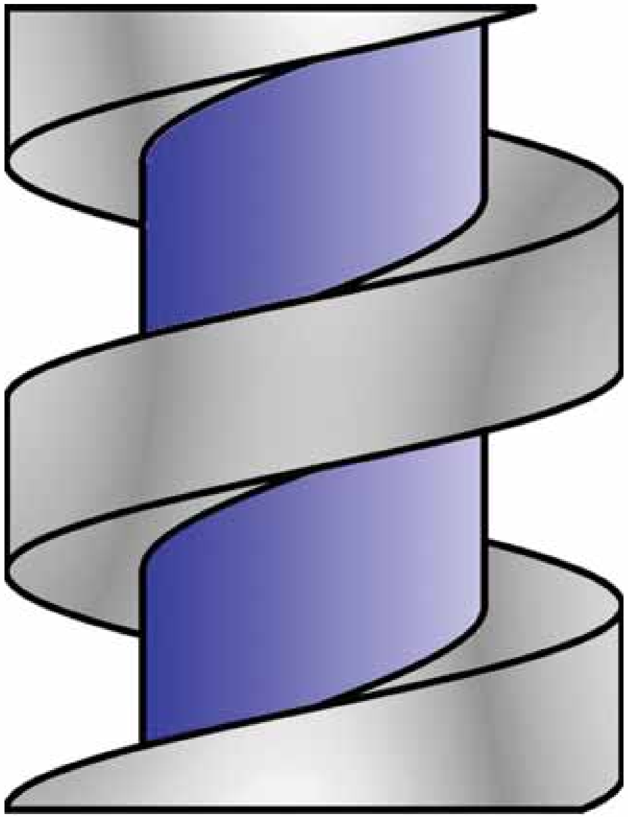
Right handed and Left handed Screw Threads



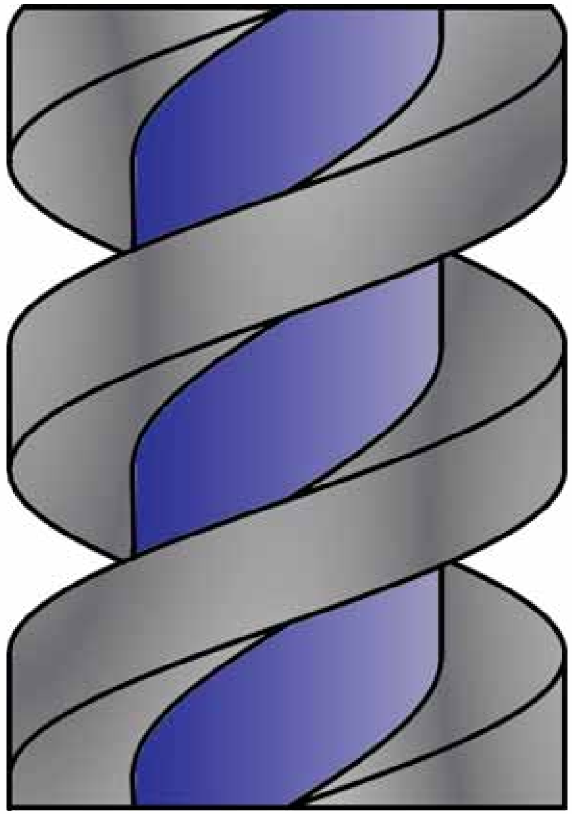
Variations of Helical Screw Threads

- A helical screw thread can consist of more than two helices
- A two start screw thread has four helices
- A three start would have six helices, etc

Single Start



Double Start

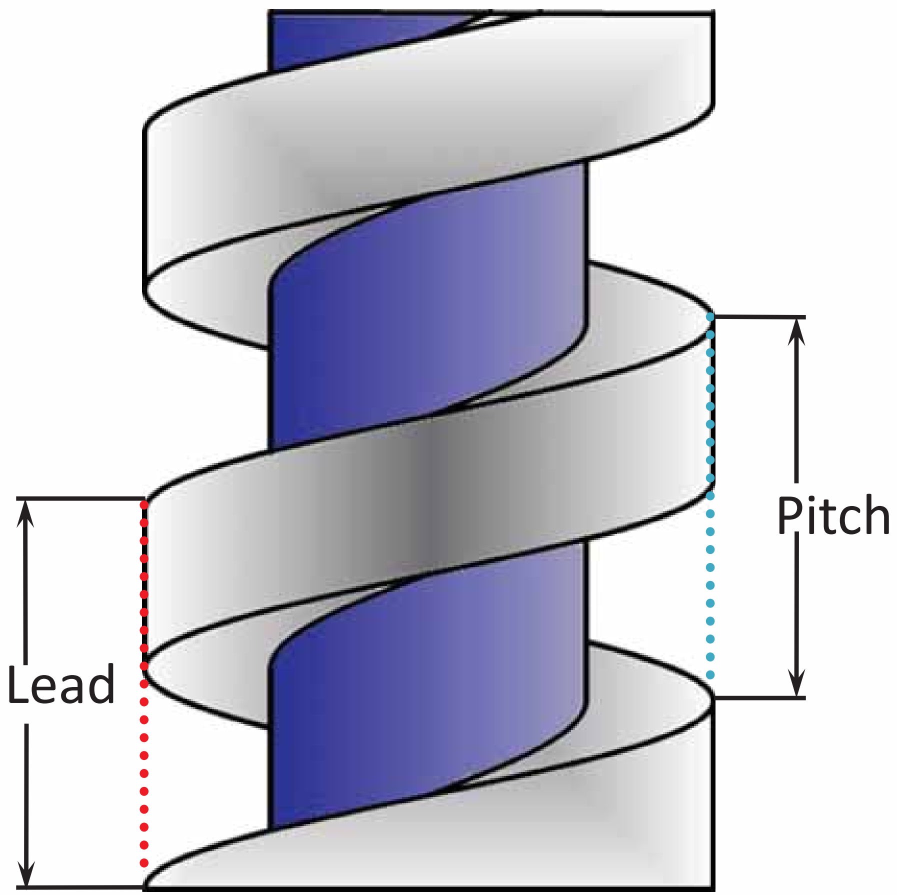


Helical Screw Thread

When large axial movement is required two or more threads may be cut on one screw

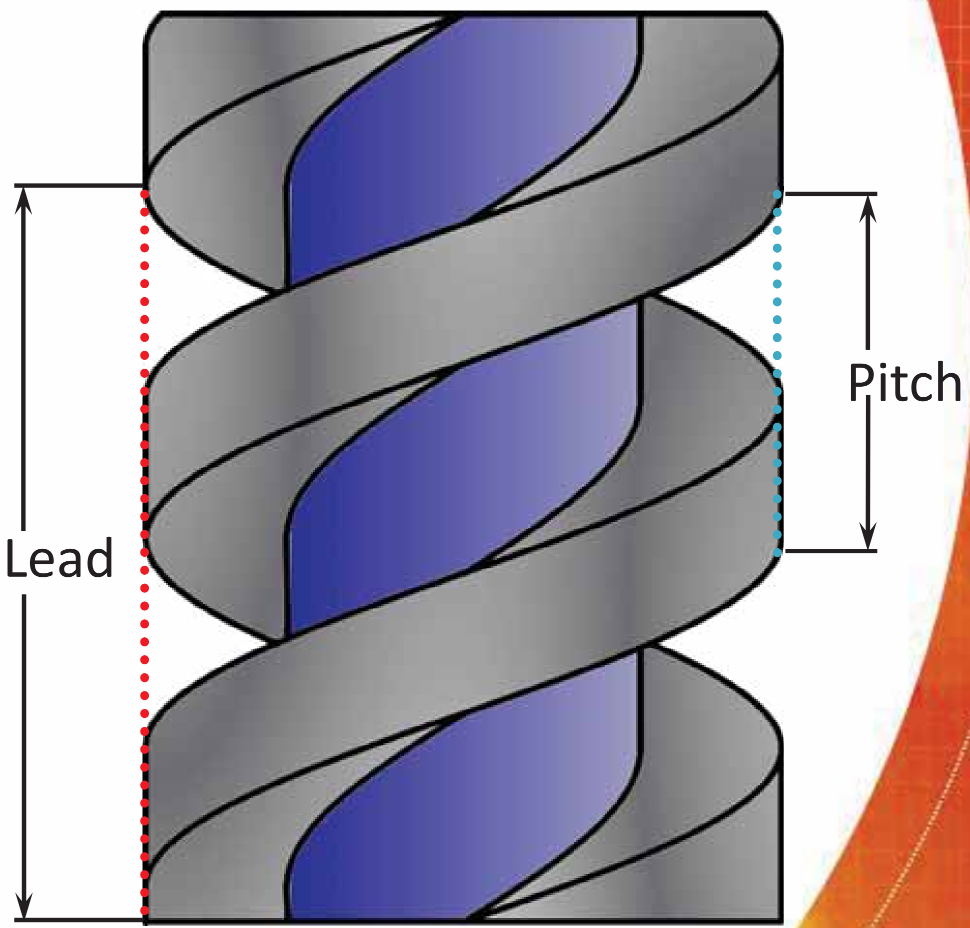
Single Start Thread

Pitch = Lead



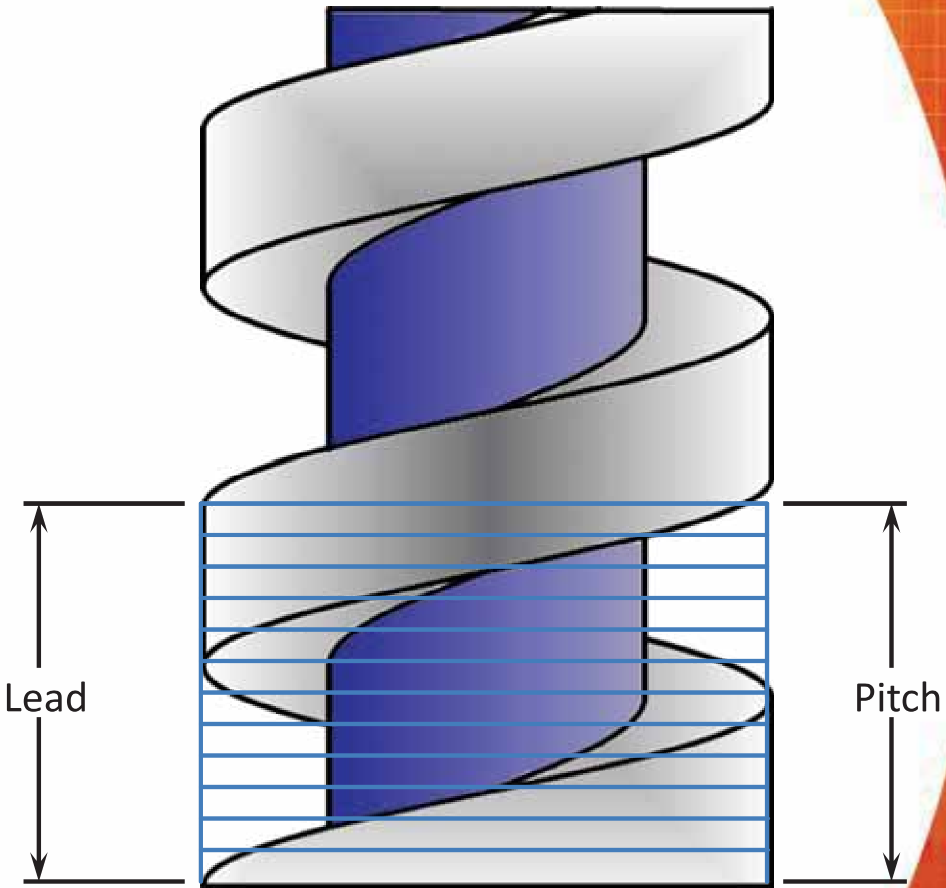
Two Start Thread

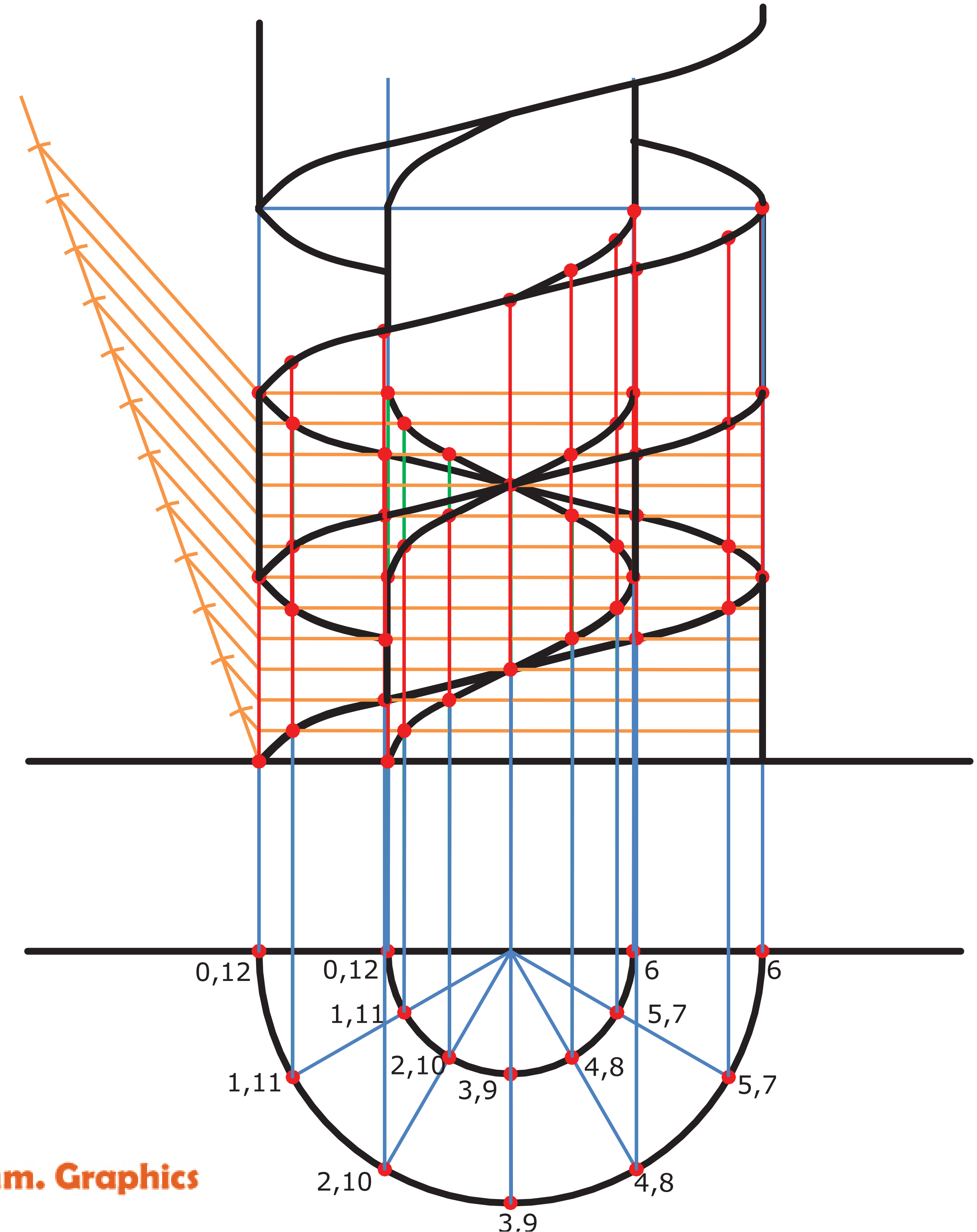
Pitch= ½ Lead



Helical Screw Thread

- Draw one revolution of a single start right-handed screw thread (½ the pitch) given
- Inside Ø: 40mm
- Outside Ø: 82mm
- Lead: 60mm
- Square Thread: 30mm

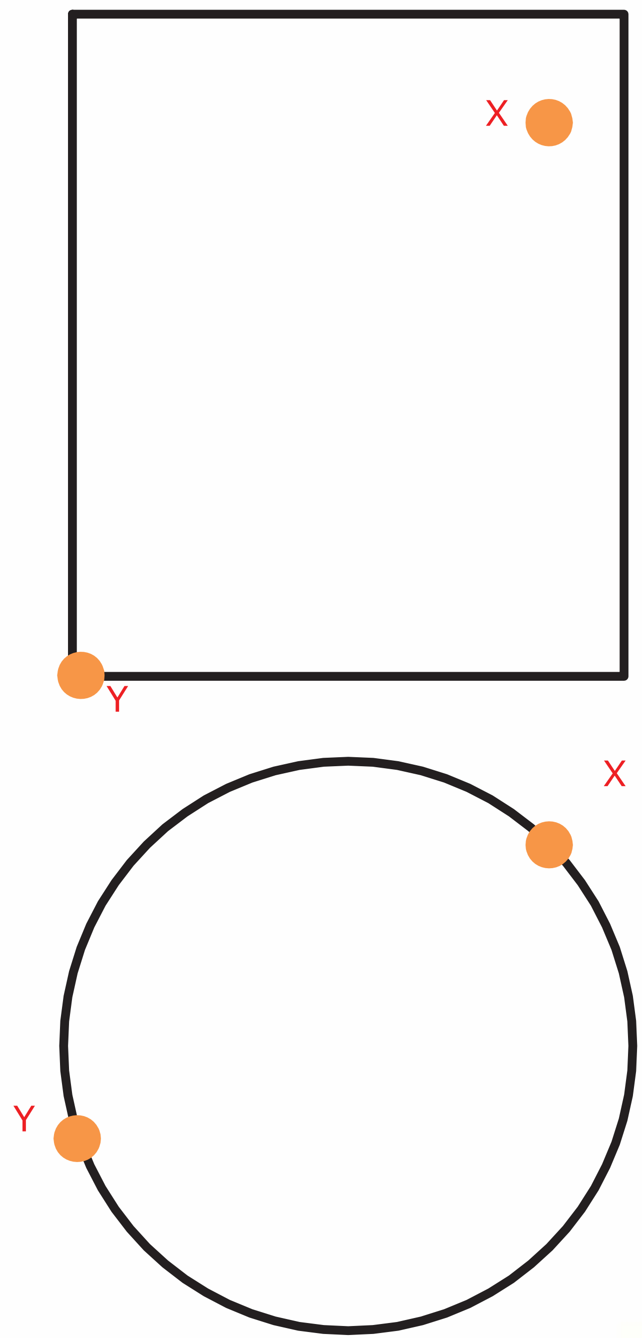


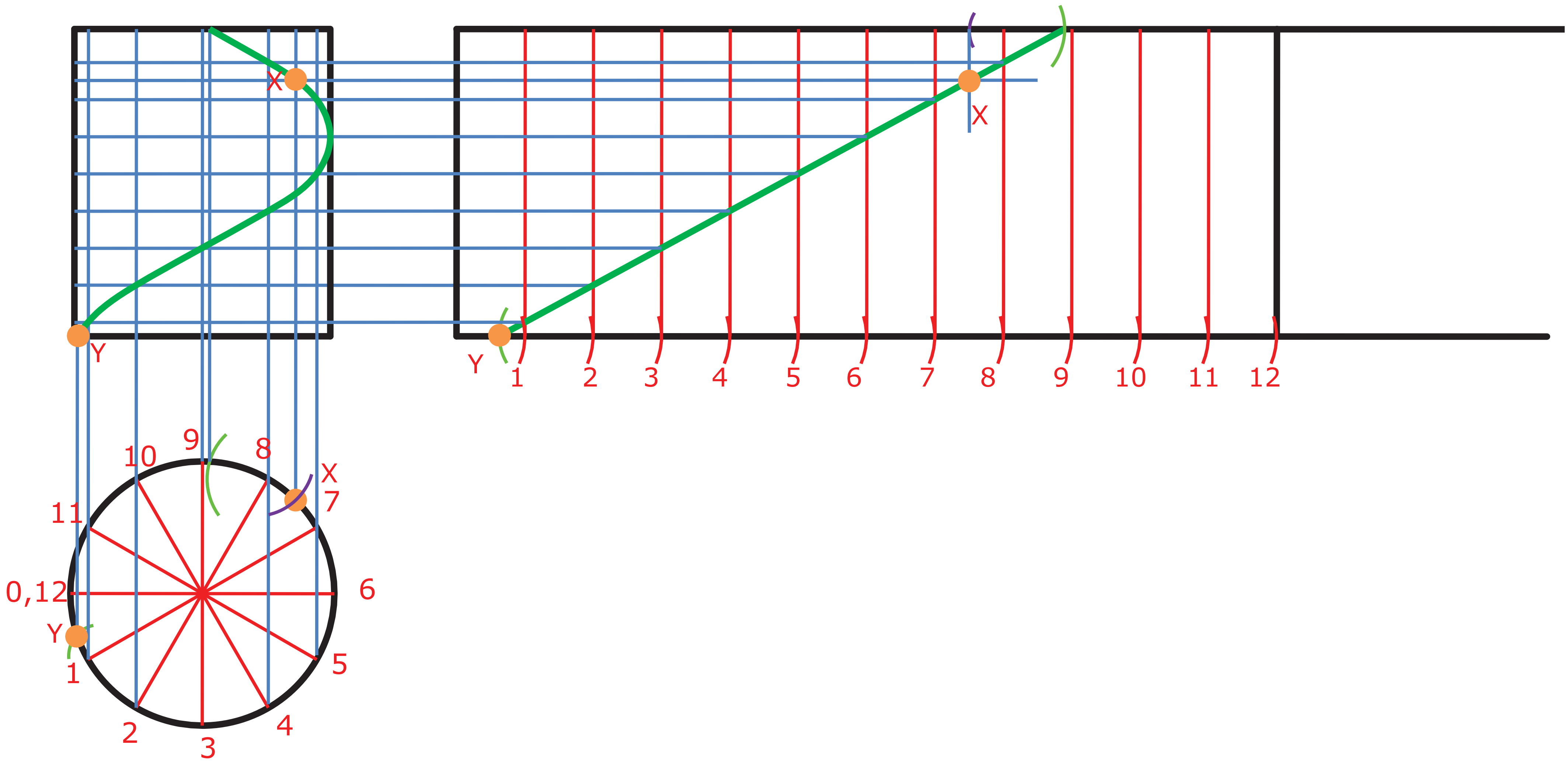


Design and Comm. Graphics

Helix Problems

- Given the plan and elevation of a cylinder with two points on its surface, X and Y. Draw a helix starting from the base of the cylinder and finishing at the top of the cylinder and passing through X and Y





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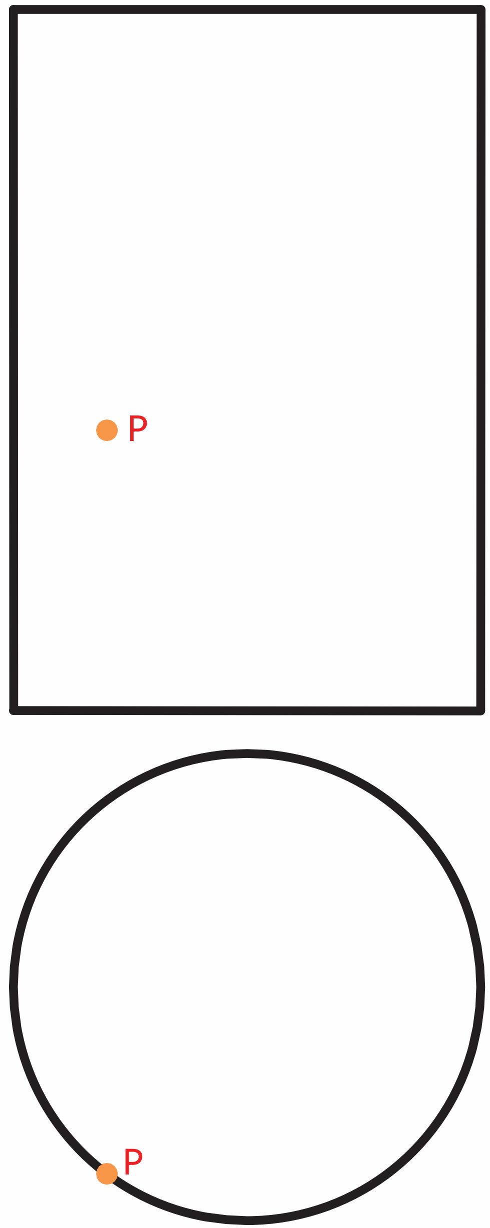
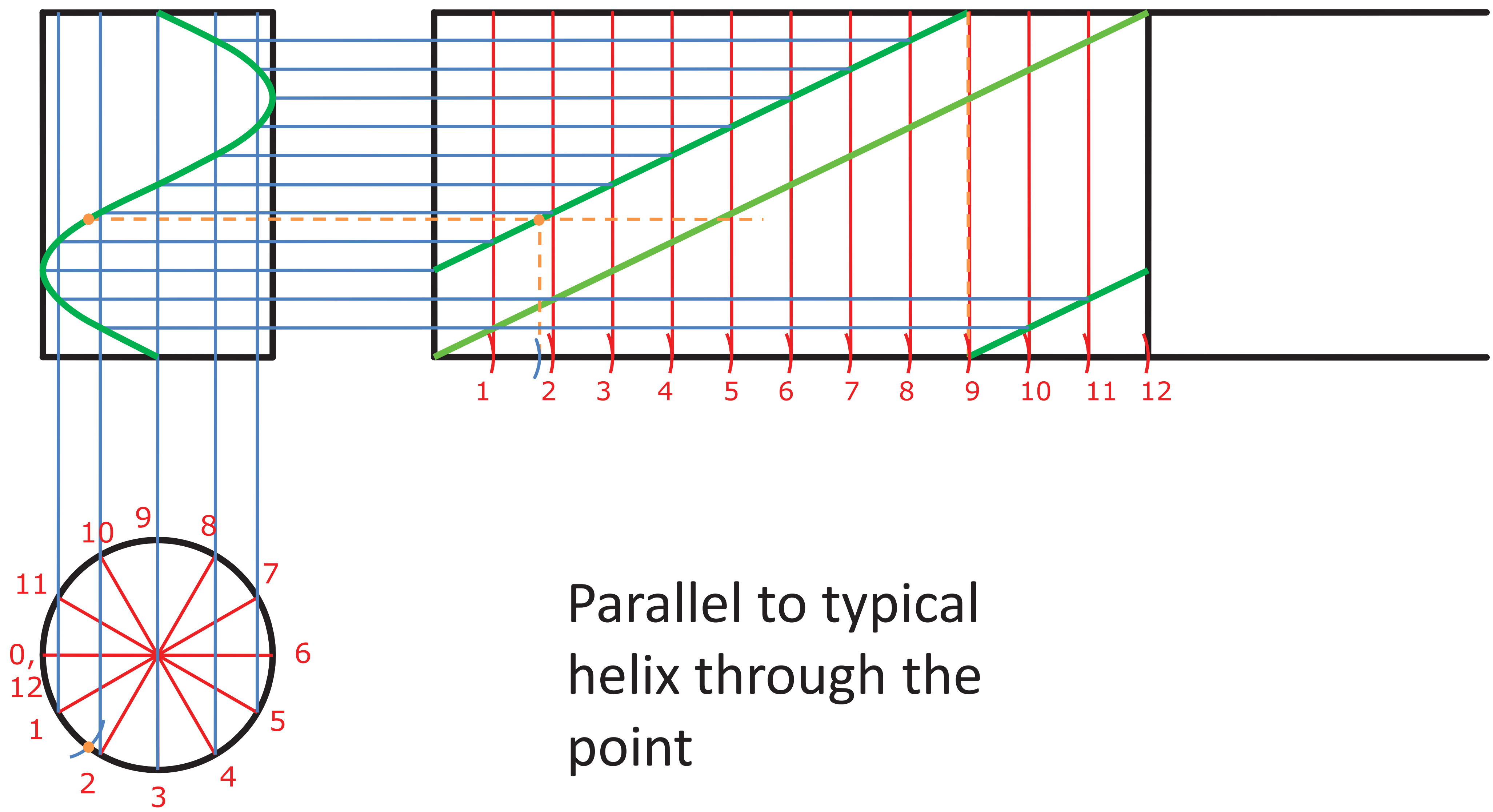
Helix Problems

- Given the plan and elevation of a cylinder, having the point P on its surface draw a helix of one revolution so as it passes through P





Parallel to typical helix through the point

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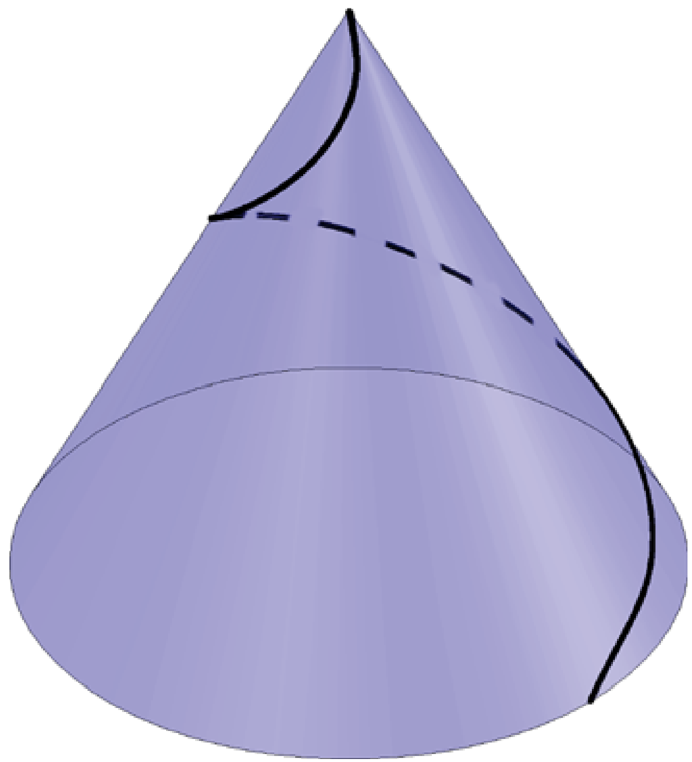
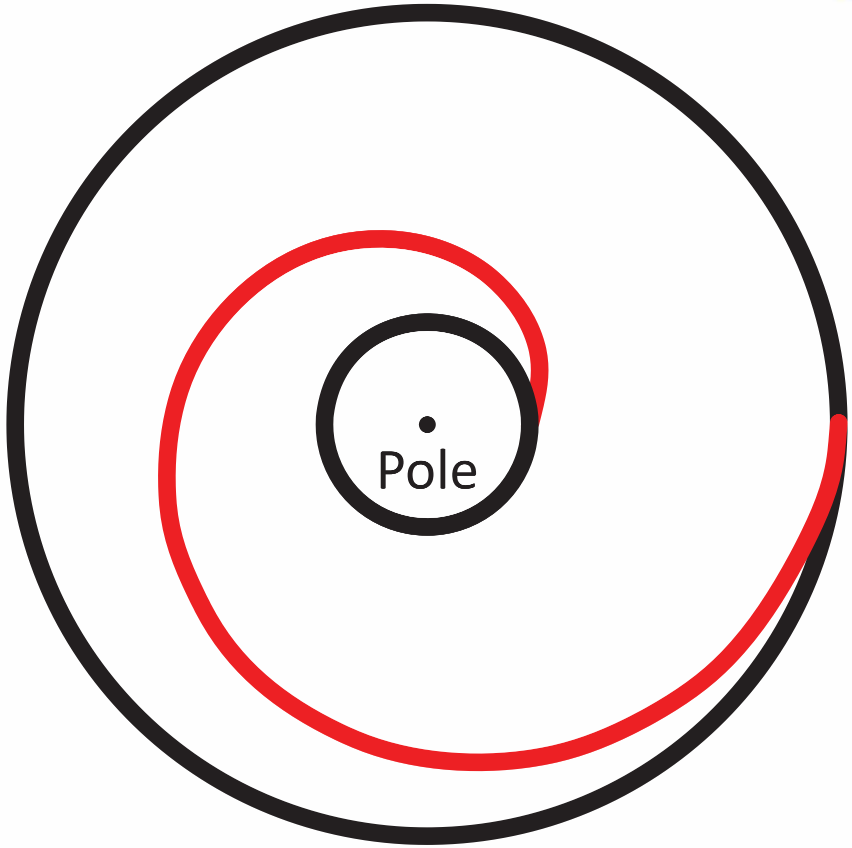
SPIRAL GEOMETRY



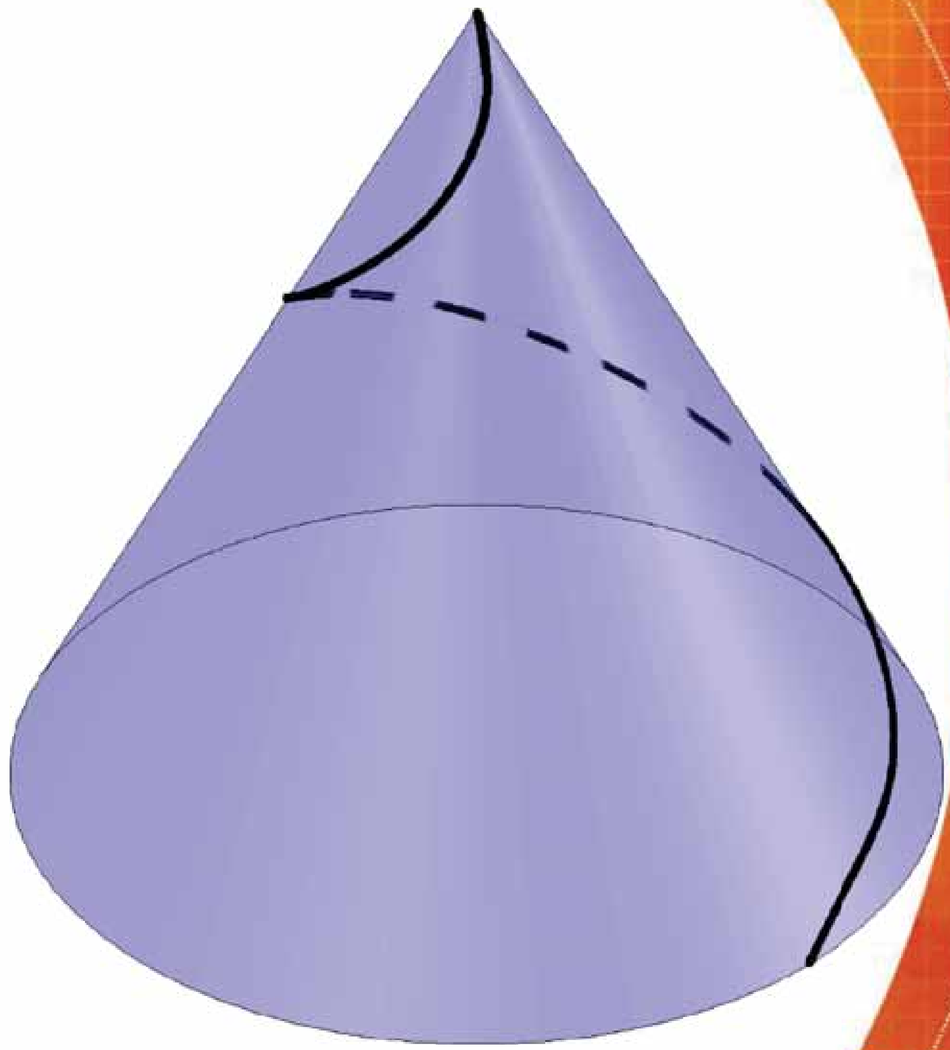
Spiral

- A spiral can be the locus of a point as it moves around a fixed point (pole) while steadily increasing its distance from the point.
- If a line rotate about one of its end points (the pole), and at the same time a point moves continuously in one direction along the line, the locus of the moving point is a spiral



Conical Helix/Conical Spiral

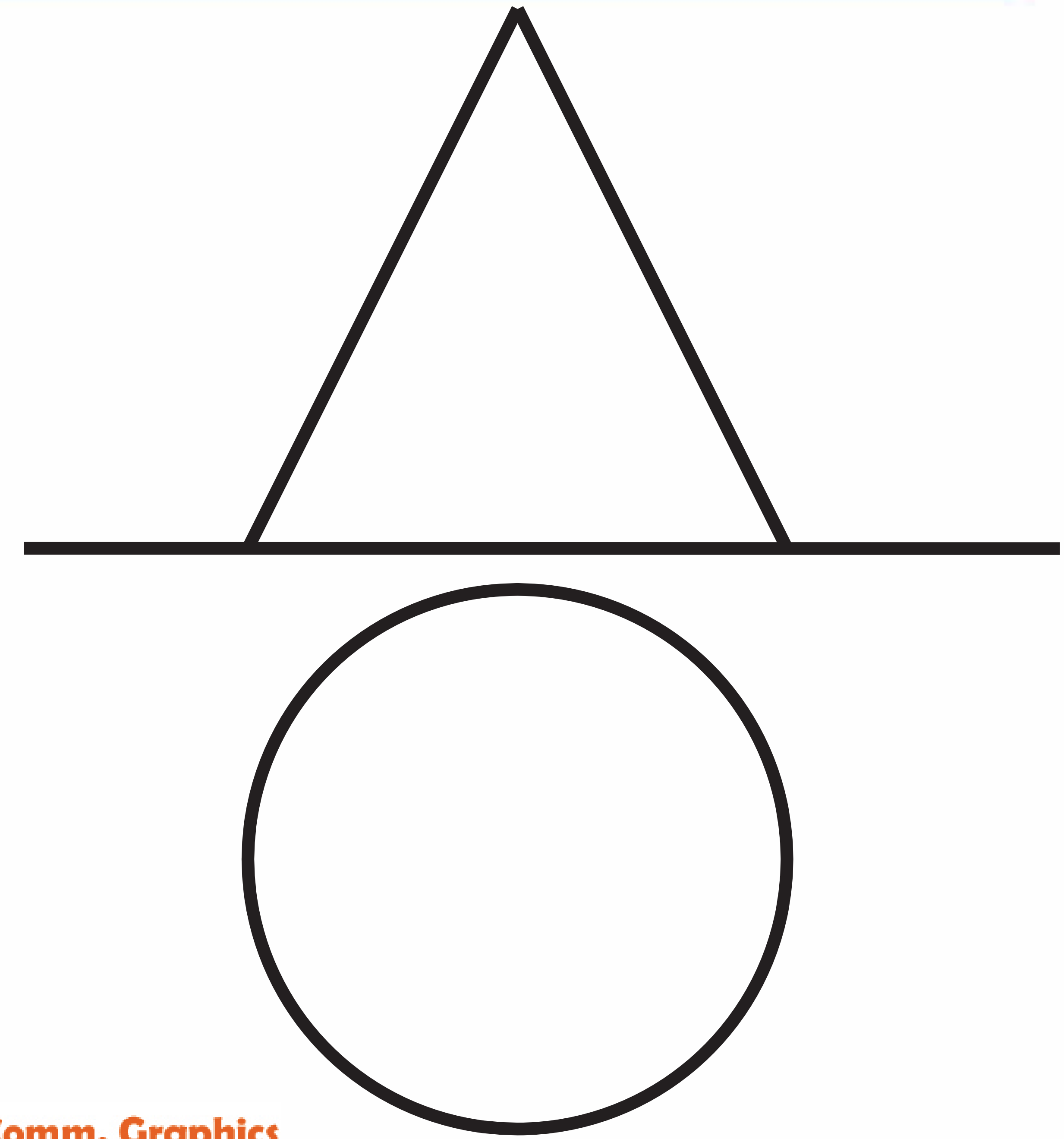
- A conical spiral is the locus of a point as it moves on the surface of a cone so that it rotates at a constant rate around the cone while also progressing in the direction of the axis at a constant rate
- The plan of a conical spiral is an Archimedean spiral

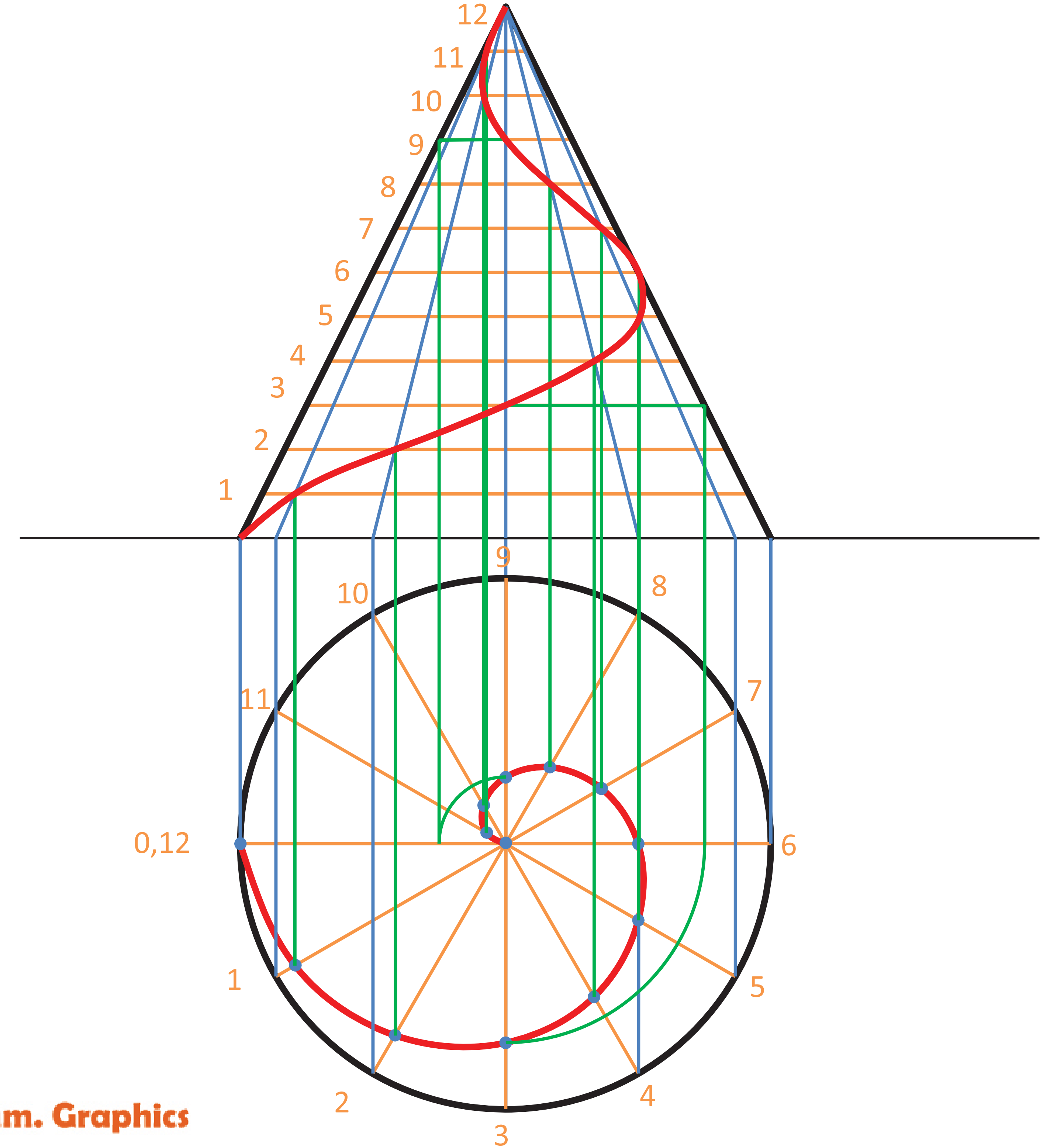


Applications of the Conical Spiral

- The conical spiral is used for augers and other boring devices such as screw tips
- It is also used for the construction of Archimedean wells

Construct a conical spiral





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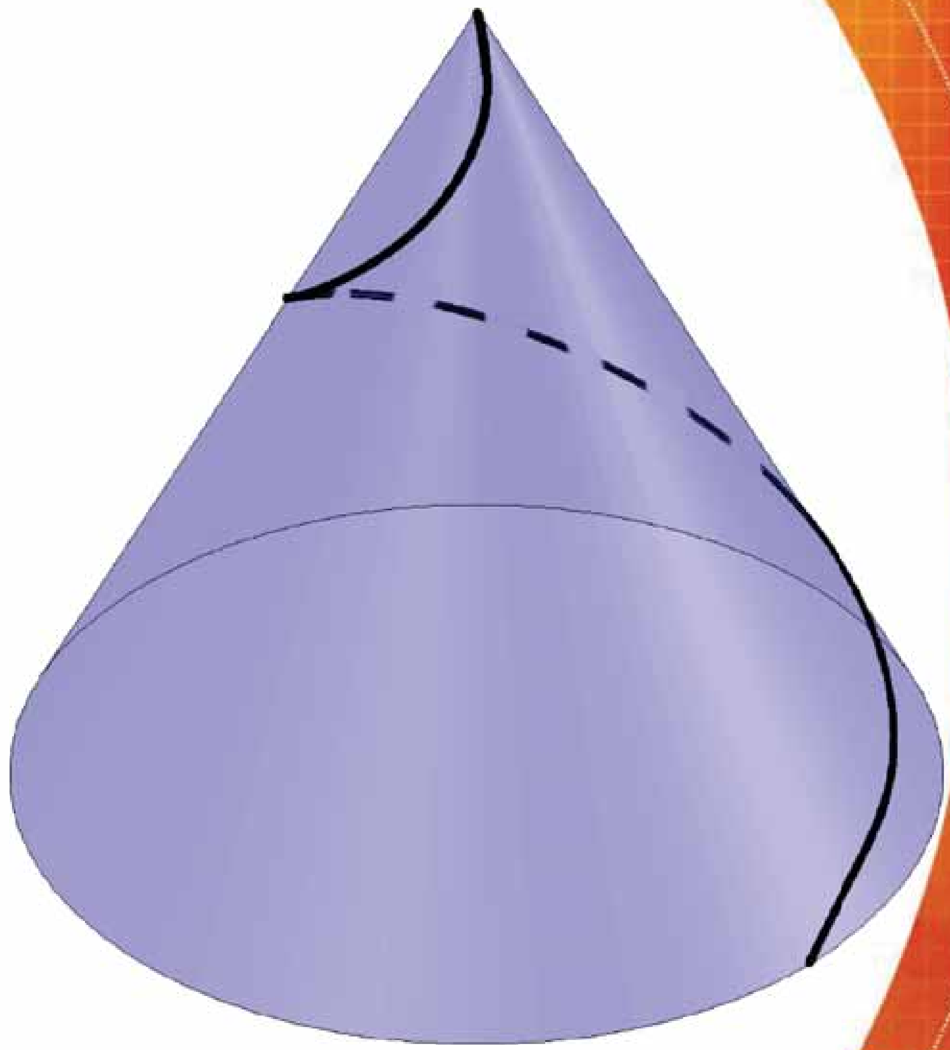
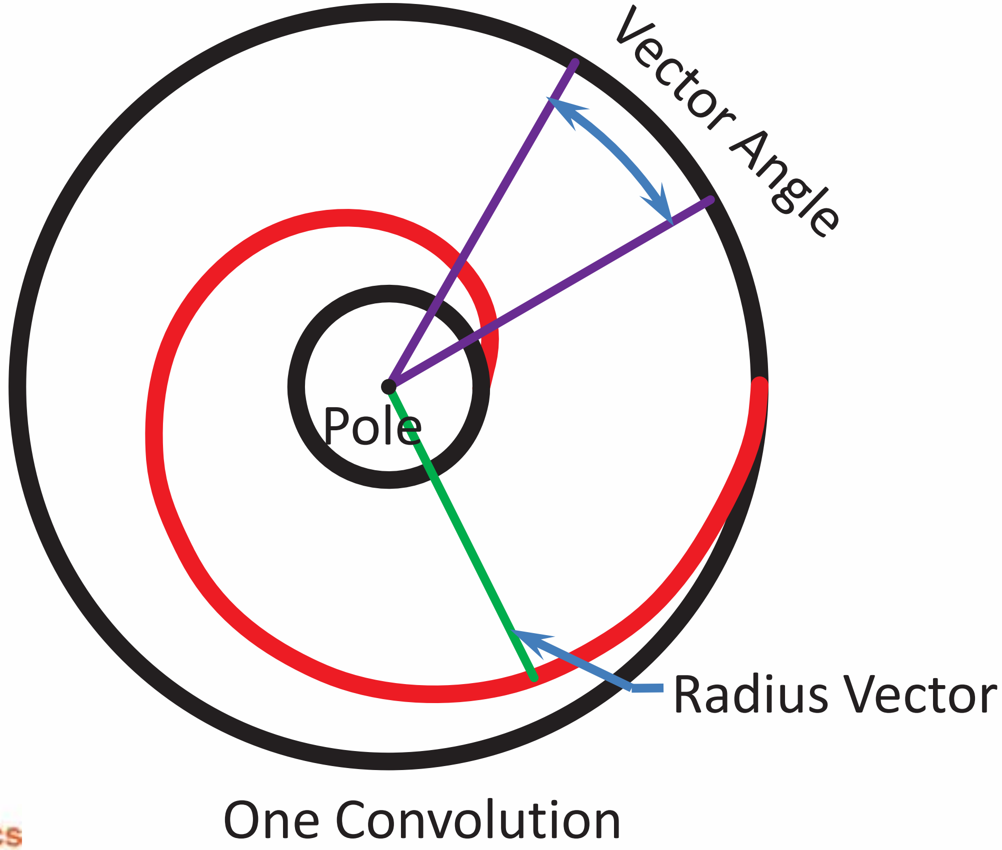
Conical Spiral

- Note: A conical spiral is not a geodesic, as the development of a conical spiral is a curve



Spiral Terminology

Term	Definition
Pole	The point at the centre
Convolution	One complete rotation of the point around the spiral
Vector Angle	The angle between any two radius vectors
Radius Vector	Any line from pole to a point on the spiral

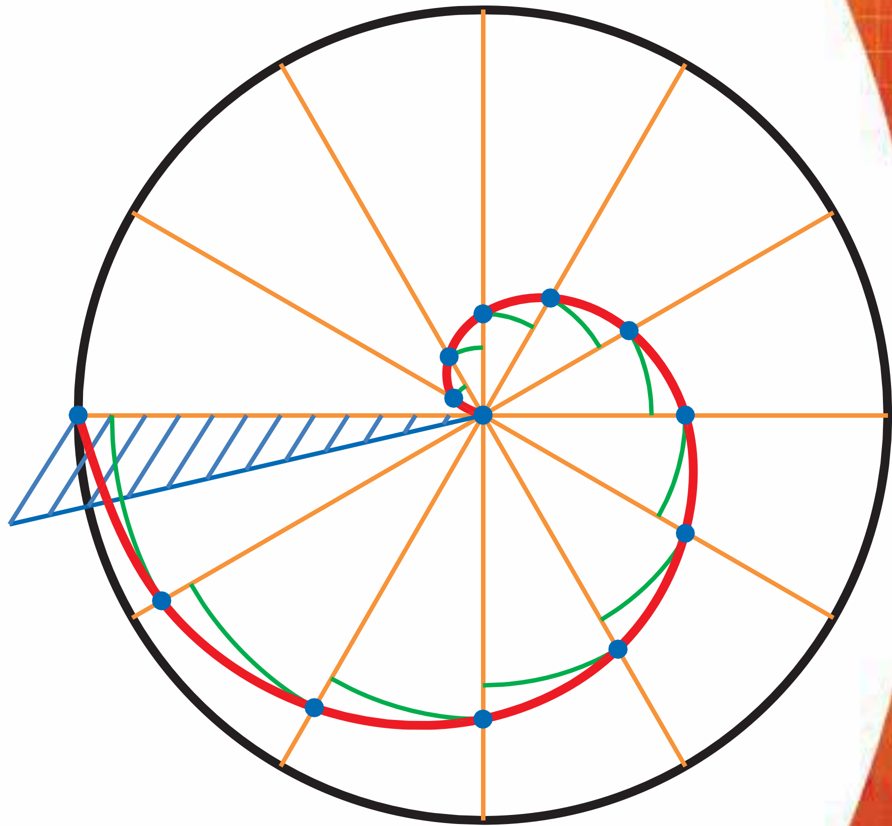


Spiral Types

- There are two main types of spirals:
- Archimedean Spirals
- Logarithmic Spirals

Archimedean Spirals

- An Archimedean spiral is the locus of a point that moves around a circle at a constant speed while also moving away from/towards the pole at a constant speed

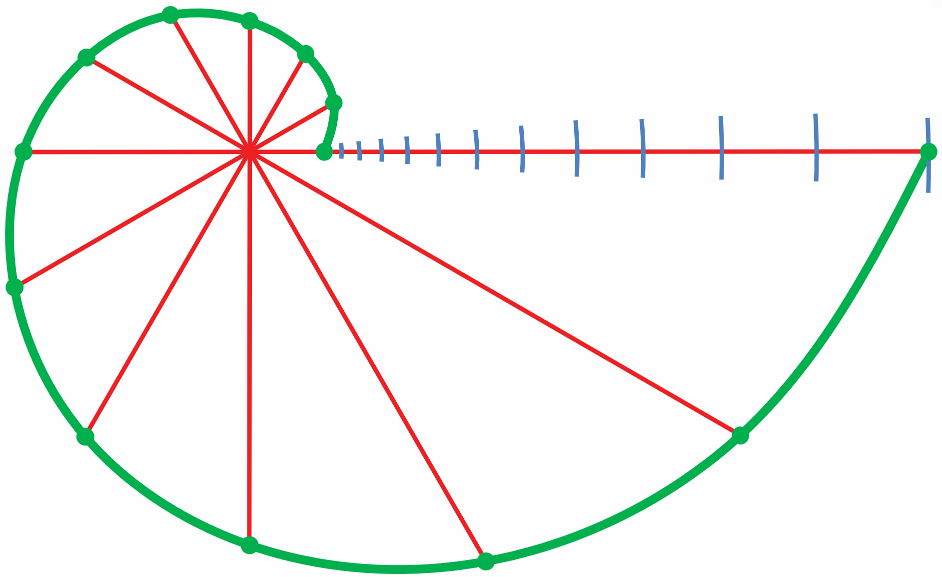


Applications of the Archimedean Spiral

- The Archimedean spiral is functional as well as aesthetic
- Archimedean springs are used in watch making and door mechanisms
- Because of the link between the conical and Archimedean spiral many of the applications of the Archimedean spiral are exhibited through the conical spiral

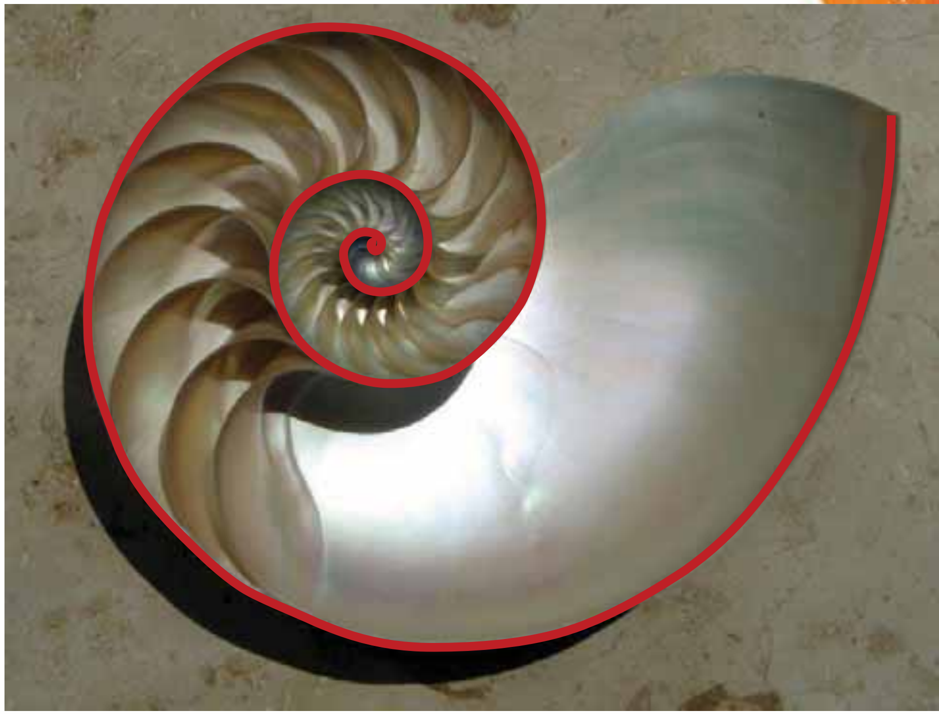
Logarithmic Spirals

- A logarithmic spiral is a spiral that increases/decreases proportionally according to a given rule.
- A logarithmic curve will never terminate at a pole



Applications of the Logarithmic Spiral

- Logarithmic Spirals are naturally occurring spirals in nature
- The sea shell on the right contains a logarithmic spiral
- The natural occurrence of manmade design in nature or visa versa is known as bio-mimicry



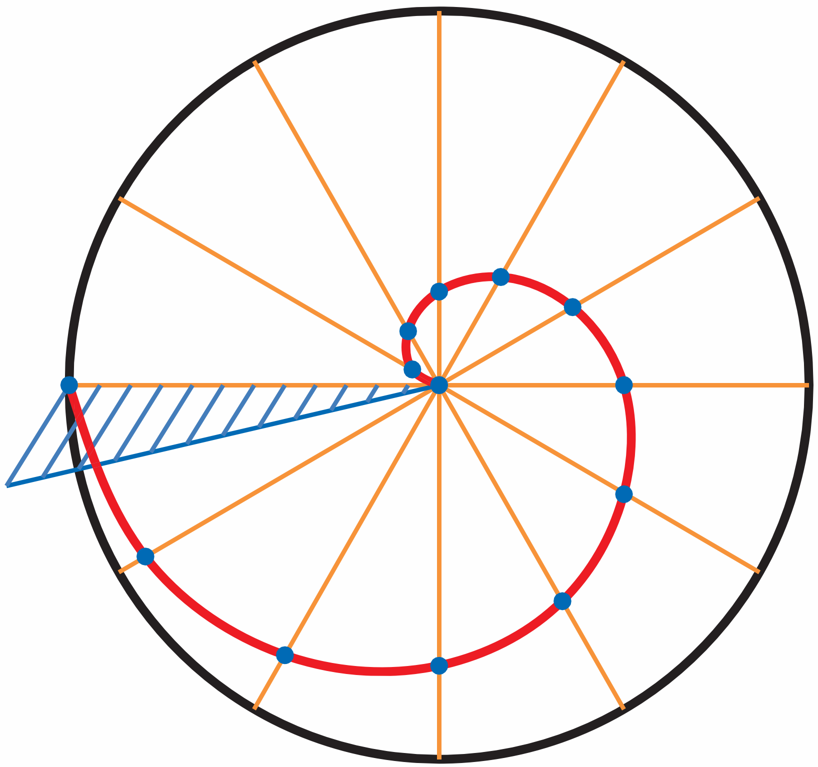
This Wikipedia and Wikimedia Commons image is from the user Chris 73 and is freely available at http://commons.wikimedia.org/wiki/Image:NautilusCutawayLogarithmicSpiral.jpg under the creative commons cc-by-sa 2.5 license

Applications of the Logarithmic Spiral

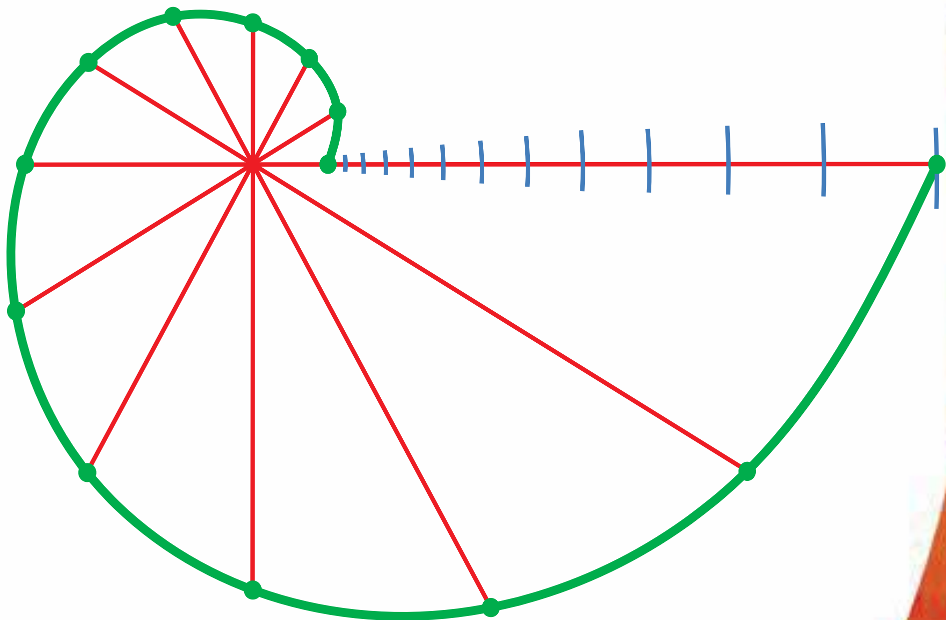
- Some universes branch out in a logarithmic spiral
- An understanding of geometrical shapes may assist scientists and engineers in furthering their research and discoveries

Spiral Types

Archimedean Spiral

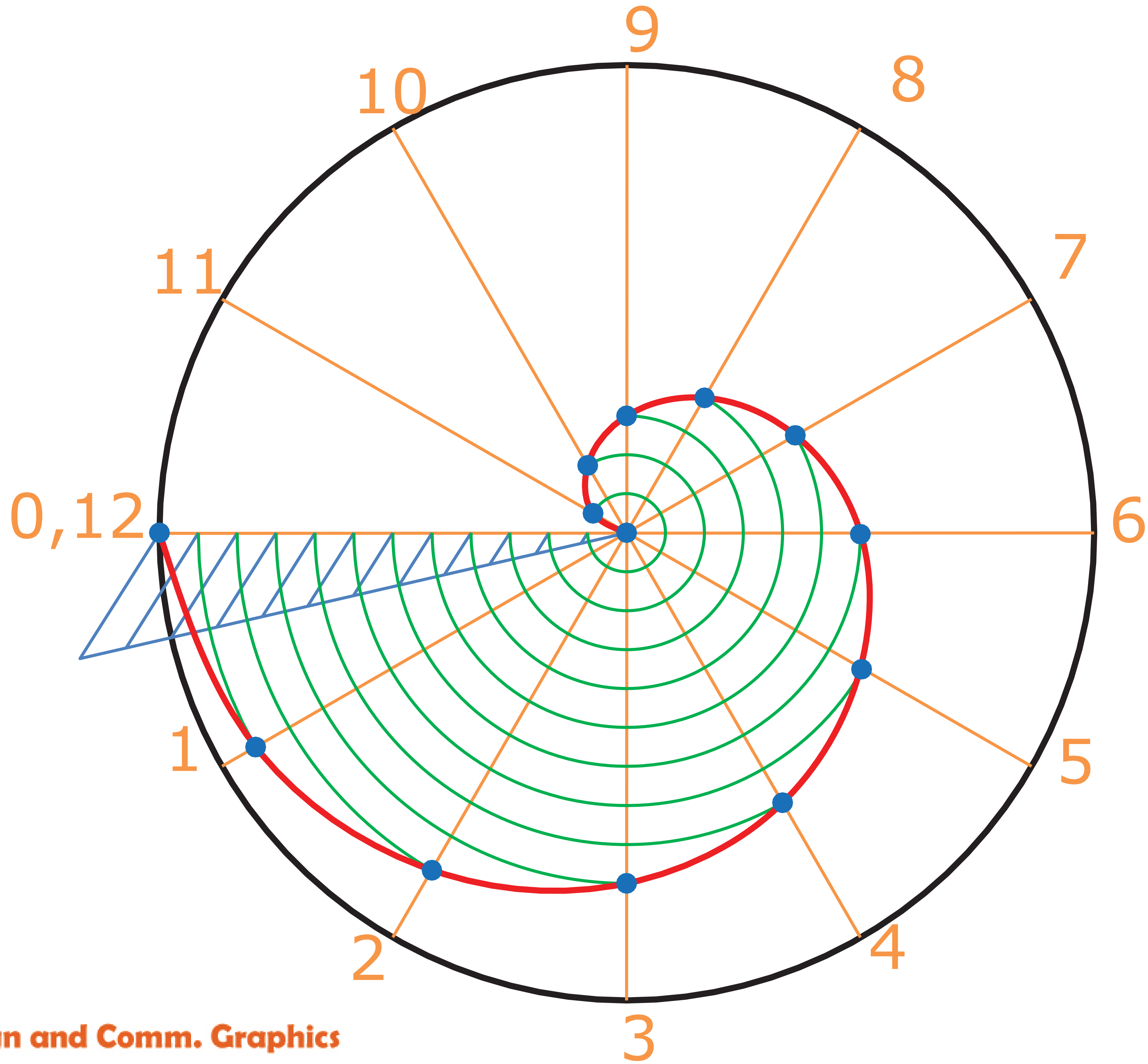


Logarithmic Spiral



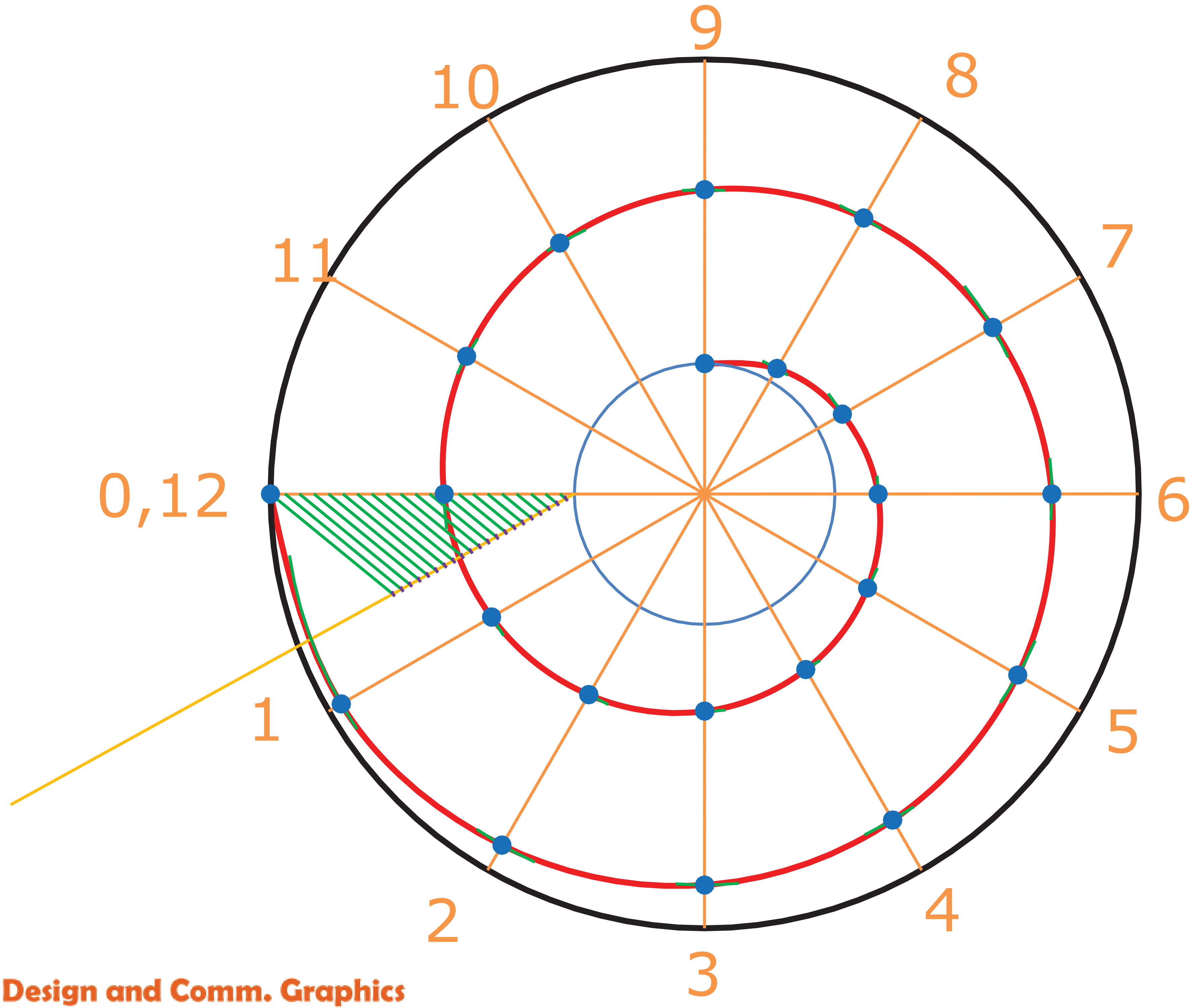
Archimedean Spirals

- Draw an archimedean spiral of one convolution given the longest radius vector as 60mm and the shortest as 0mm


9
8
10
7
11
0,12
6
1
5
2
4
3

Archimedean Spirals

- Draw an archimedian spiral of 1 ¾ convolutions given the longest radius vector as 60mm and the shortest as 20mm

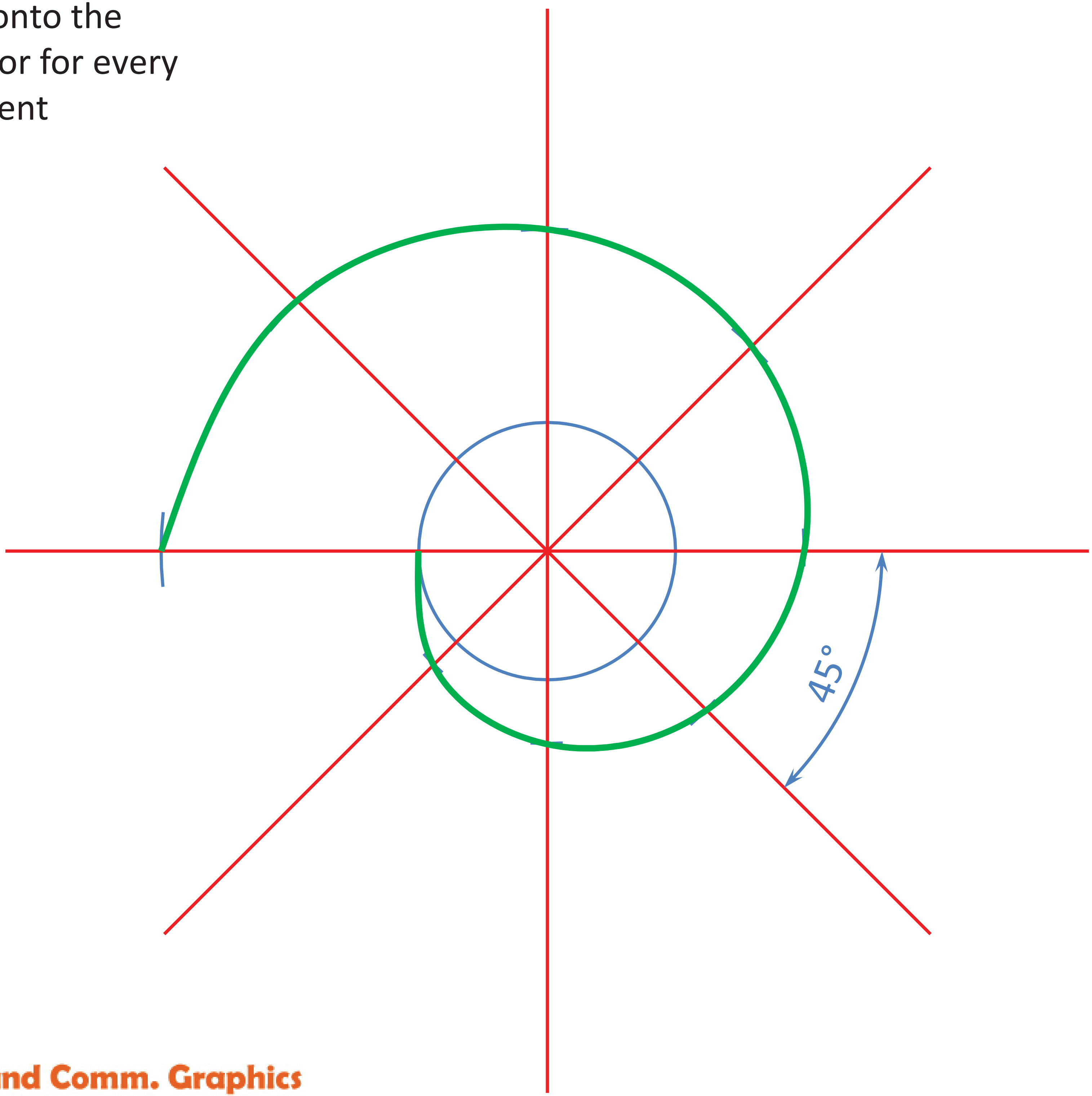


9
8
10
7
11
0,12
6
1
5
2
4
3

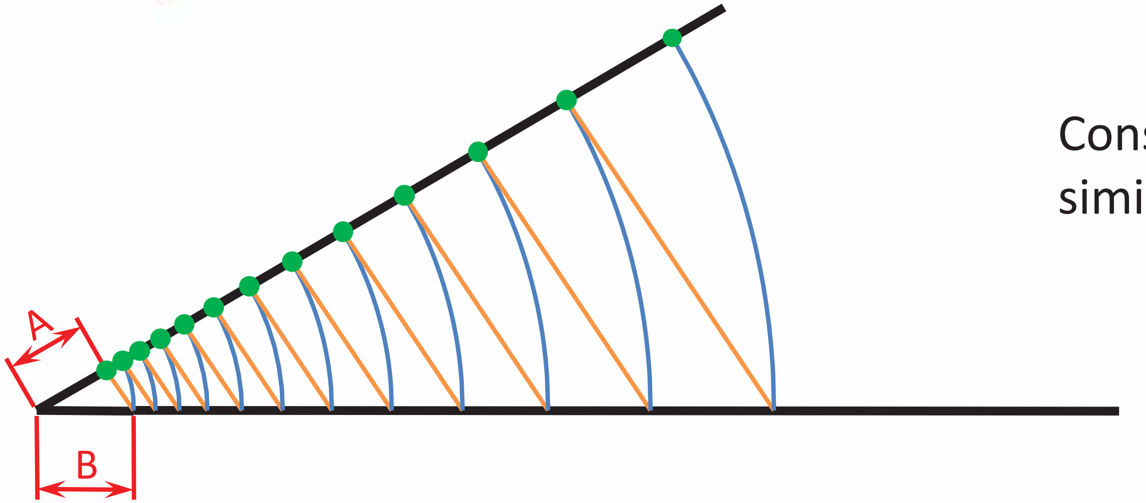
Archimedean Spirals

- Construct one convolution of an archimedean spiral given the shortest radius vector of 20mm and an increase in vector length of 5mm every 45°

Add 5mm onto the radius vector for every 45° increment

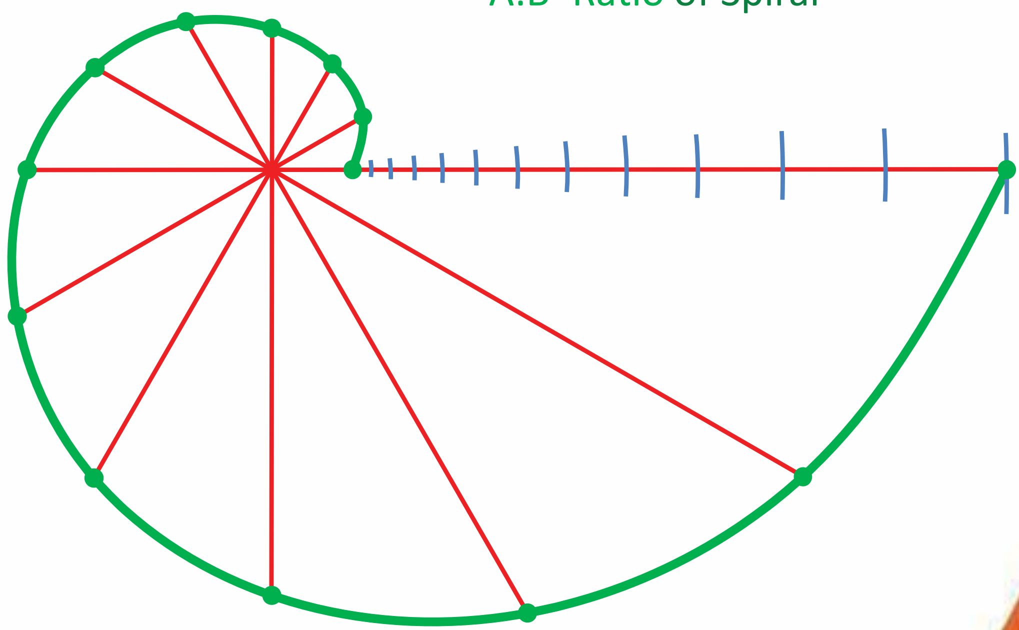


Logarithmic Spirals



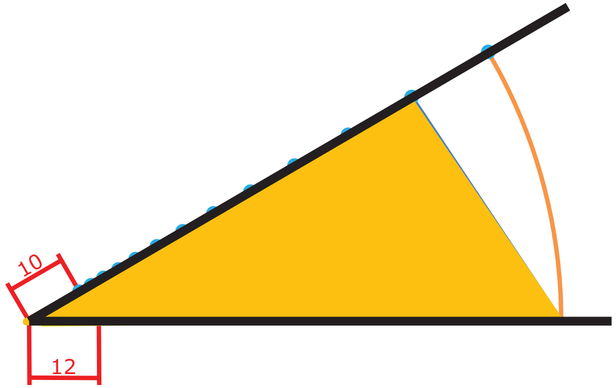
Construction derived from similar traingles

A:B=Ratio of Spiral

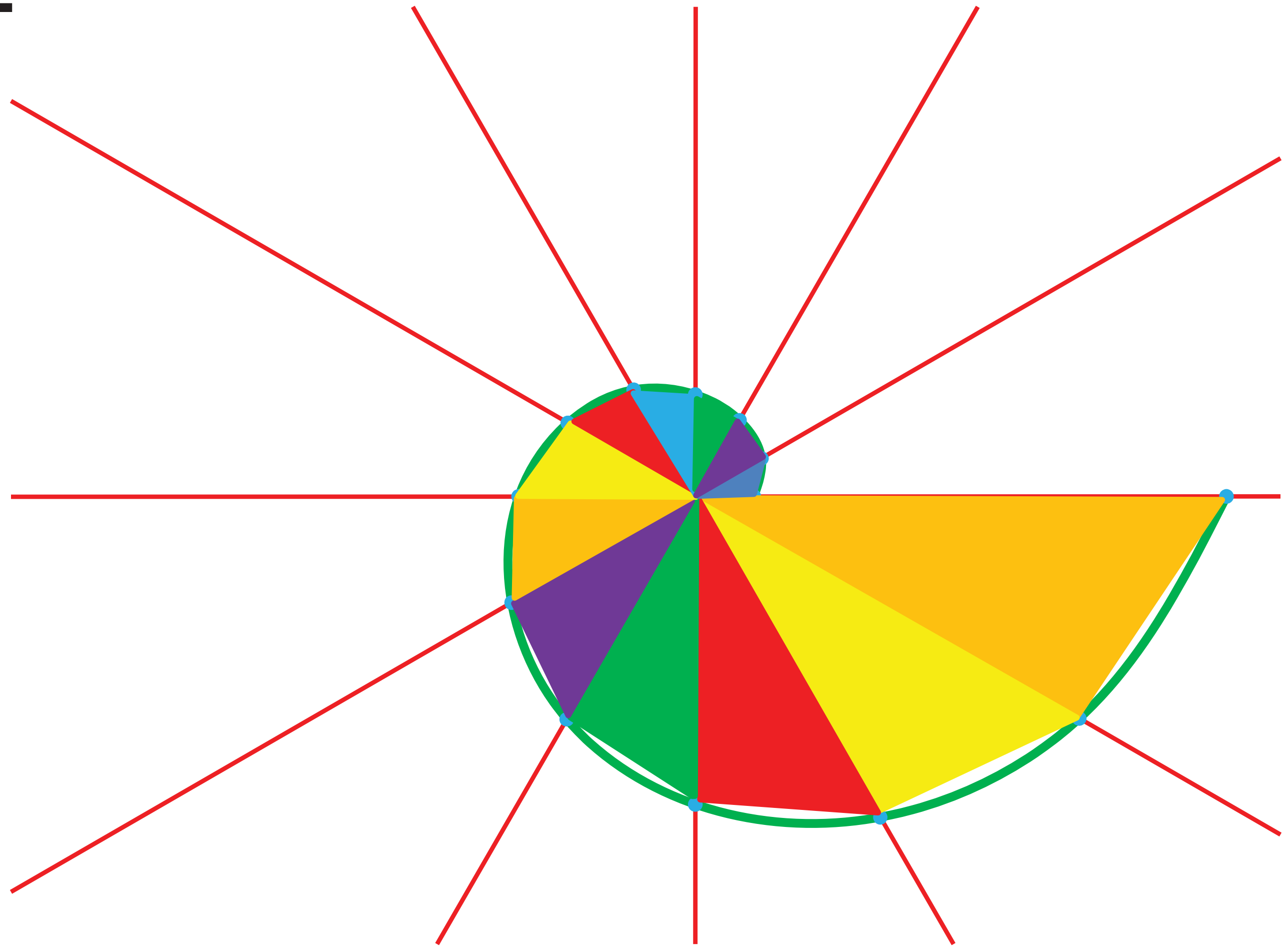


Logarithmic Spirals

- Construct one convolution of an logarithmic spiral given a vector angle of 30° and the ratio of the vector lengths as 10:12 (Initial radius not specified)

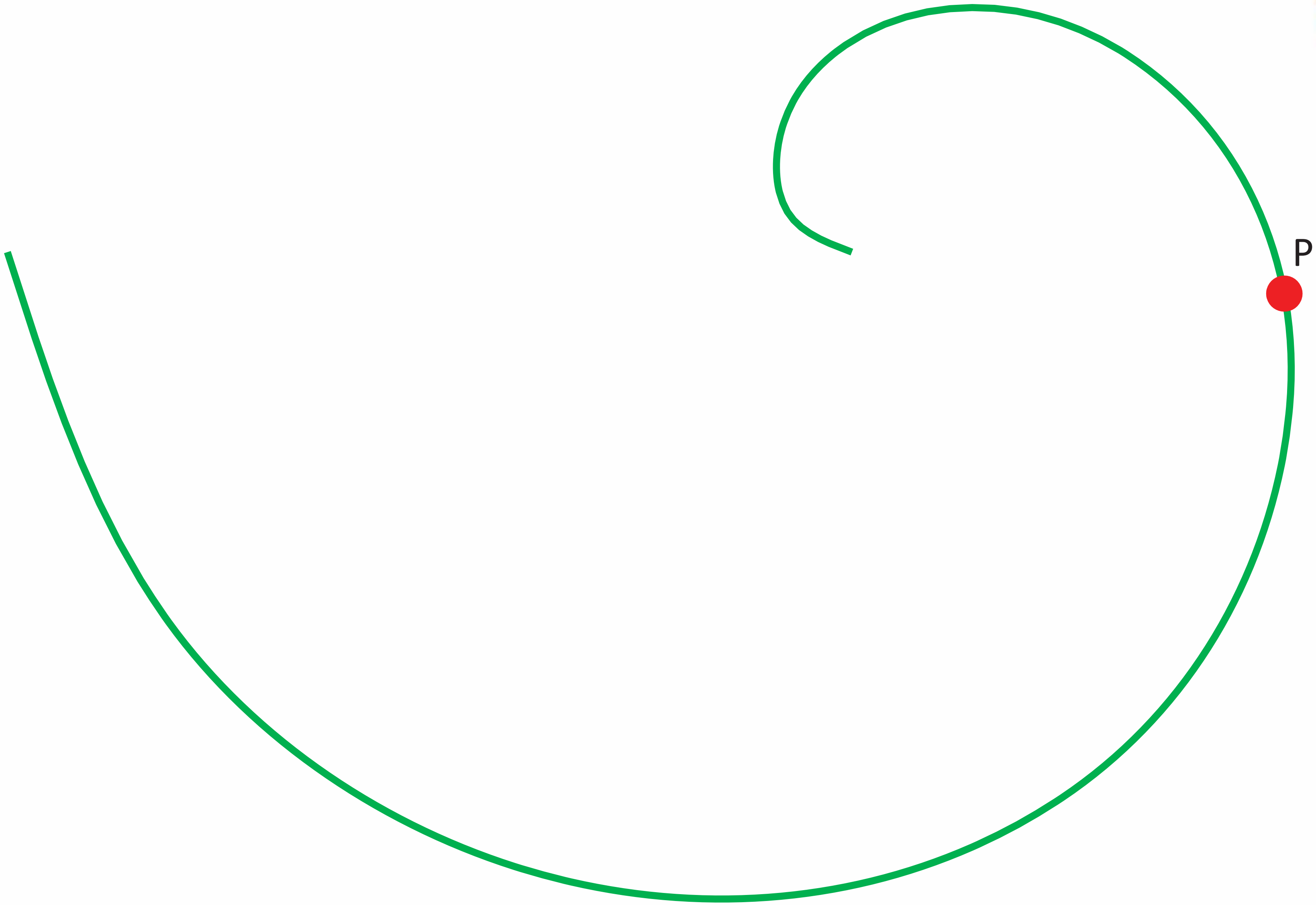


Logarithmic spiral derived from similar triangles

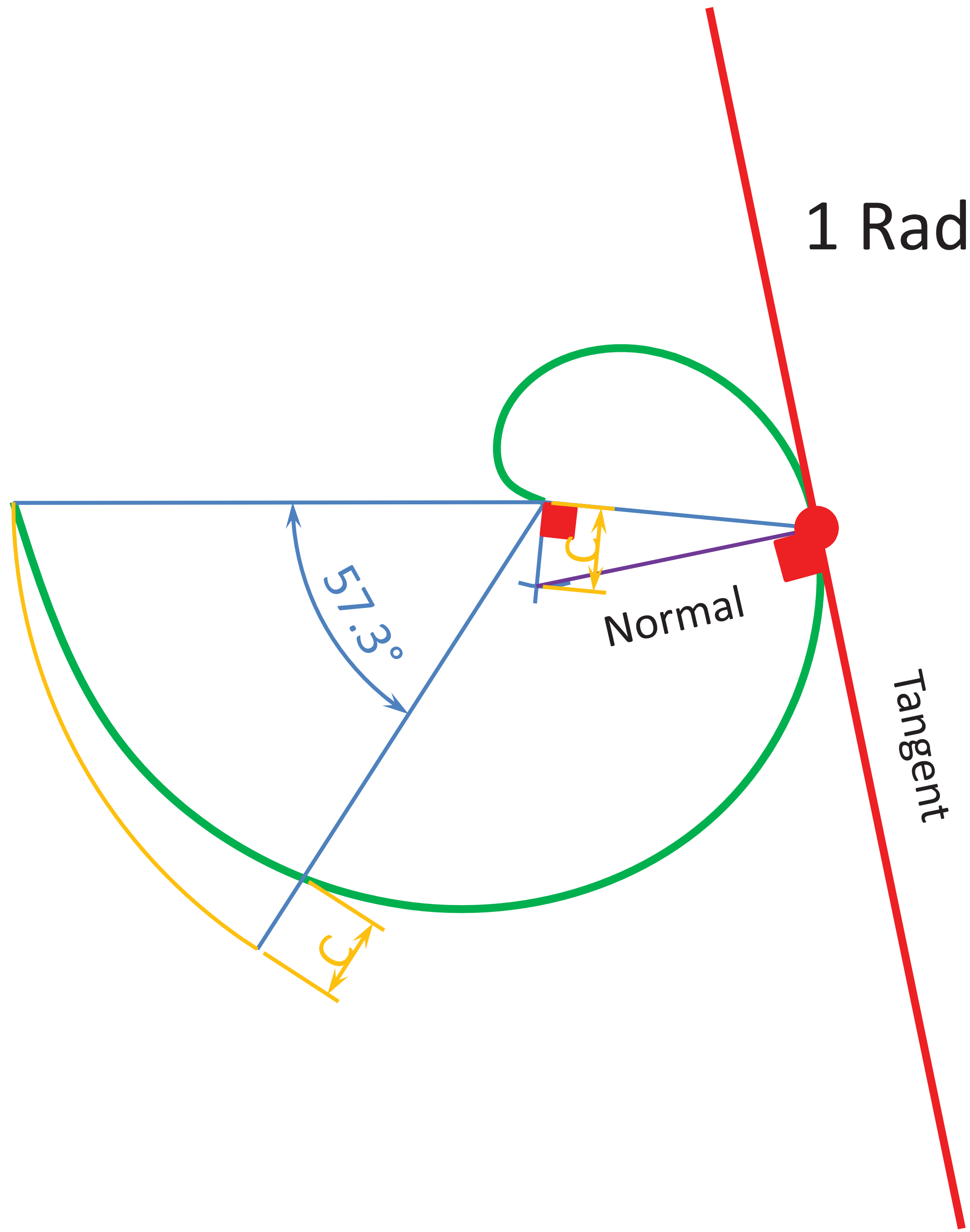


TANGENTS TO SPIRALS

Tangent to an Archimedean spiral at a point P

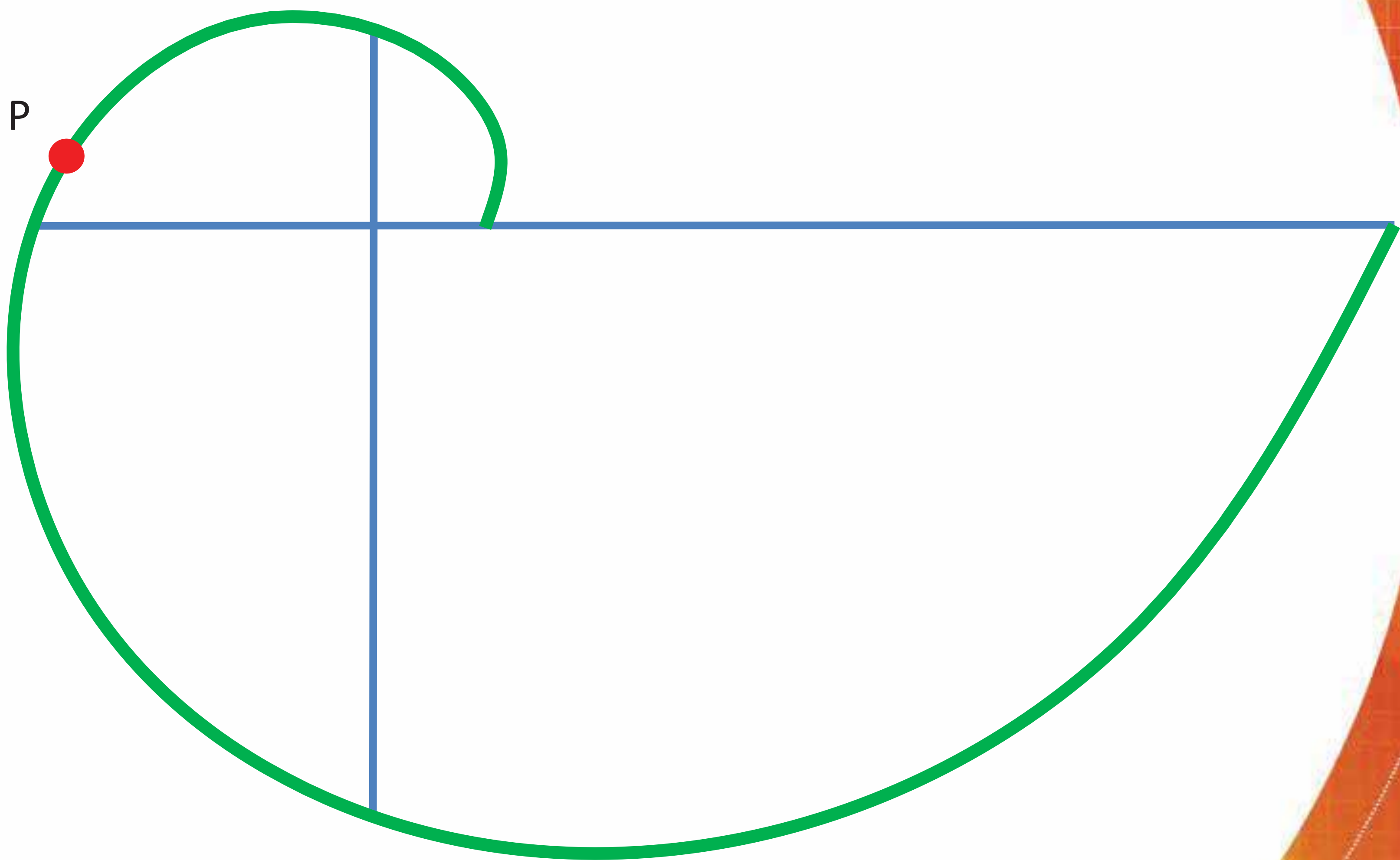


1 Radian = 57.3°

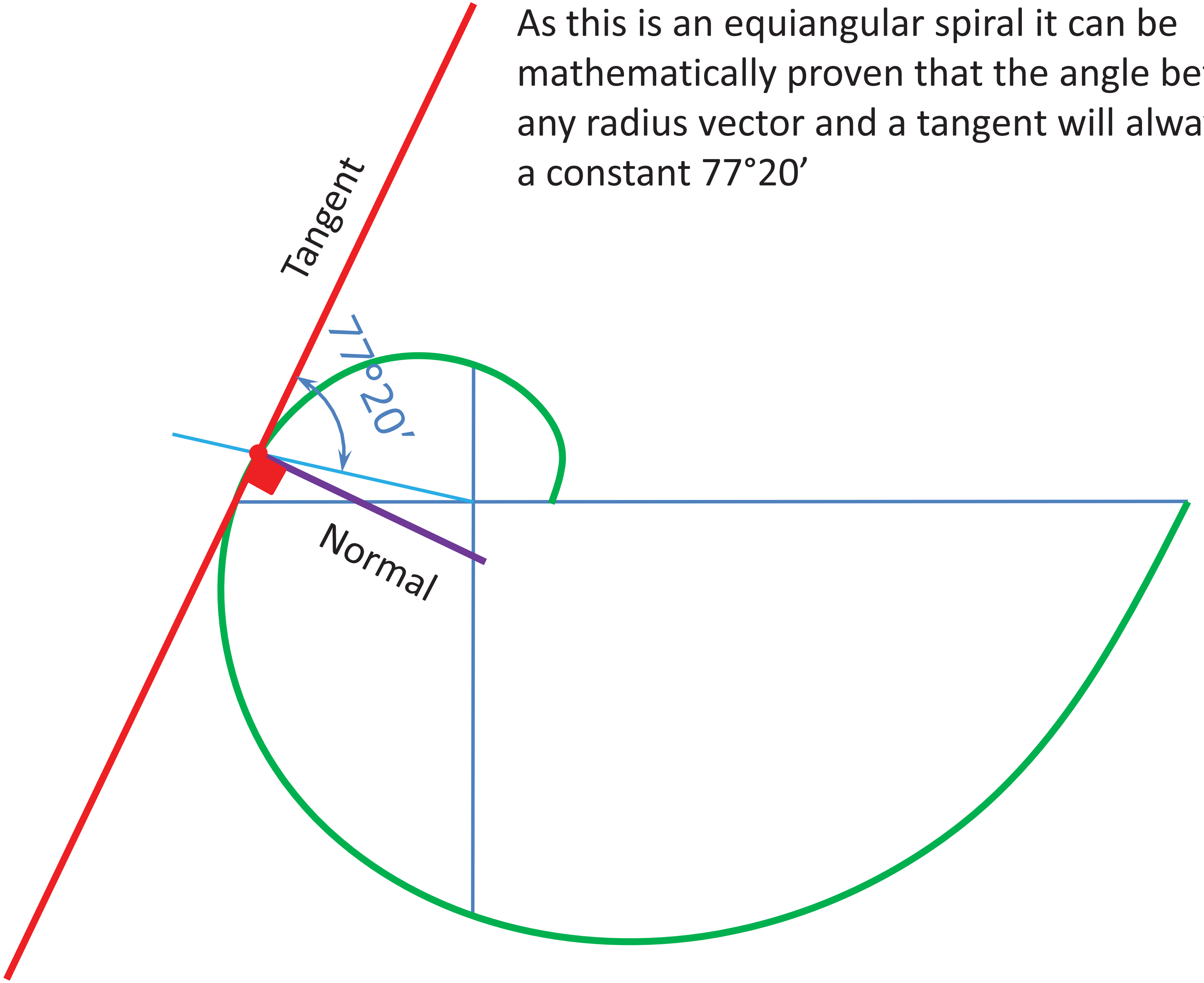


Design and Comm. Graphics

Tangent to a Logarithmic spiral at a point P



As this is an equiangular spiral it can be mathematically proven that the angle between any radius vector and a tangent will always be a constant 77°20’



Animation of a logarithmic spiral involute

- http://mathworld.wolfram.com/LogarithmicSpiralInvolute.html

LOCI

Applications of Loci

- A locus is the movement of a point as it follows certain conditions
- A locus may be used to ensure that moving parts in machinery do not collide

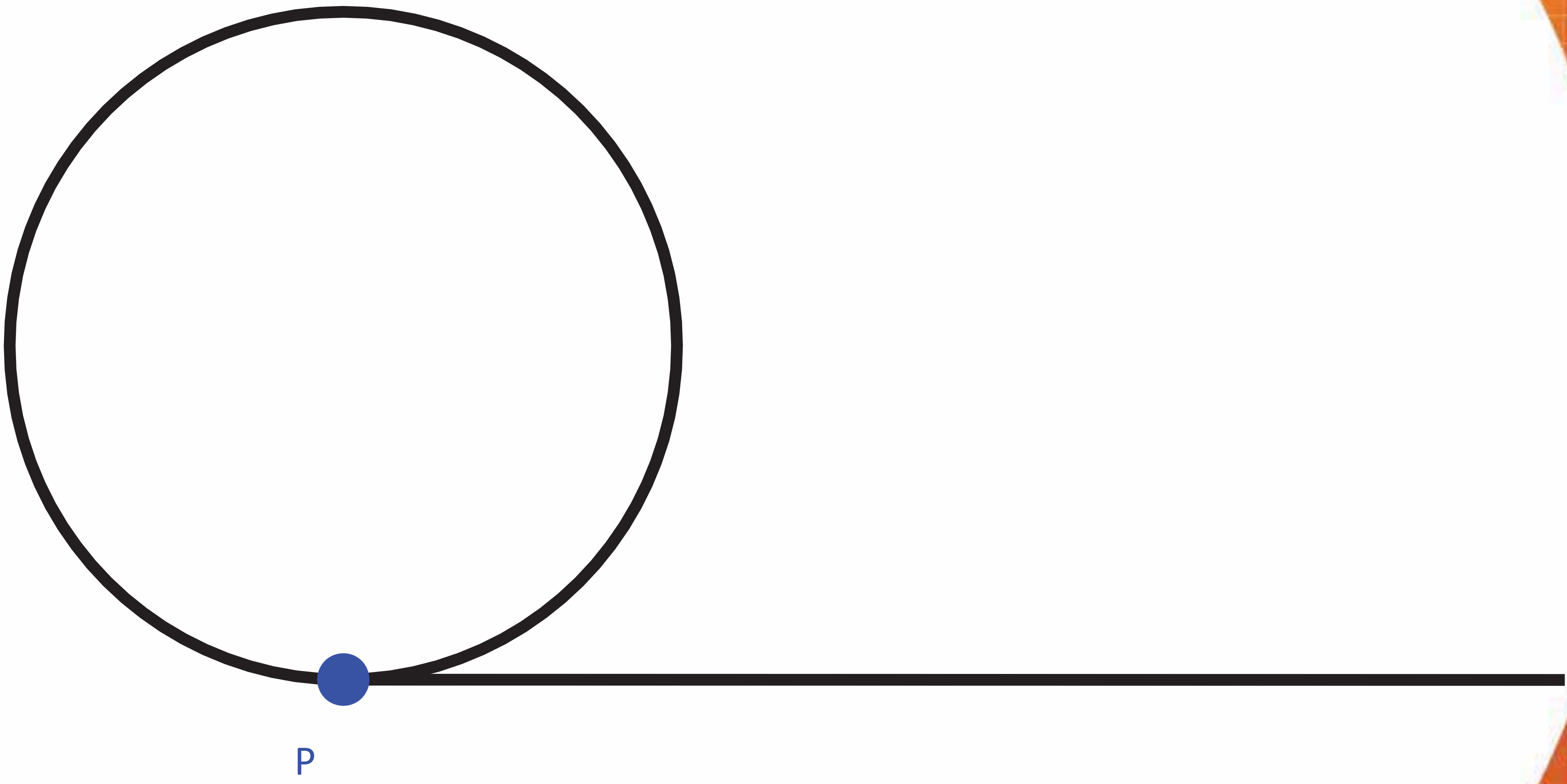
Cycloid

- A cycloid is the locus of a point **on the circumference** of a circle which rolls without slipping along a straight line
- The valve on a car tyre generates a cycloid as the car moves

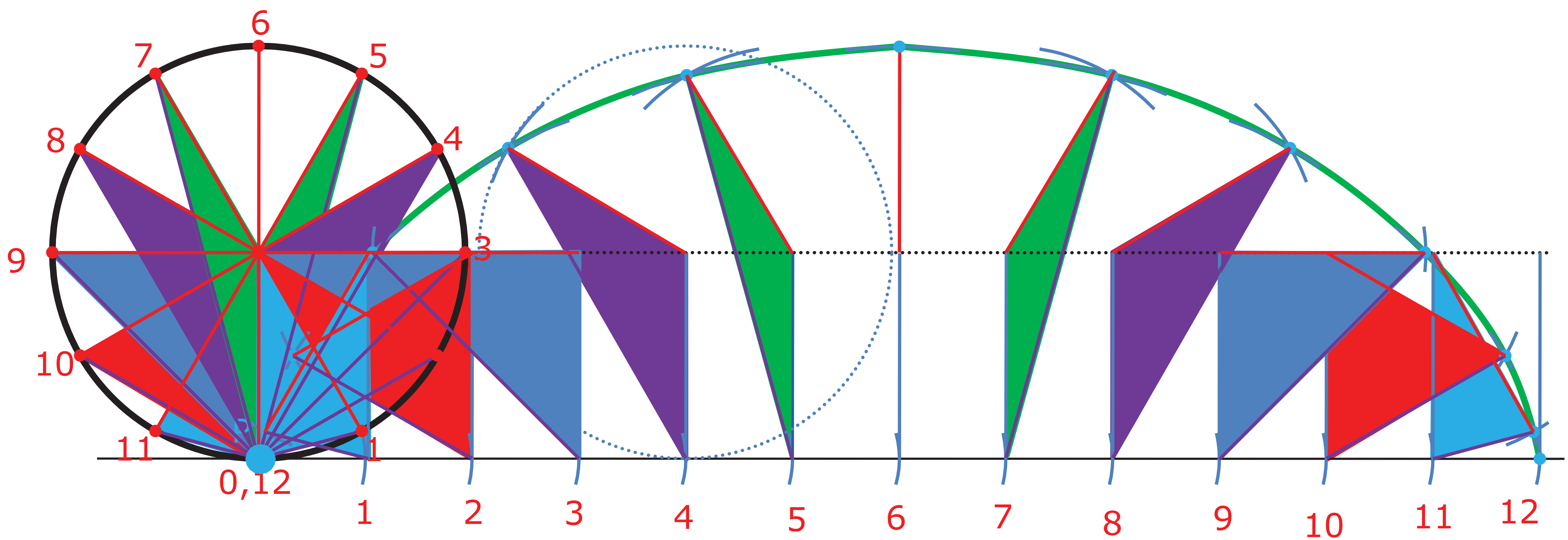
Other cycloid animations

- http://www.edumedia-sciences.com/a325_l2-cycloid.html

Draw a cycloid given the circle, the base line and the point on the circumference

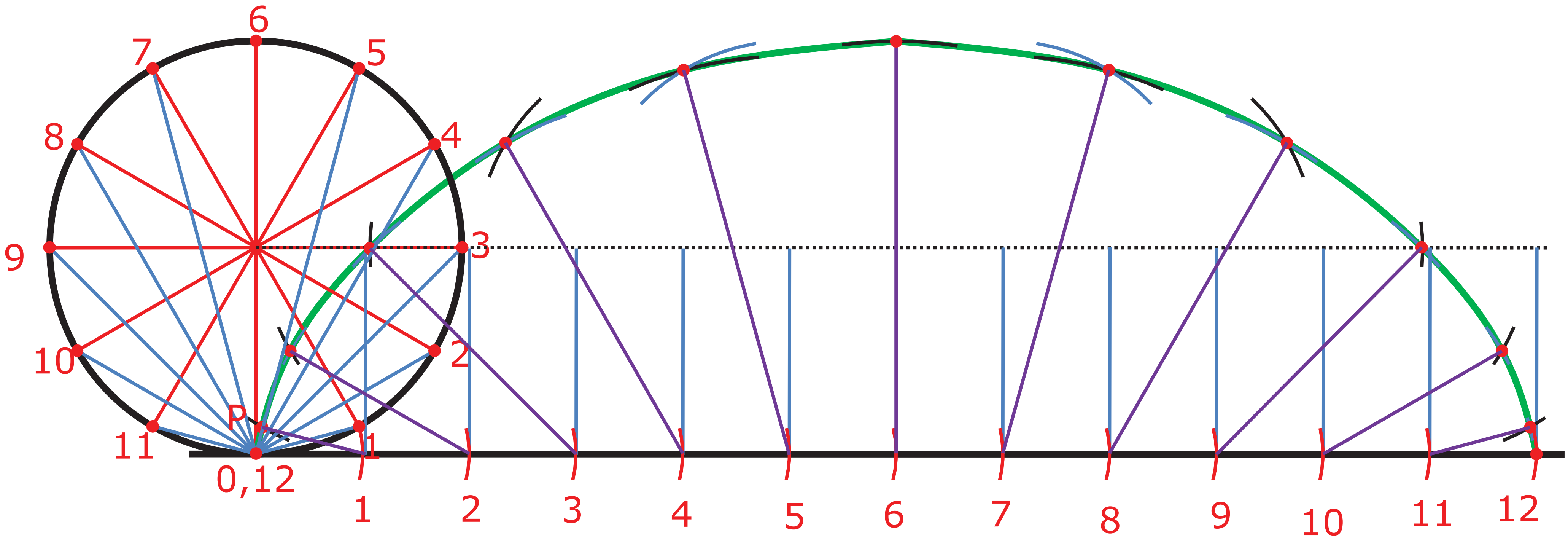


Triangulation Method



The cycloid is the locus of a point on the circumference of a circle which rolls without slipping along a straight line

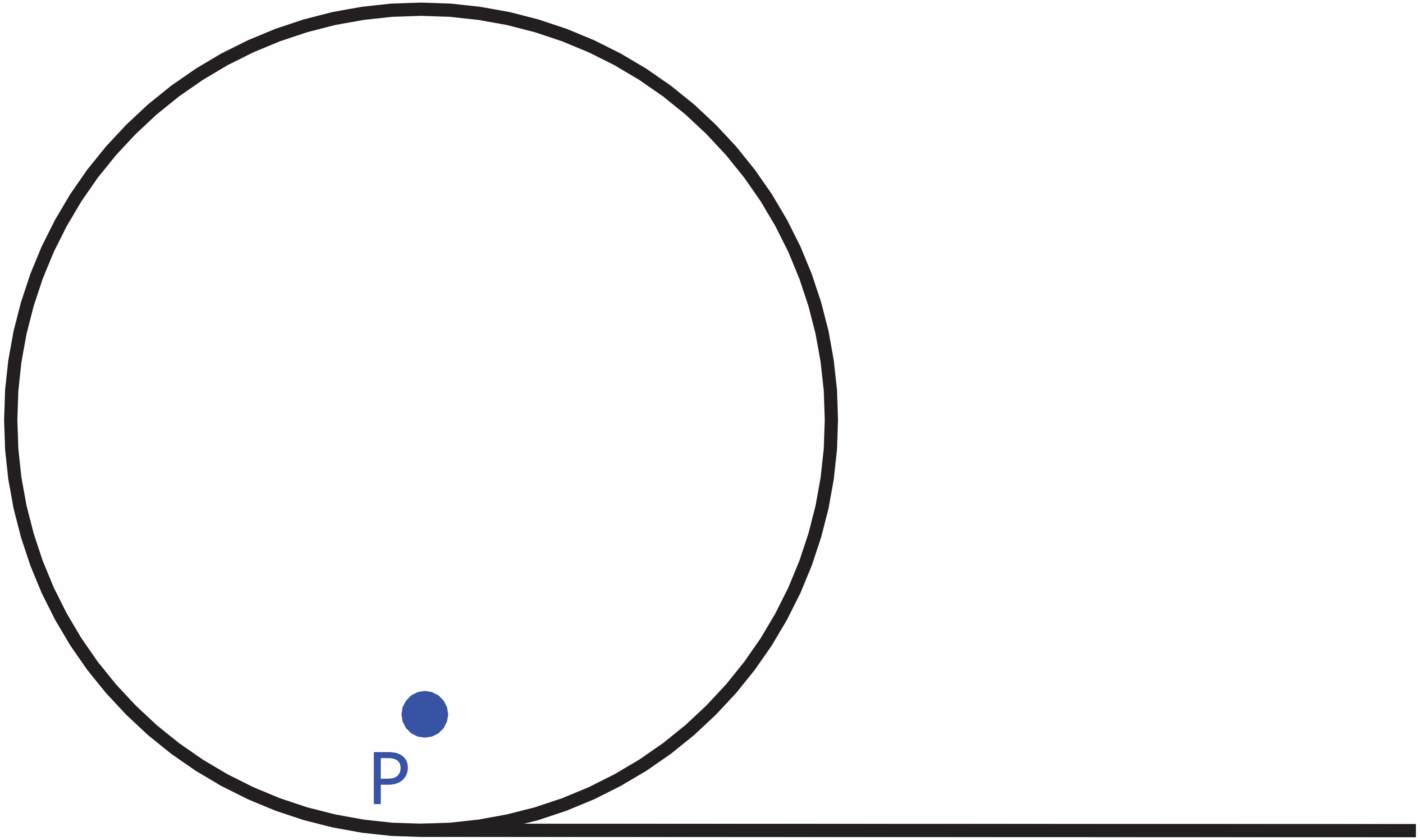
Triangulation Method with lines omitted for clarity

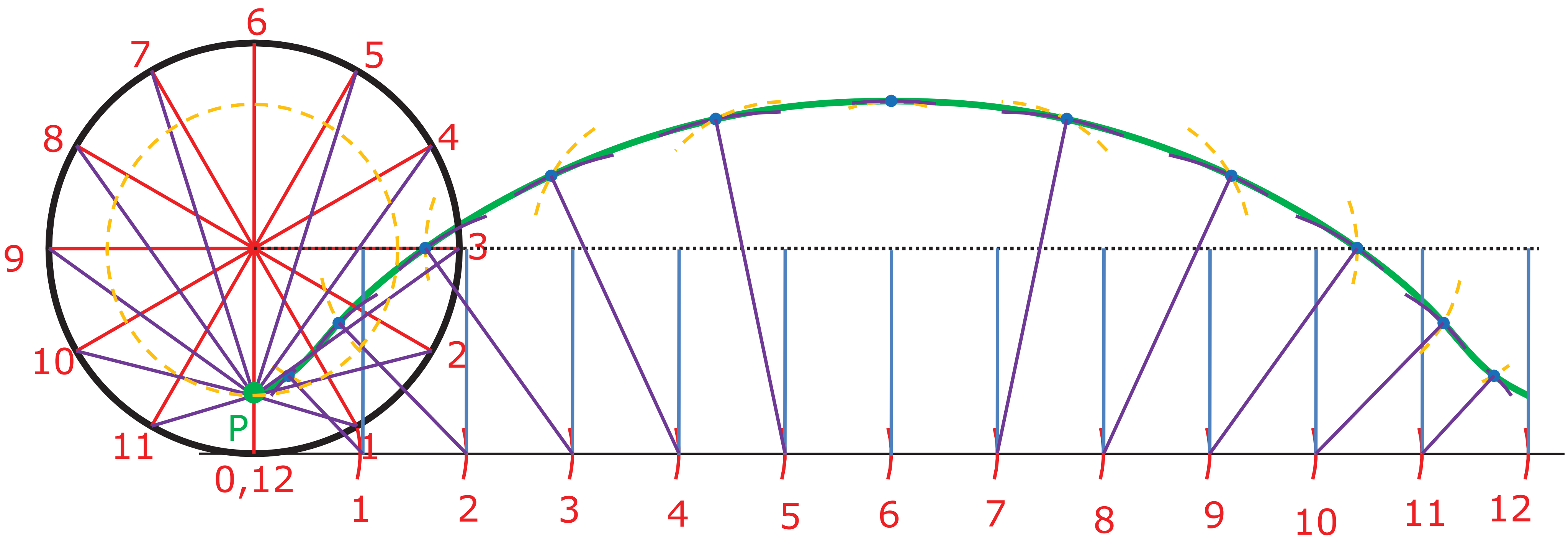


Inferior Trochoid

- An inferior trochoid is the path of a point which lies **inside a circle** which rolls, without slipping, along a straight line
- The reflector on a bicycle generates an inferior trochoid as the bike moves along a flat surface

Draw an inferior trochoid given the circle, the base line and the point P inside the circumference

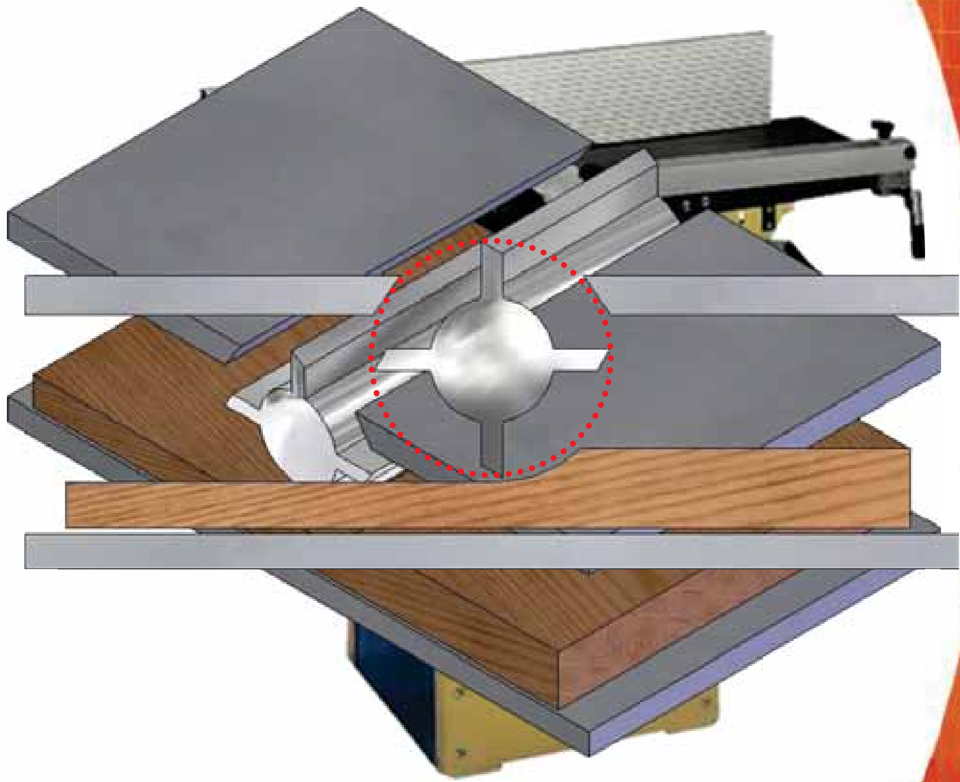




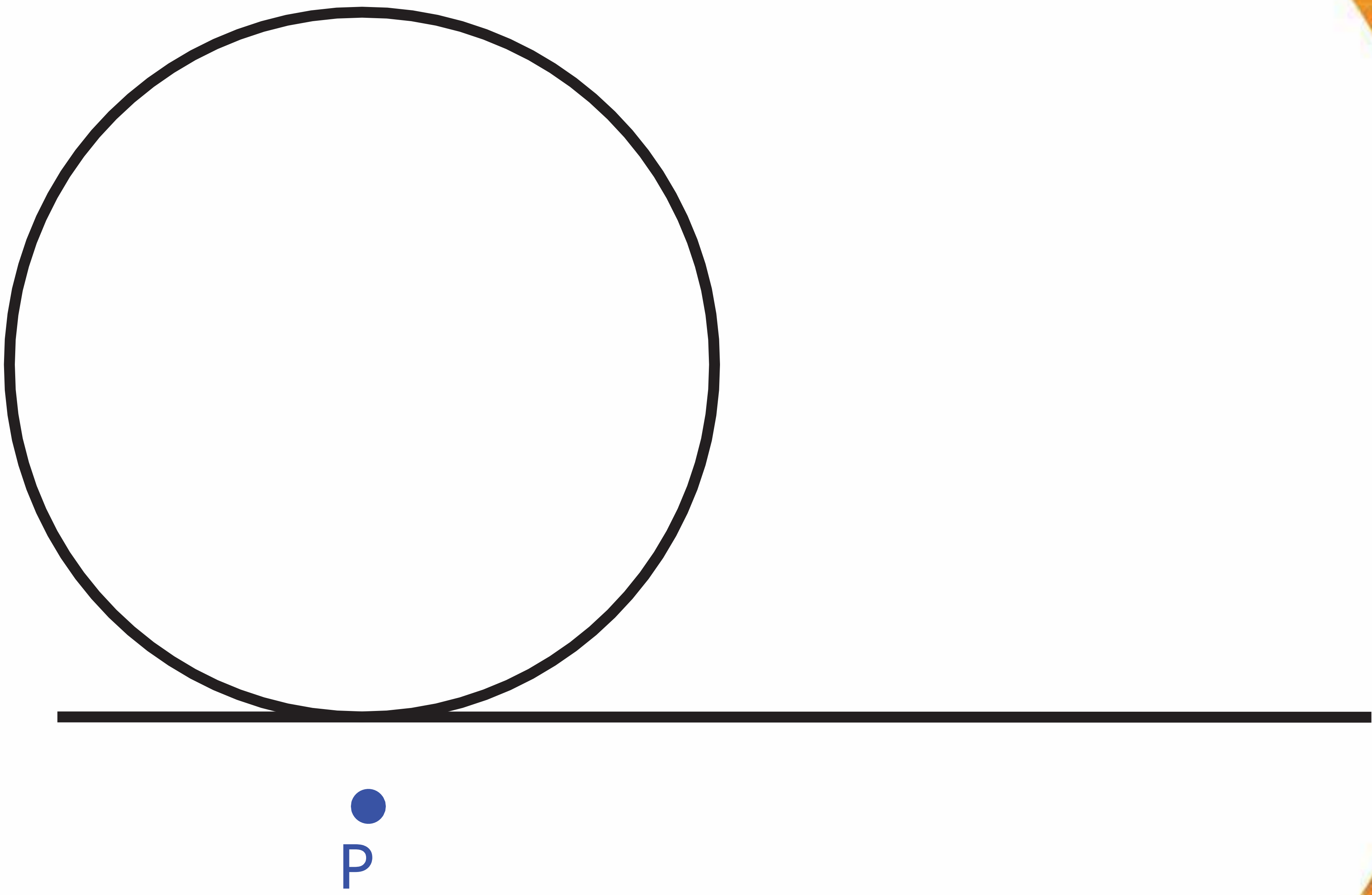
An inferior trochoid is the path of a point which lies inside a circle, which rolls, without slipping along a straight line.

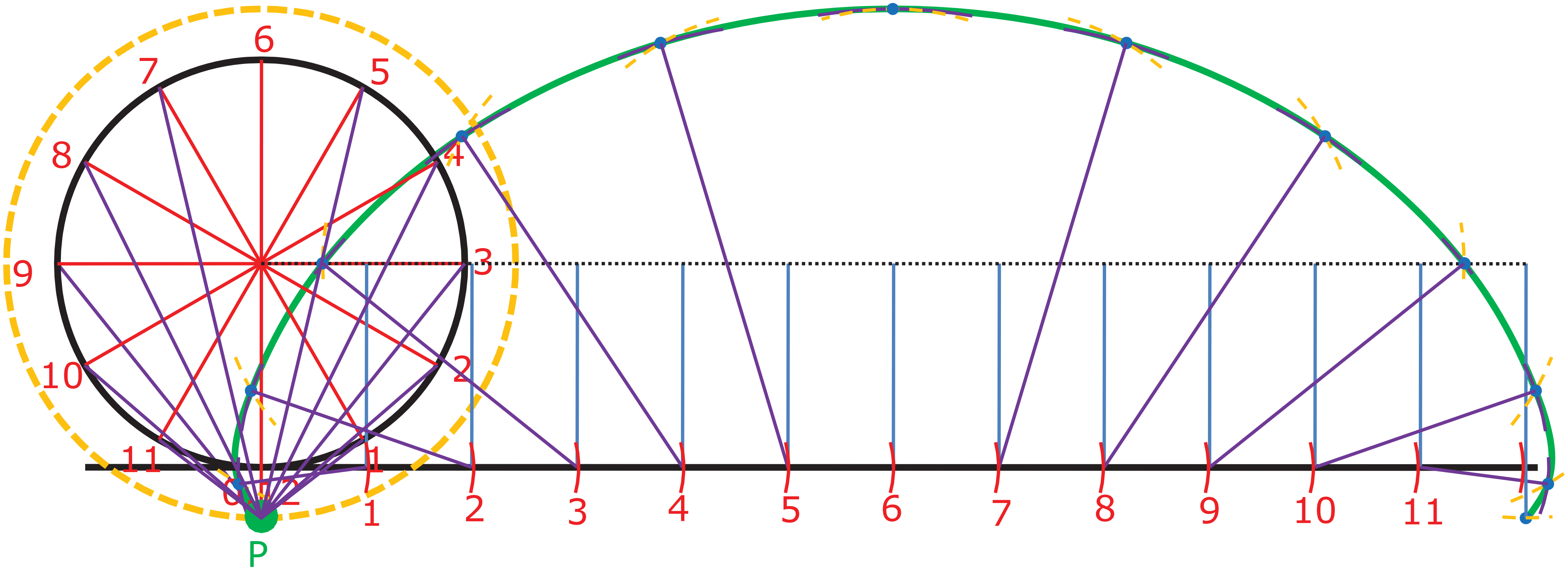
Superior Trochoid

- A superior trochoid is the path of a point which lies **outside a circle** which rolls, without slipping, along a straight line
- Timber moving against the cutter knife of a planer thicknesser generates a superior trochoid



Draw a superior trochoid given the circle, the base line and the point P outside the circumference



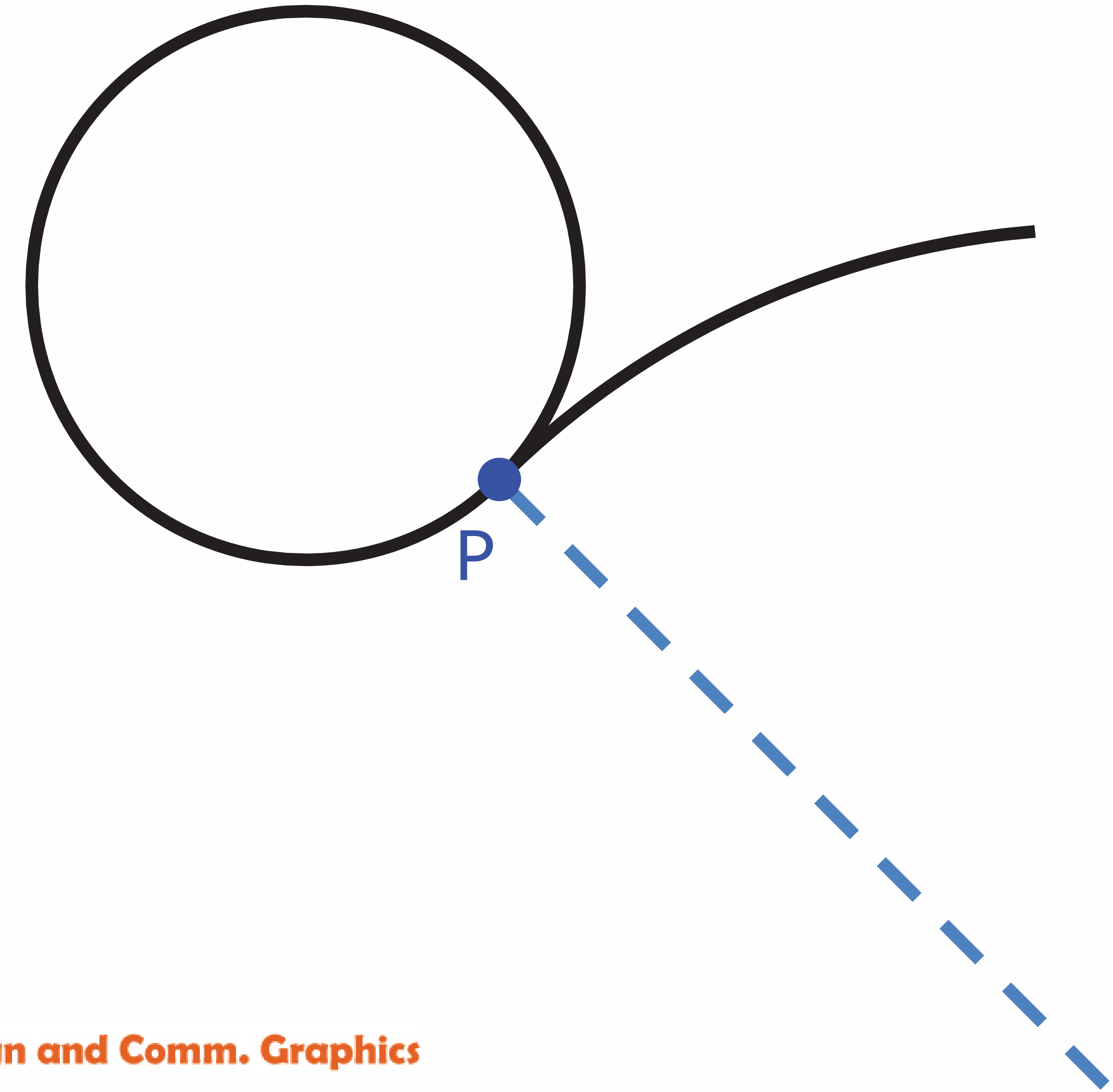


A superior trochoid is the path of a point which lies inside a circle, which rolls, without slipping around the inside of a fixed circle

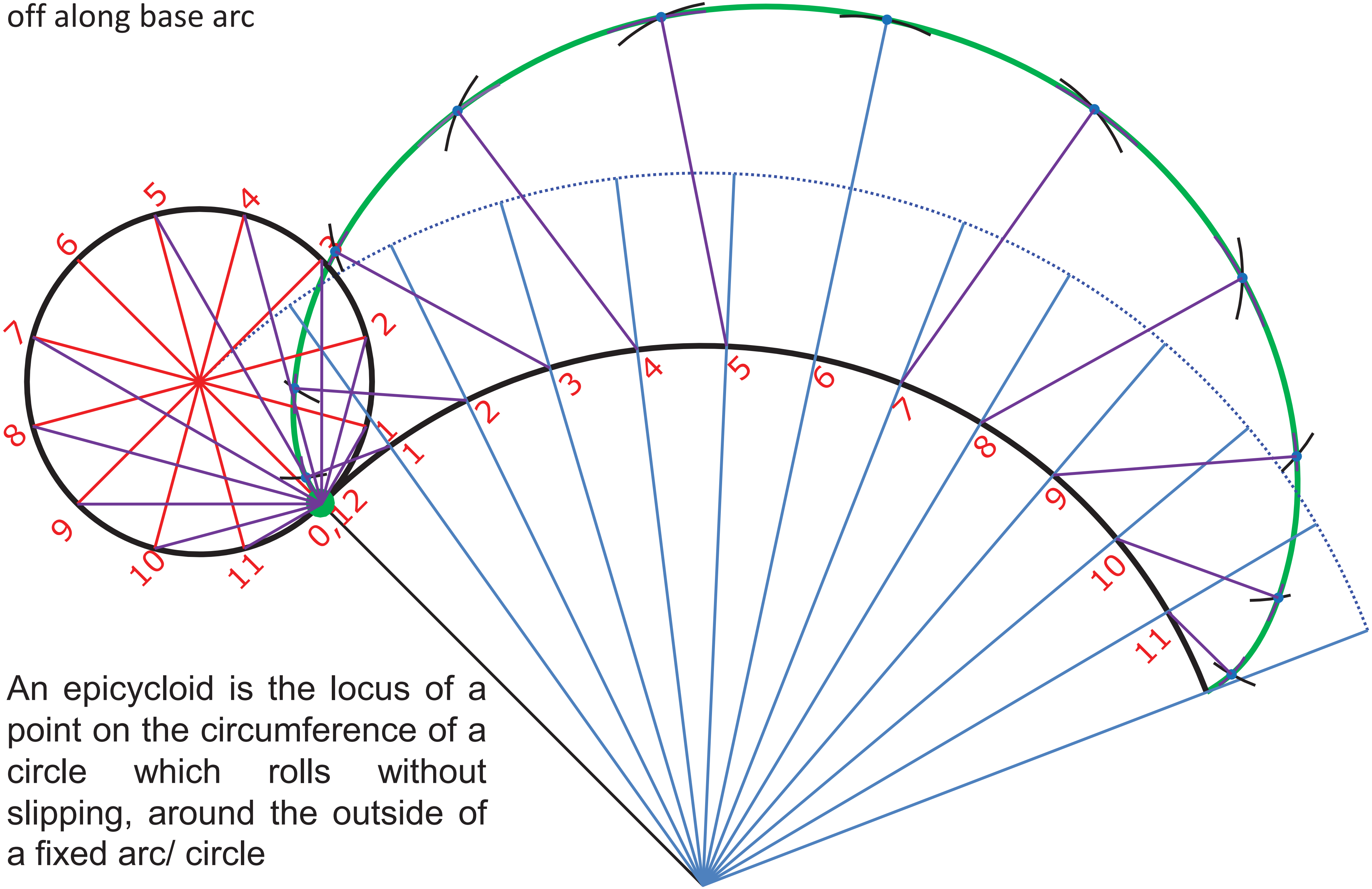
Epicycloid

- An epicycloid is the locus of a point **on the circumference** of a circle which rolls without slipping, around the outside of a fixed arc/ circle
- The applications and principles of a cycloid apply to the epicycloid
- Various types of cycloids are evident in amusement rides

If a circle rolls without slipping round the outside of a fixed circle then a point P on the circumference of the the rolling circle will produce an epicycloid



Segment lengths stepped off along base arc

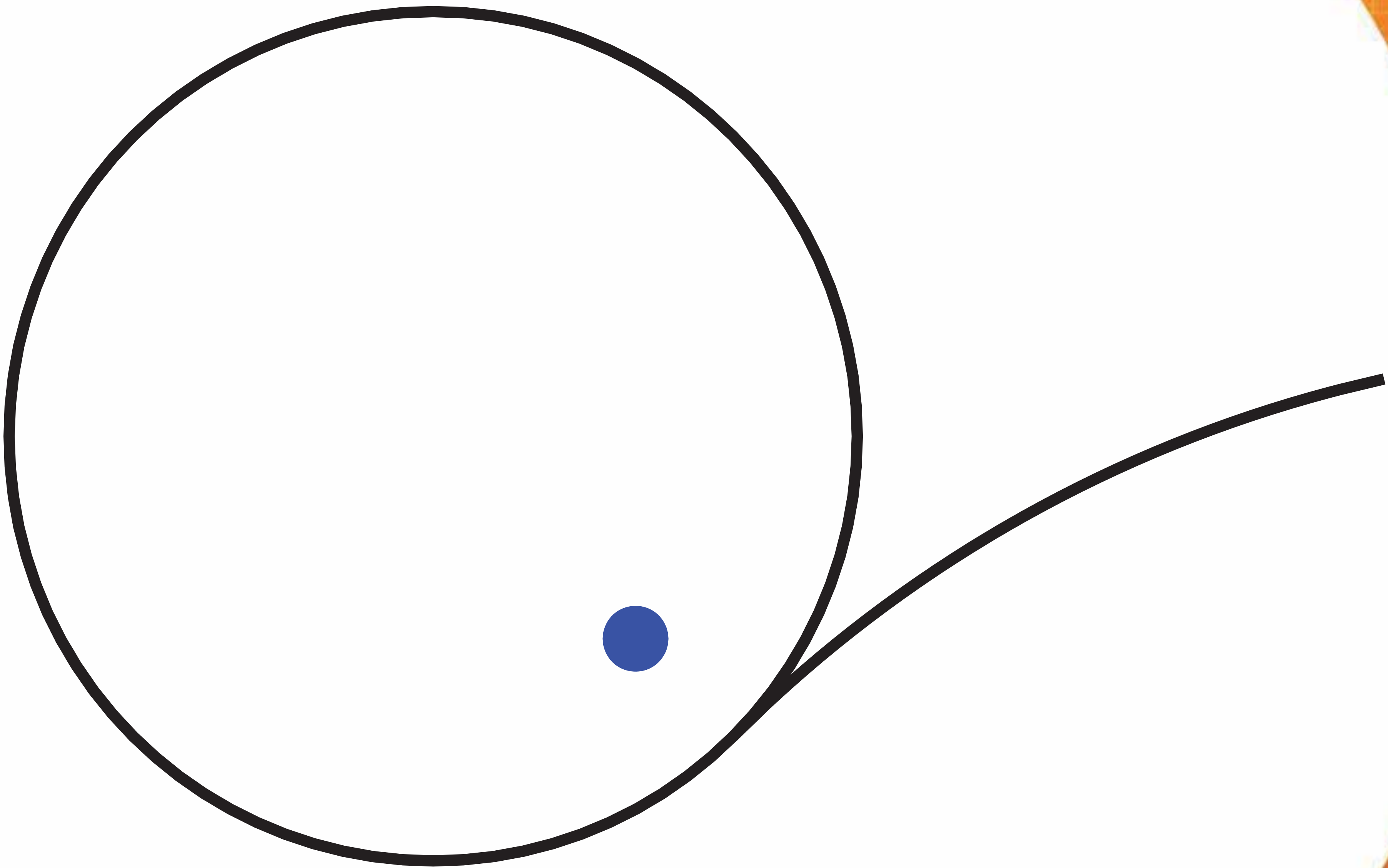


An epicycloid is the locus of a point on the circumference of a circle which rolls without slipping, around the outside of a fixed arc/ circle

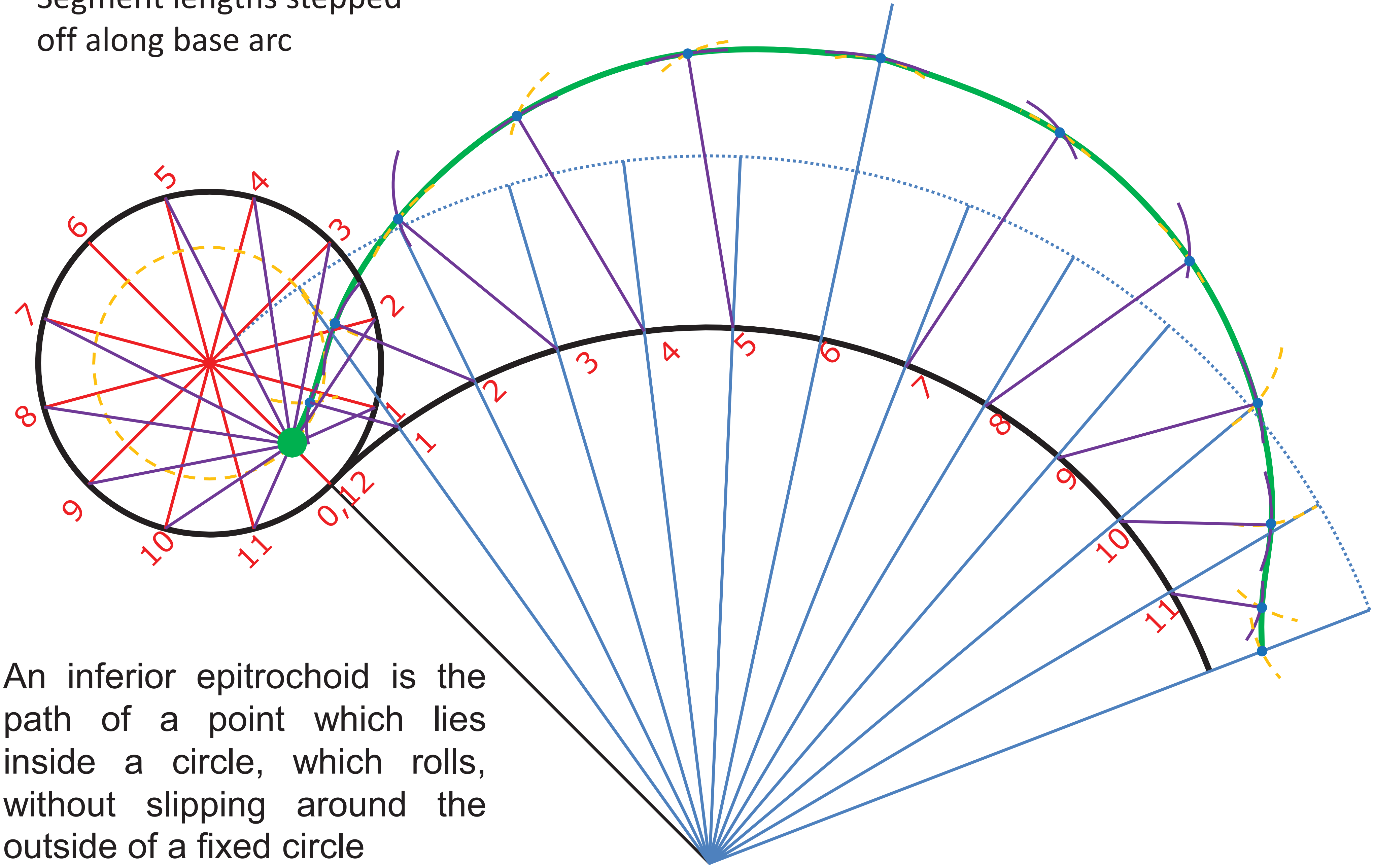
Inferior Epitrochoid

- An inferior epitrochoid is the path of a point which lies **inside a circle** which rolls, without slipping, around the outside of a fixed circle
- The applications and principles of the inferior trochoid apply to the inferior epitrochoid

If a circle rolls without slipping round the inside of a fixed circle then a point P inside the circumference of the the rolling circle will produce an inferior epitrochoid



Segment lengths stepped off along base arc

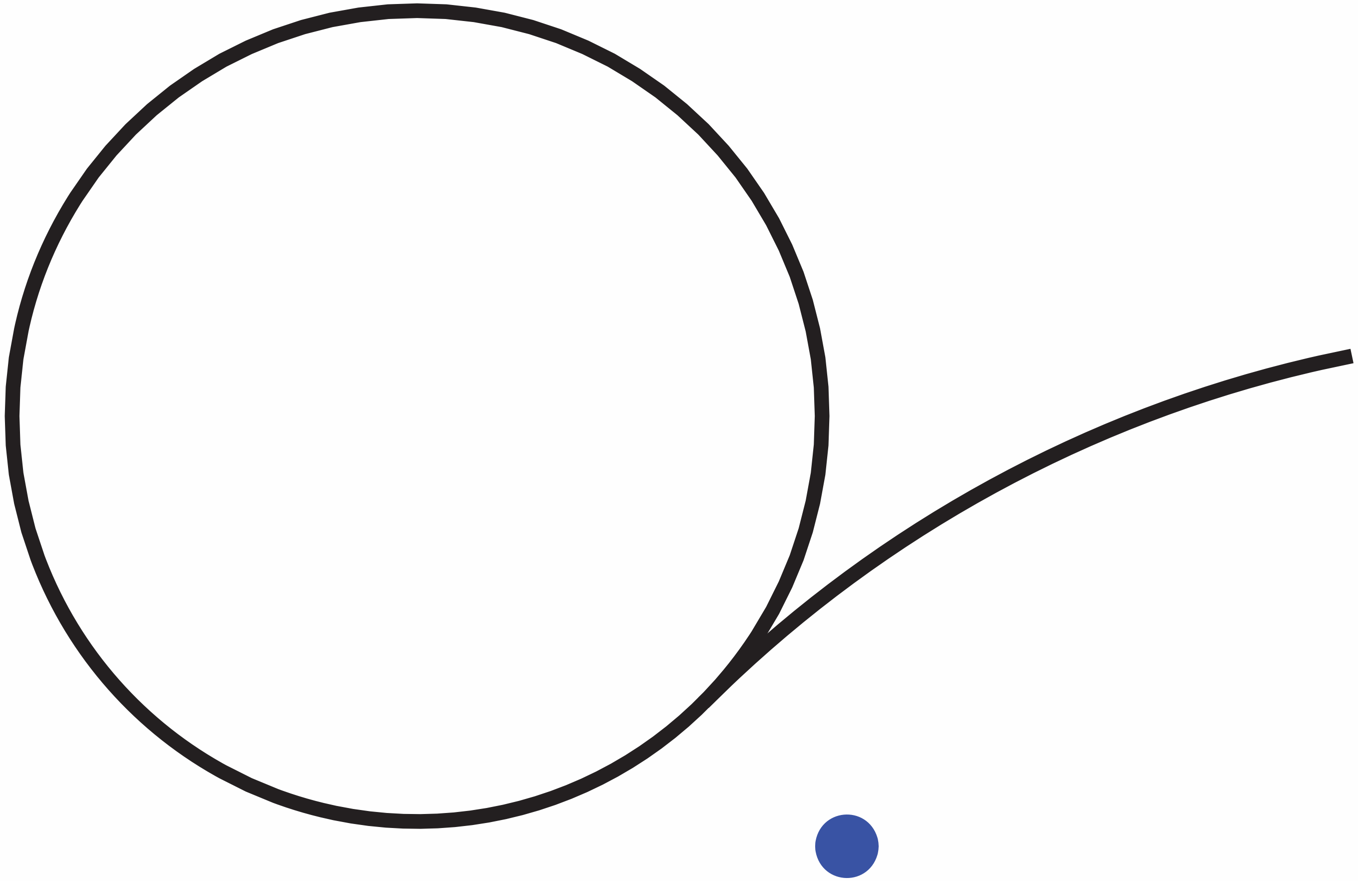


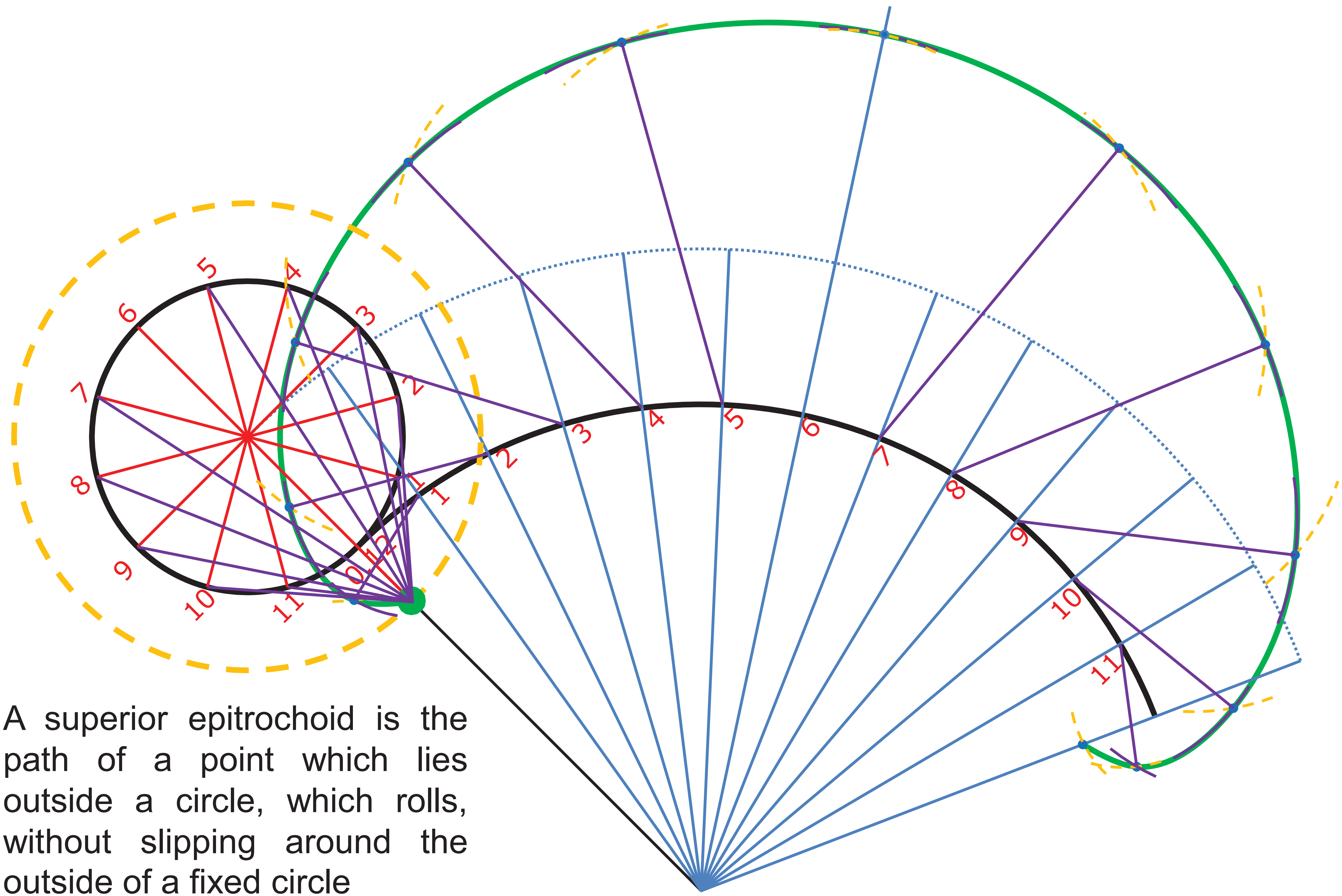
An inferior epitrochoid is the path of a point which lies inside a circle, which rolls, without slipping around the outside of a fixed circle

Superior Epitrochoid

- A superior epitrochoid is the path of a point which lies **outside a circle** which rolls, without slipping, around the outside of a fixed circle
- The applications and principles of the superior trochoid apply to the superior epitrochoid

If a circle rolls without slipping round the inside of a fixed circle then a point P outside the circumference of the the rolling circle will produce a superior epitrochoid



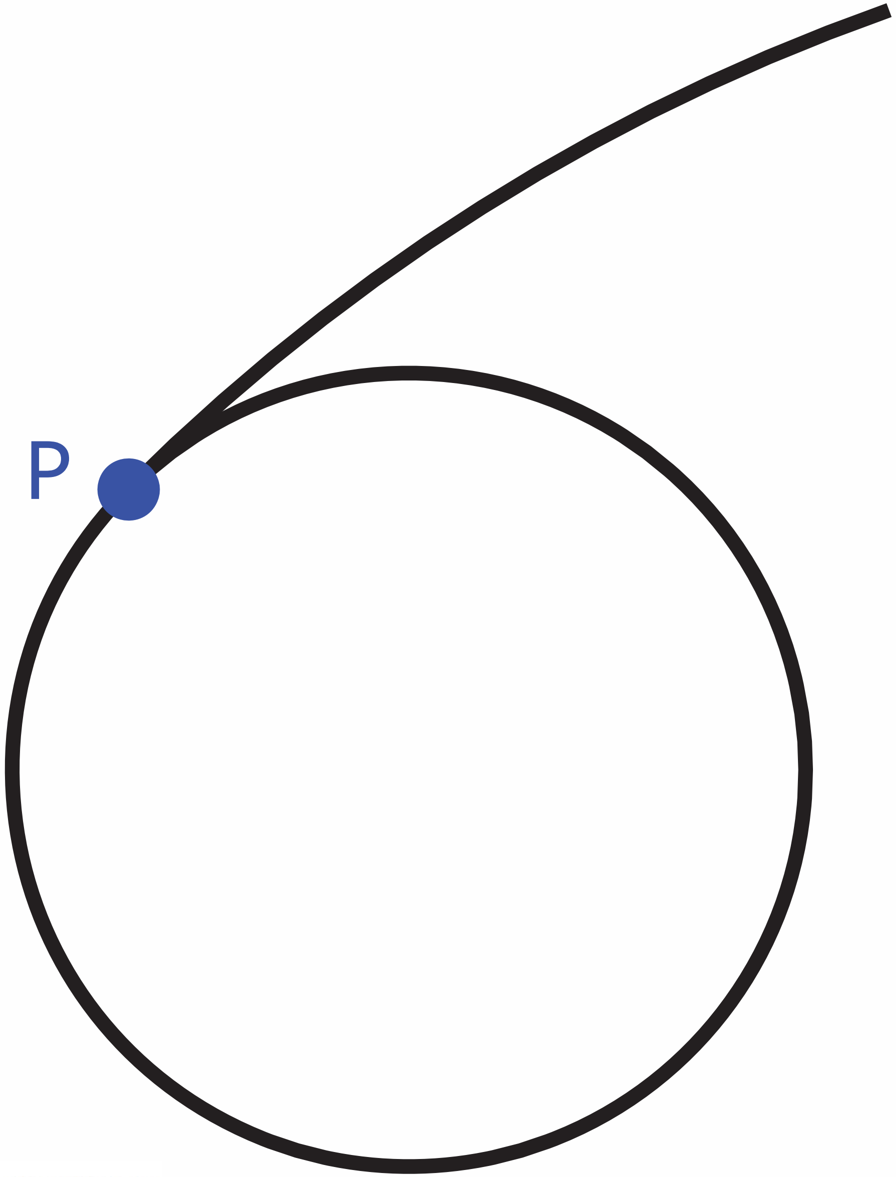


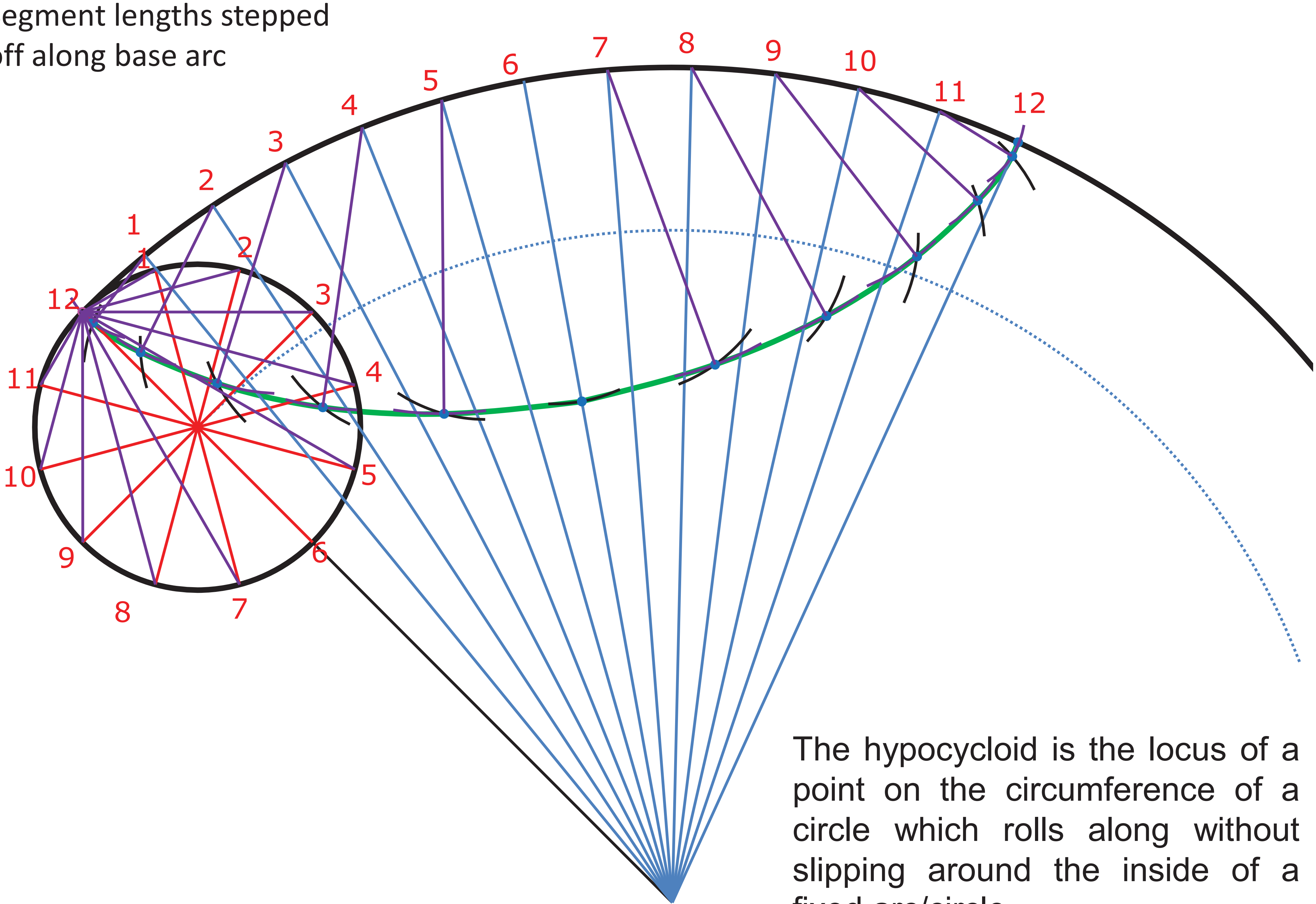
A superior epitrochoid is the path of a point which lies outside a circle, which rolls, without slipping around the outside of a fixed circle

Hypocycloid

- A hypocycloid is the locus of a point on the circumference of a circle which rolls along without slipping around the inside of a fixed arc/circle.
- The applications of the cycloid apply to the hypocycloid

If a circle rolls without slipping round the inside of a fixed circle then a point P on the circumference of the the rolling circle will produce a hypocycloid



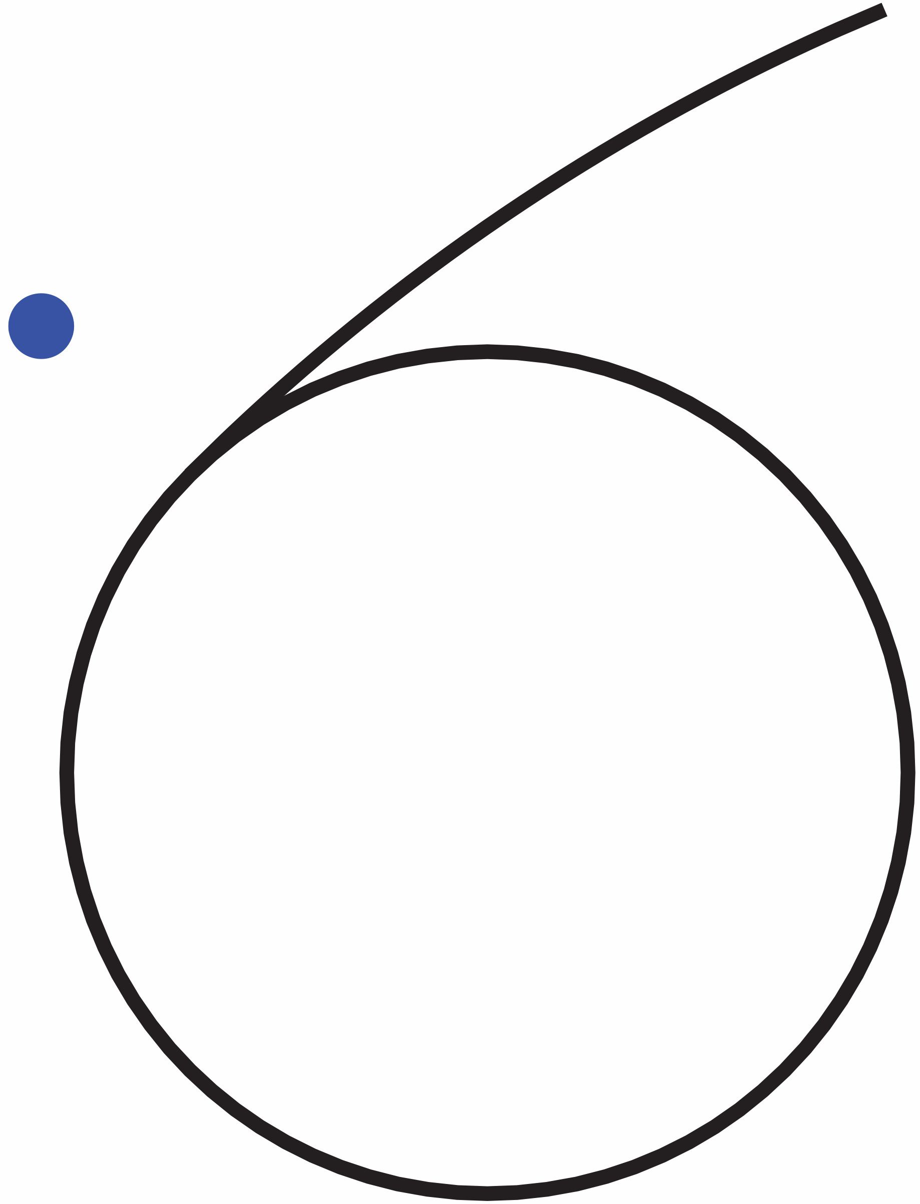


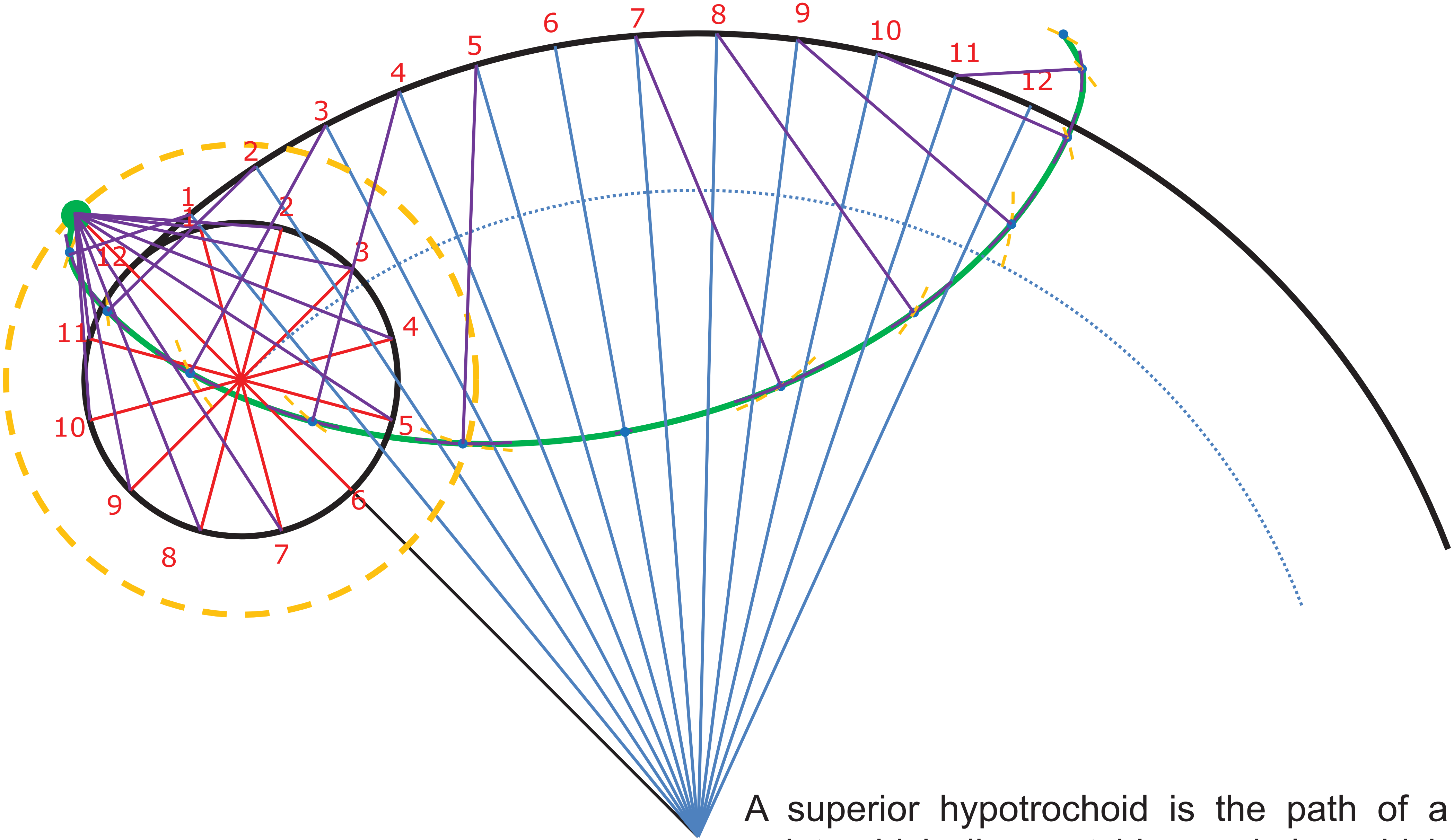
The hypocycloid is the locus of a point on the circumference of a circle which rolls along without slipping around the inside of a fixed arc/circle

Superior Hypotrochoid

- A superior hypotrochoid is the path of a point which lies **outside a circle** which rolls, without slipping, around the inside of a fixed circle
- The applications and principles of the superior trochoid apply to the superior hypotrochoid

If a circle rolls without slipping round the inside of a fixed circle then a point P outside the circumference of the the rolling circle will produce a superior hypotrochoid



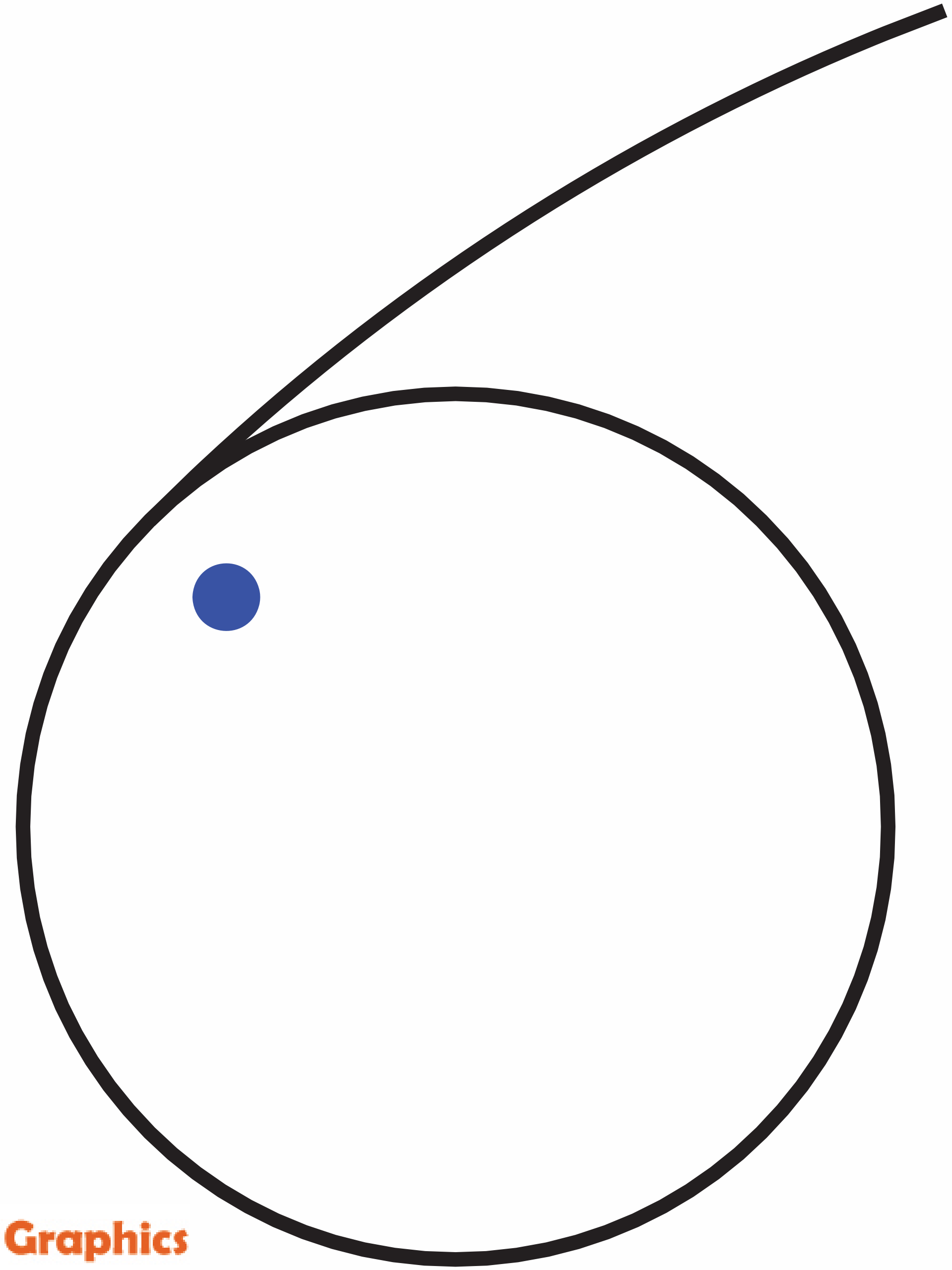


A superior hypotrochoid is the path of a point which lies outside a circle, which rolls, without slipping around the inside of a fixed circle

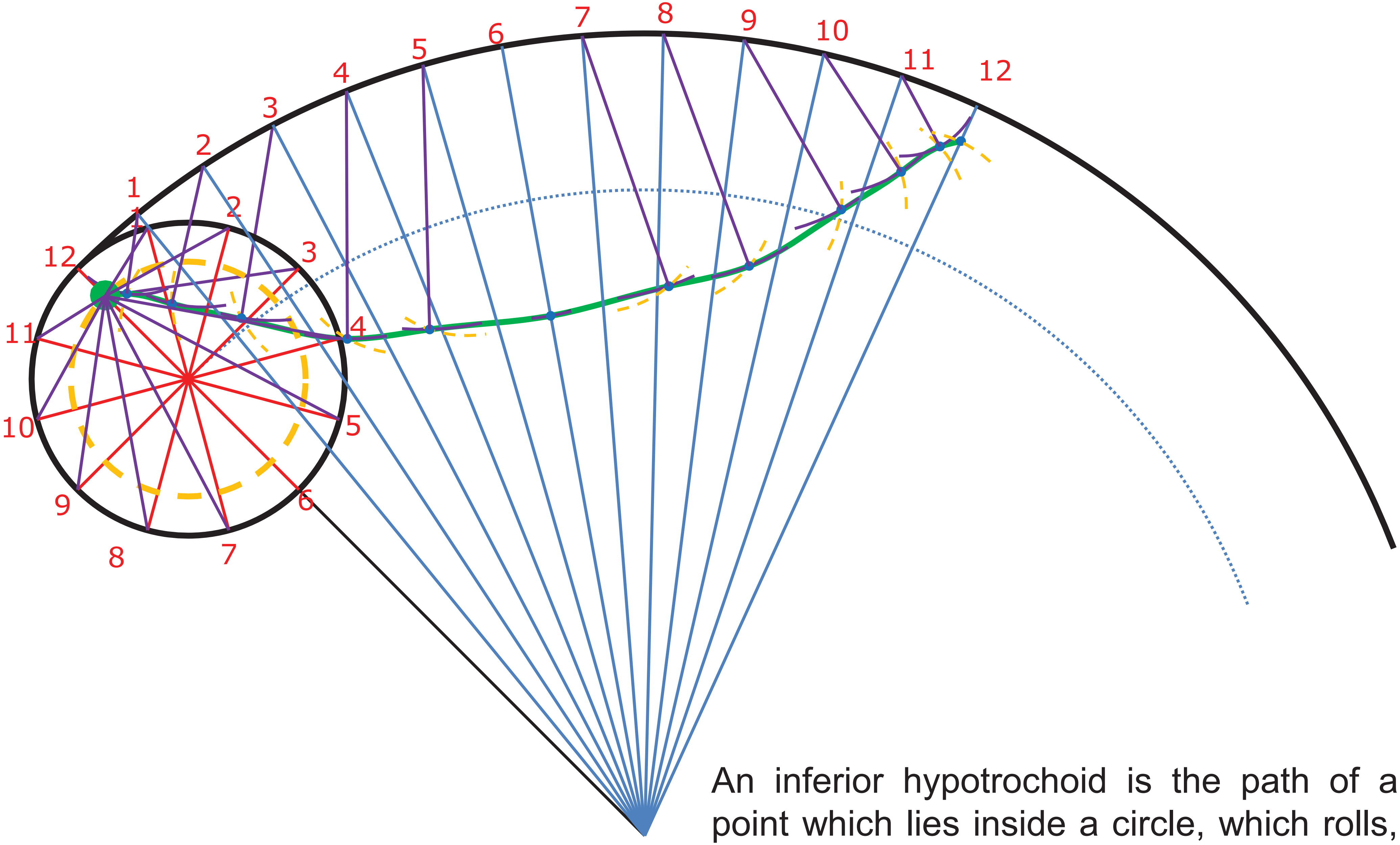
Inferior Hypotrochoid

- An inferior hypotrochoid is the path of a point which lies **inside a circle** which rolls, without slipping, around the inside of a fixed circle
- The applications and principles of the inferior trochoid apply to the inferior hypotrochoid

If a circle rolls without slipping round the inside of a fixed circle then a point P inside the circumference of the the rolling circle will produce an inferior hypotrochoid



Segment lengths stepped off along base arc

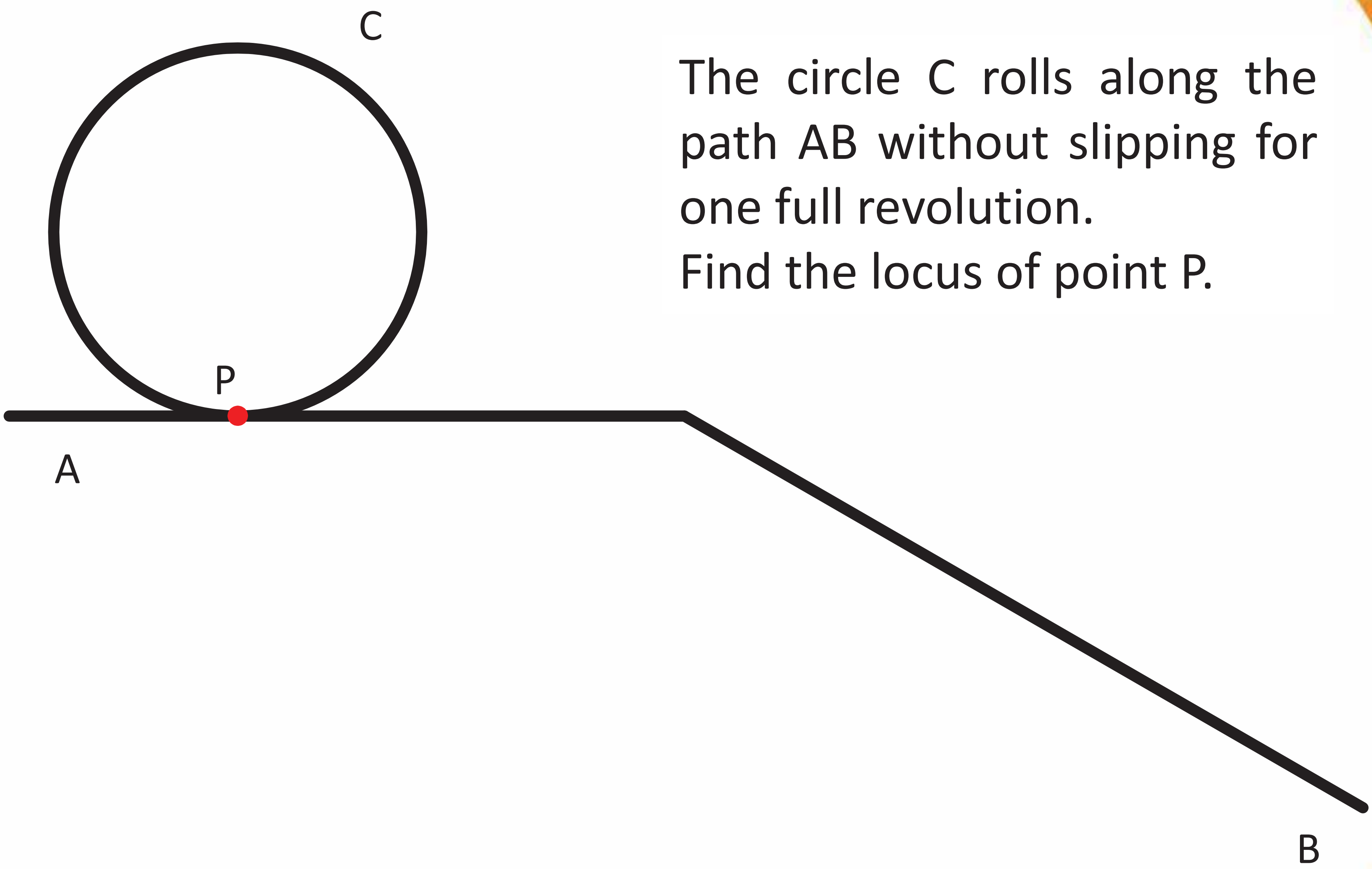


An inferior hypotrochoid is the path of a point which lies inside a circle, which rolls, without slipping around the inside of a fixed circle

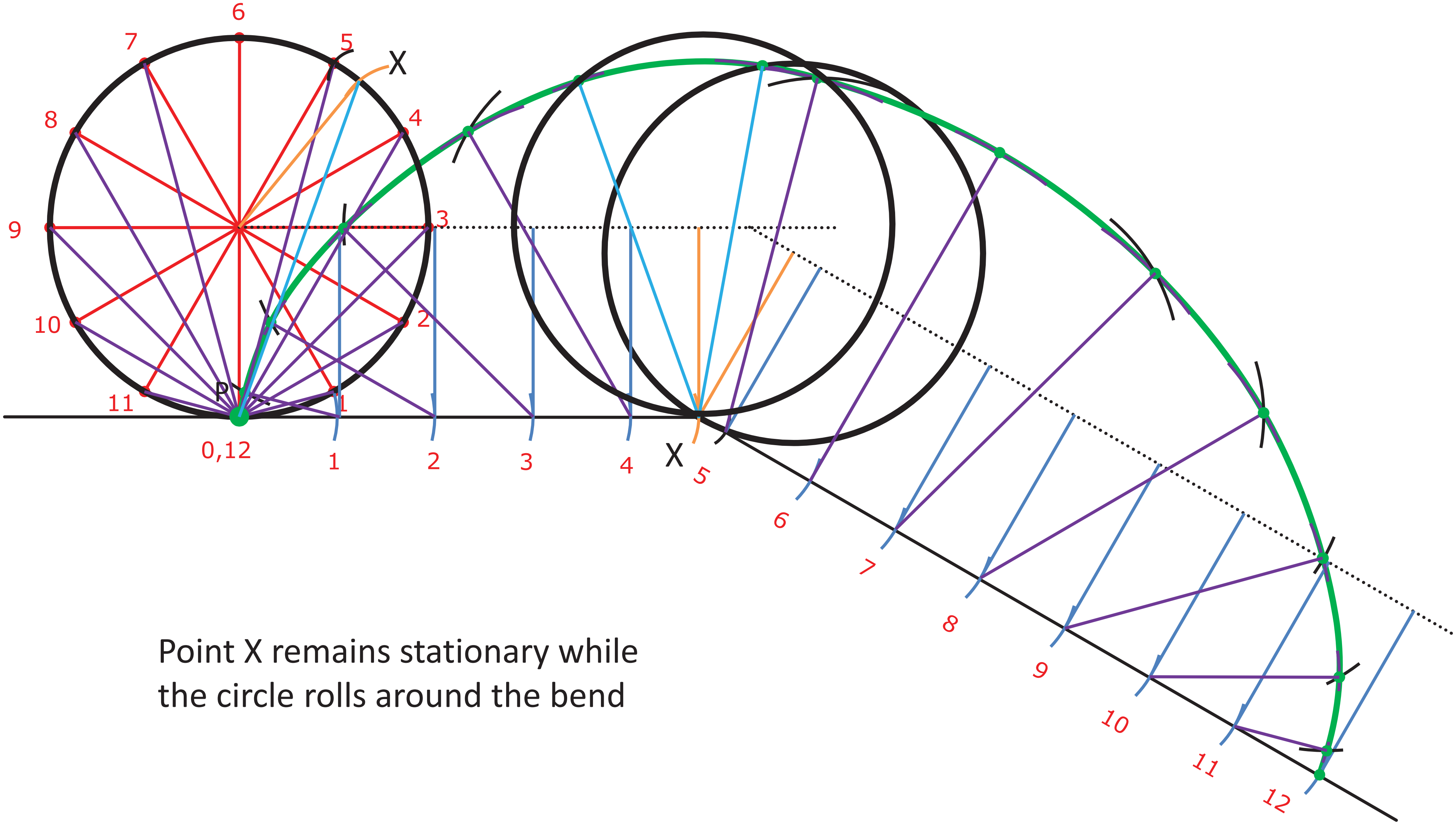
Loci of irregular paths

- The path the object follows can change as the object rolls
- The principle for solving these problems is similar ie. triangulation
- Treat each section of the path as a separate movement
- Any corner has two distinctive loci points

Loci of irregular paths



The circle C rolls along the path AB without slipping for one full revolution.
Find the locus of point P.

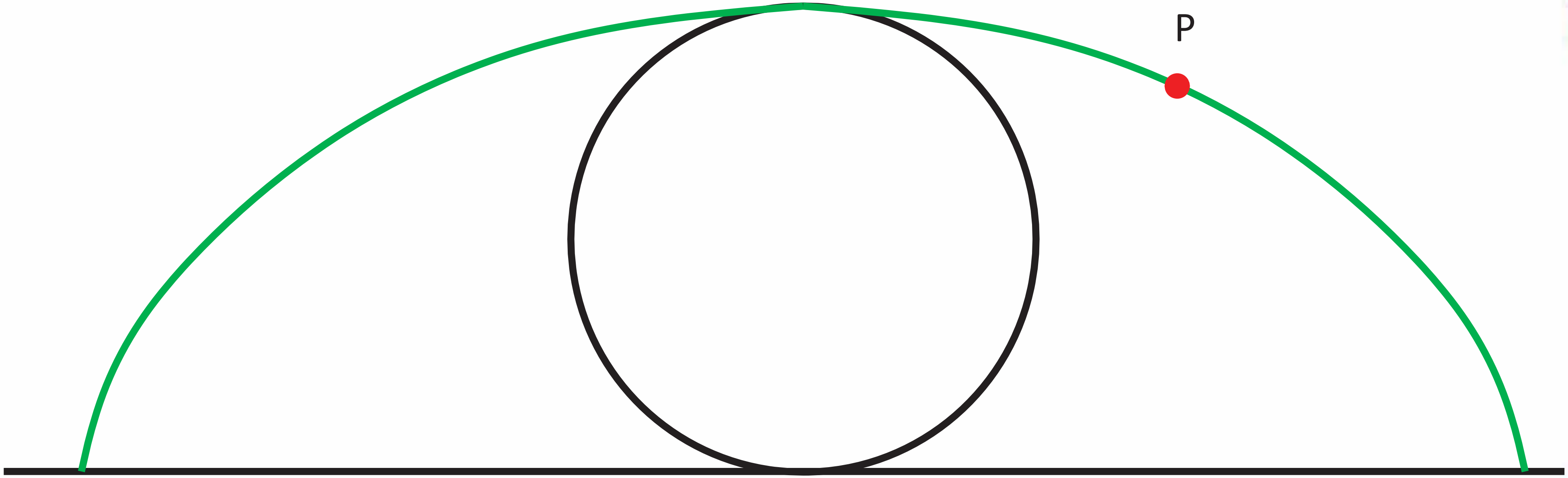


Point X remains stationary while
the circle rolls around the bend

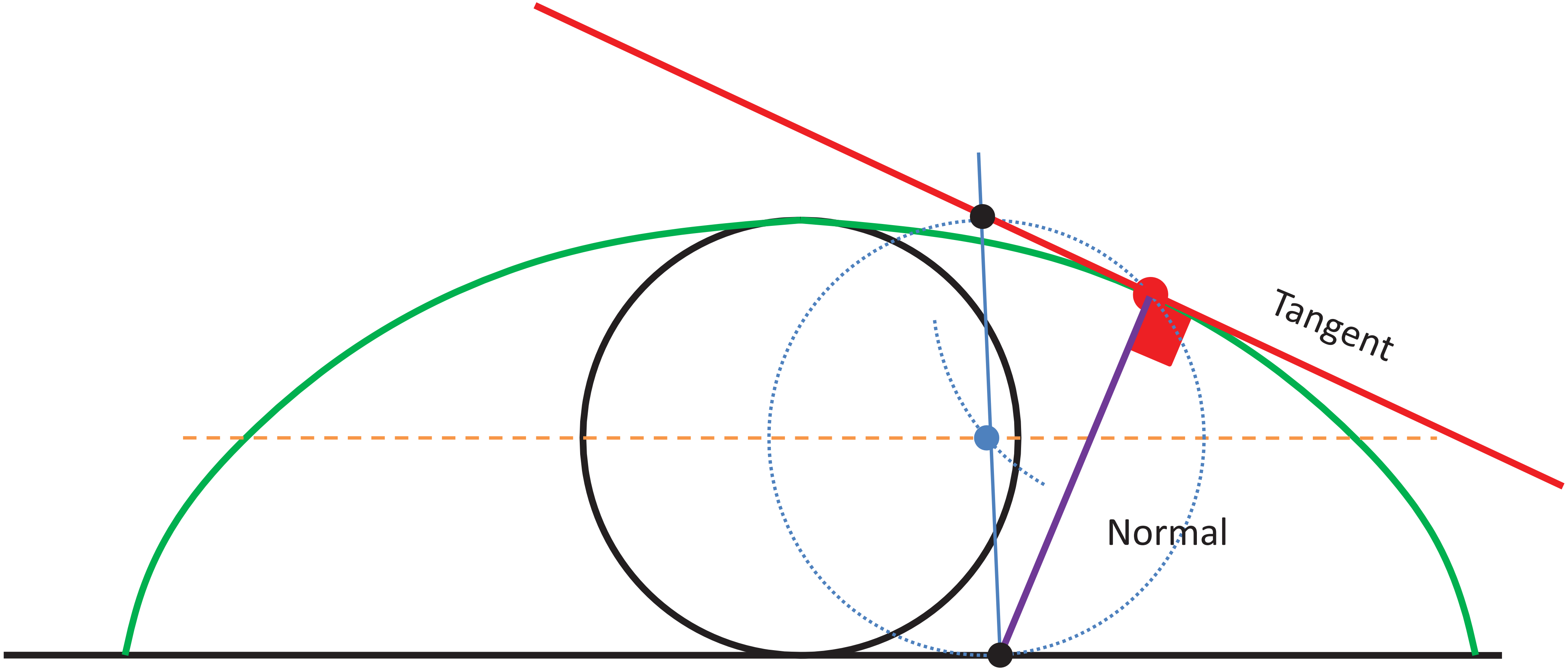
TECHNOLOGY SUBJECTS SUPPORT SERVICE

TANGENTS TO LOCI

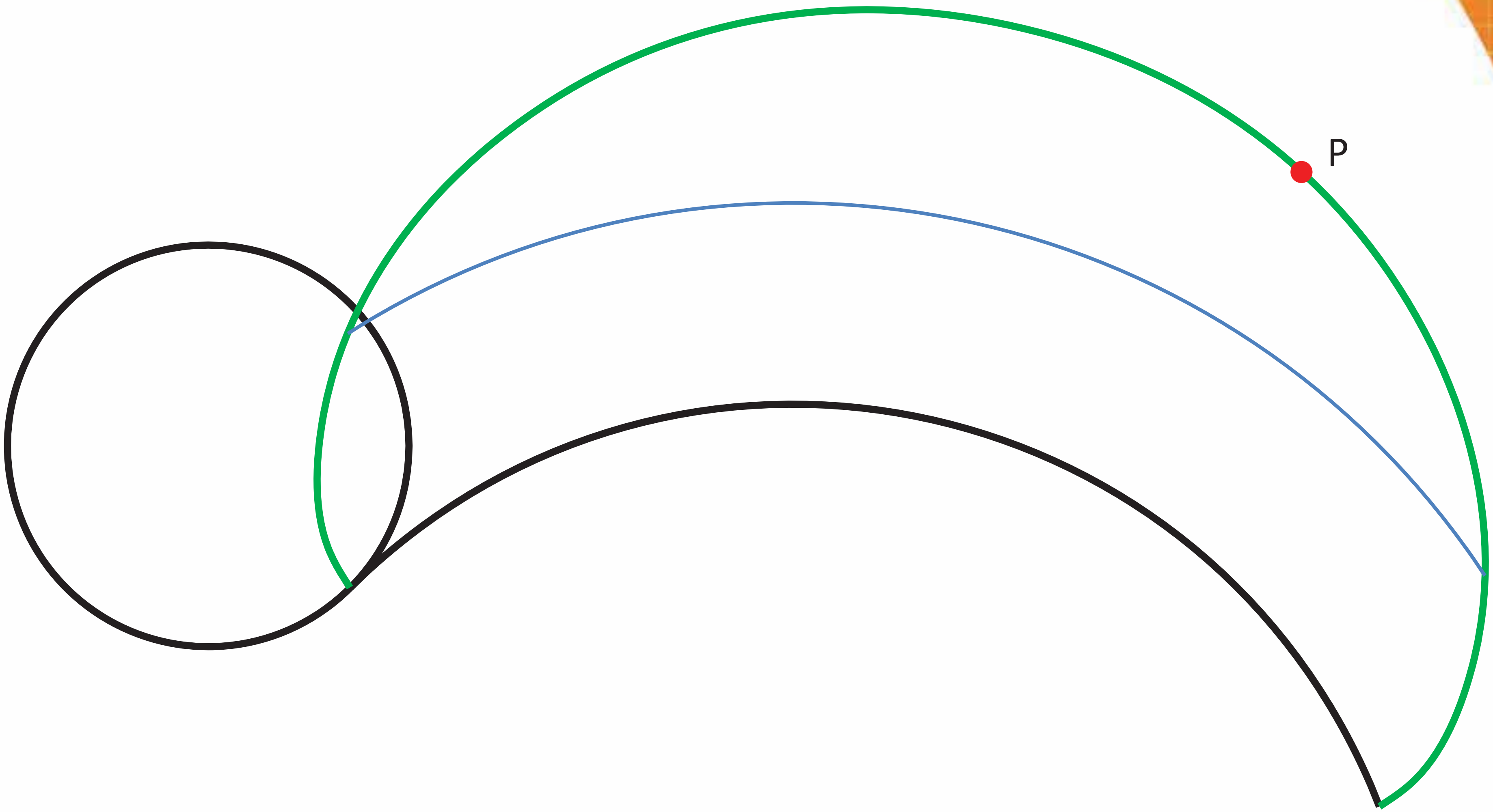
Tangent to a cycloid at a point P

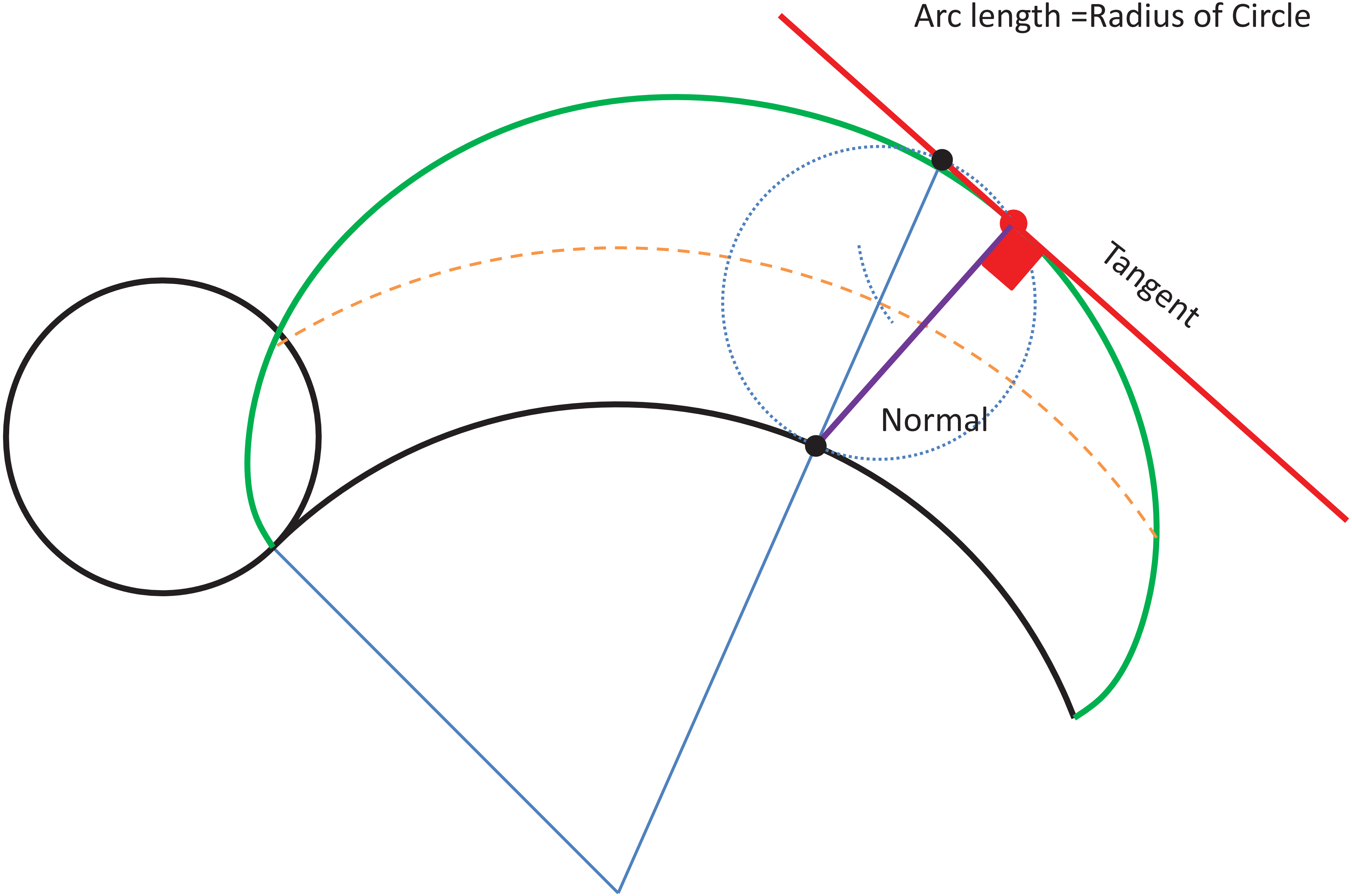


Arc length =Radius of Circle

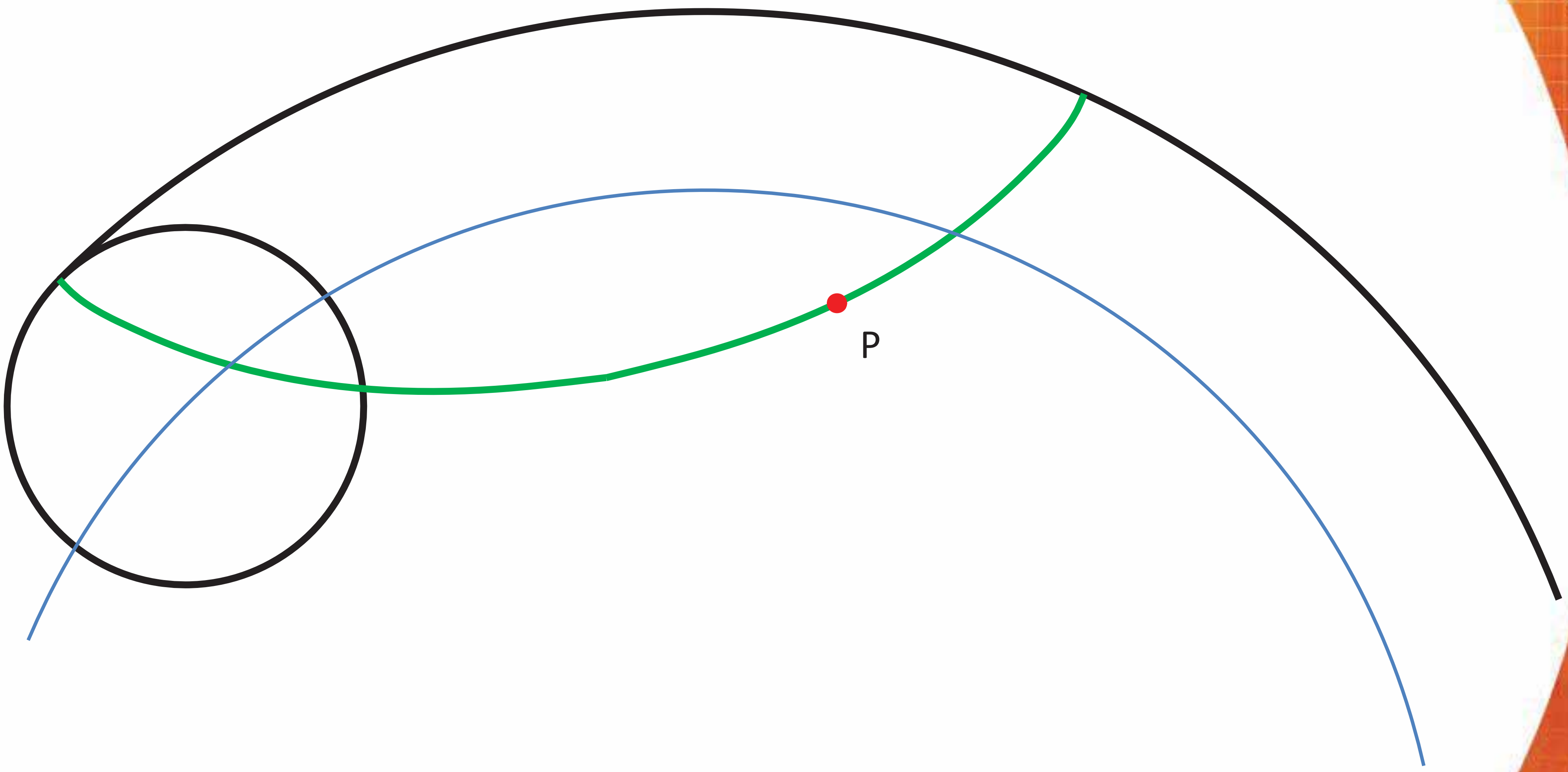


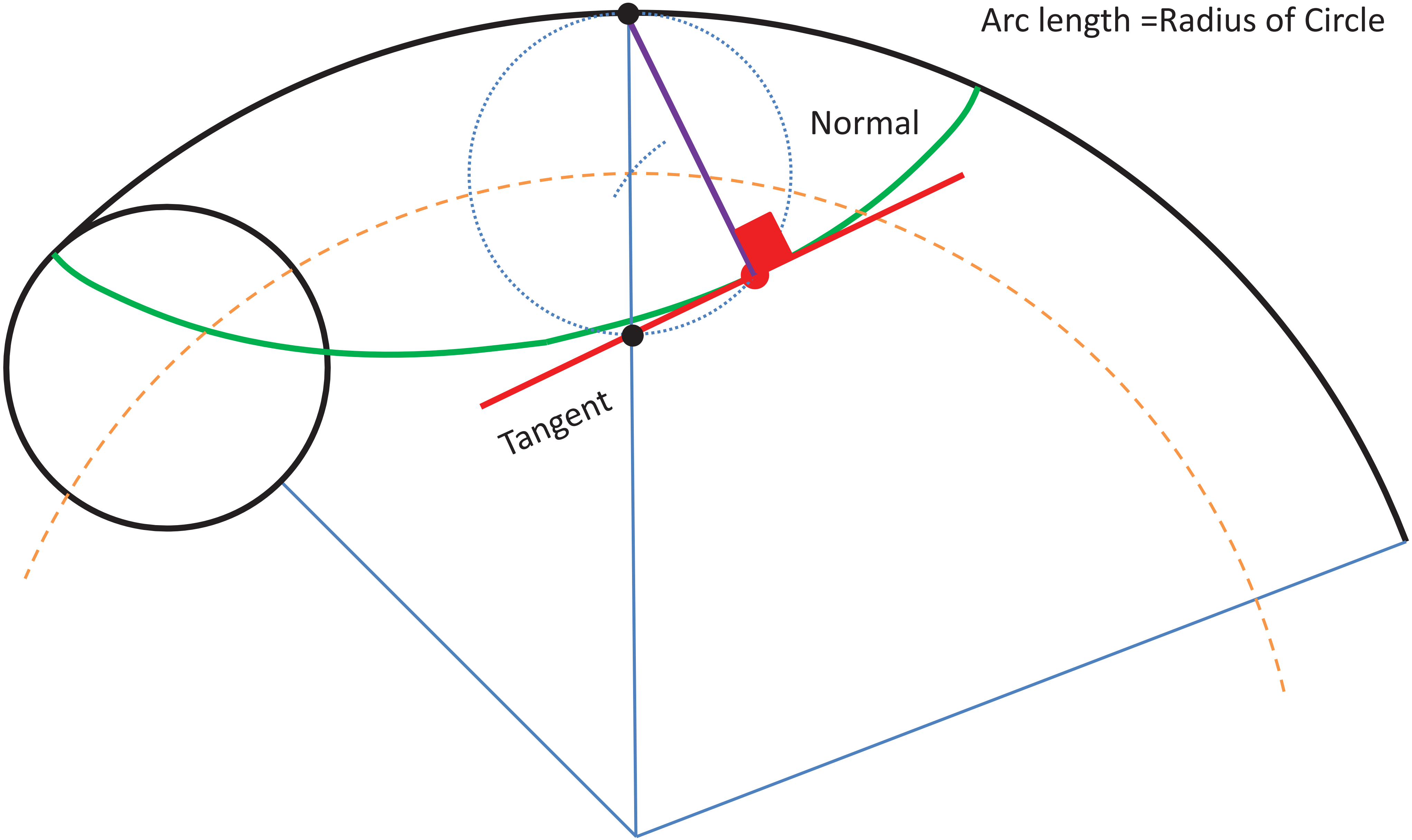
Tangent to an epicycloid at a point P




Arc length =Radius of Circle
Tangent
Normal

Tangent to the hypocycloid at a point P




Arc length =Radius of Circle
Normal
Tangent

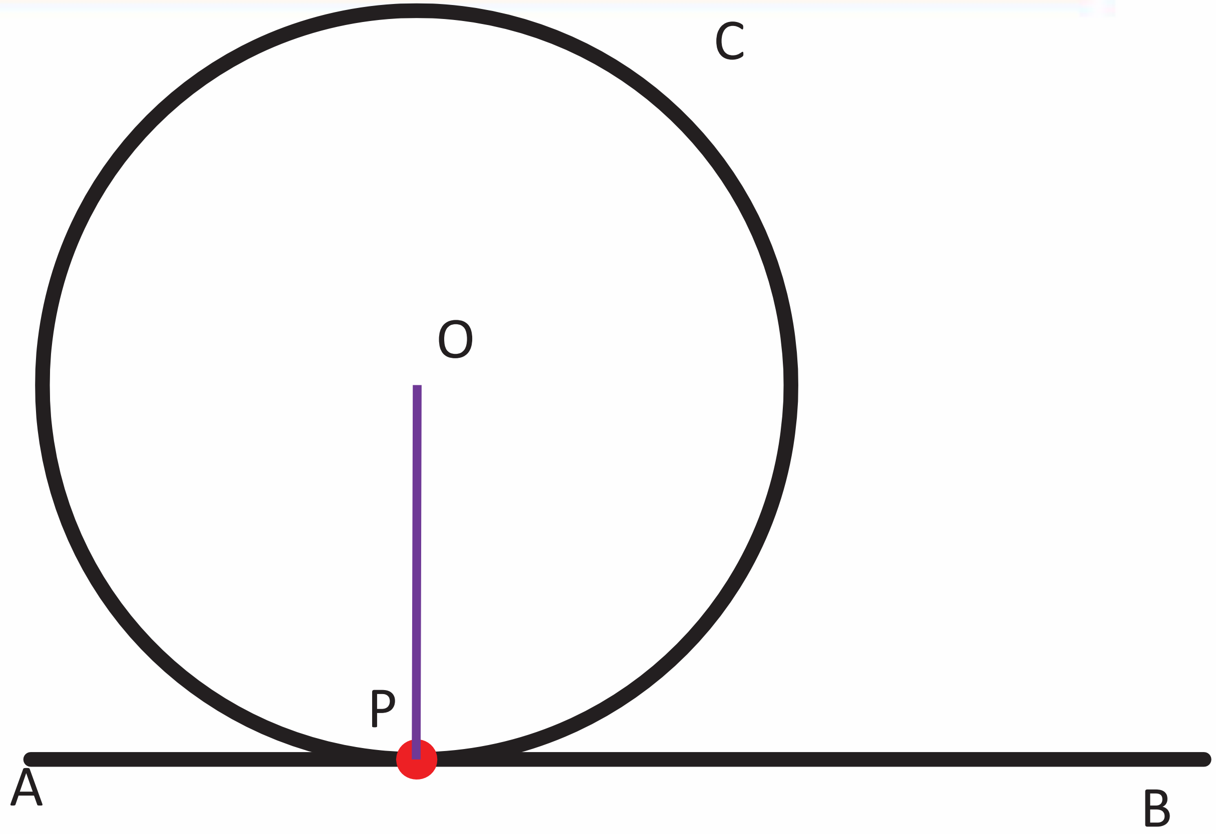
Further Information on Loci

- http://curvebank.calstatela.edu/cycloidmaple/cycloid.htm

TECHNOLOGY
SUBJECTS
SUPPORT
SERVICE

COMBINED MOVEMENT

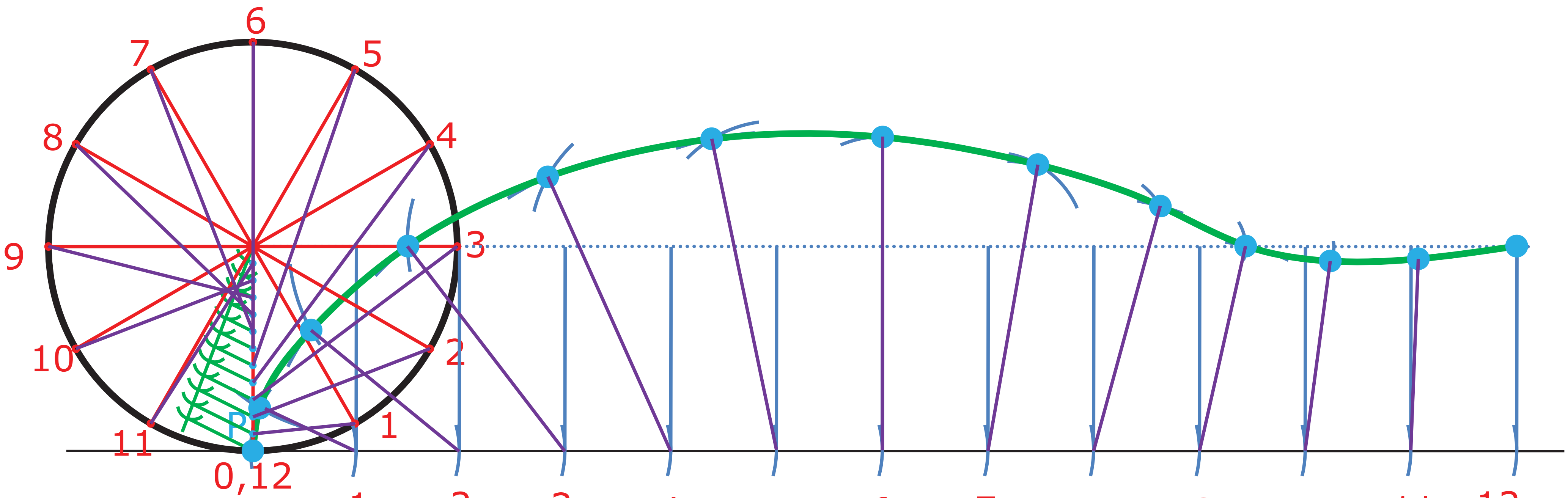
Combined Movement



Shown is a circle C, which rolls clockwise along the line AB for one full revolution.

Also shown is the initial position of a point P on the circle. During the rolling of the circle, the point P moves along the radial line PO until it reaches O.

Draw the locus of P for the combined movement.


6
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P
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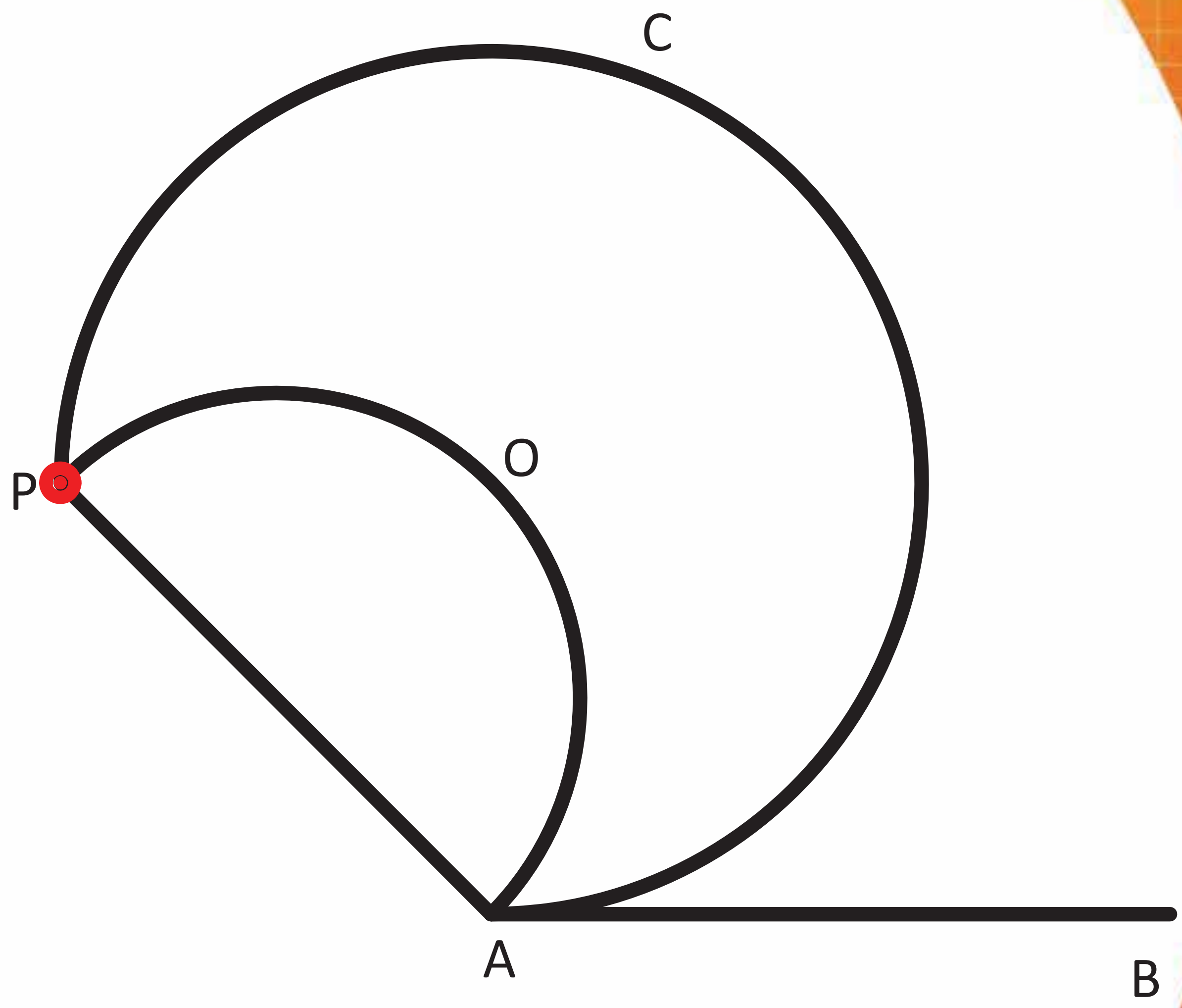
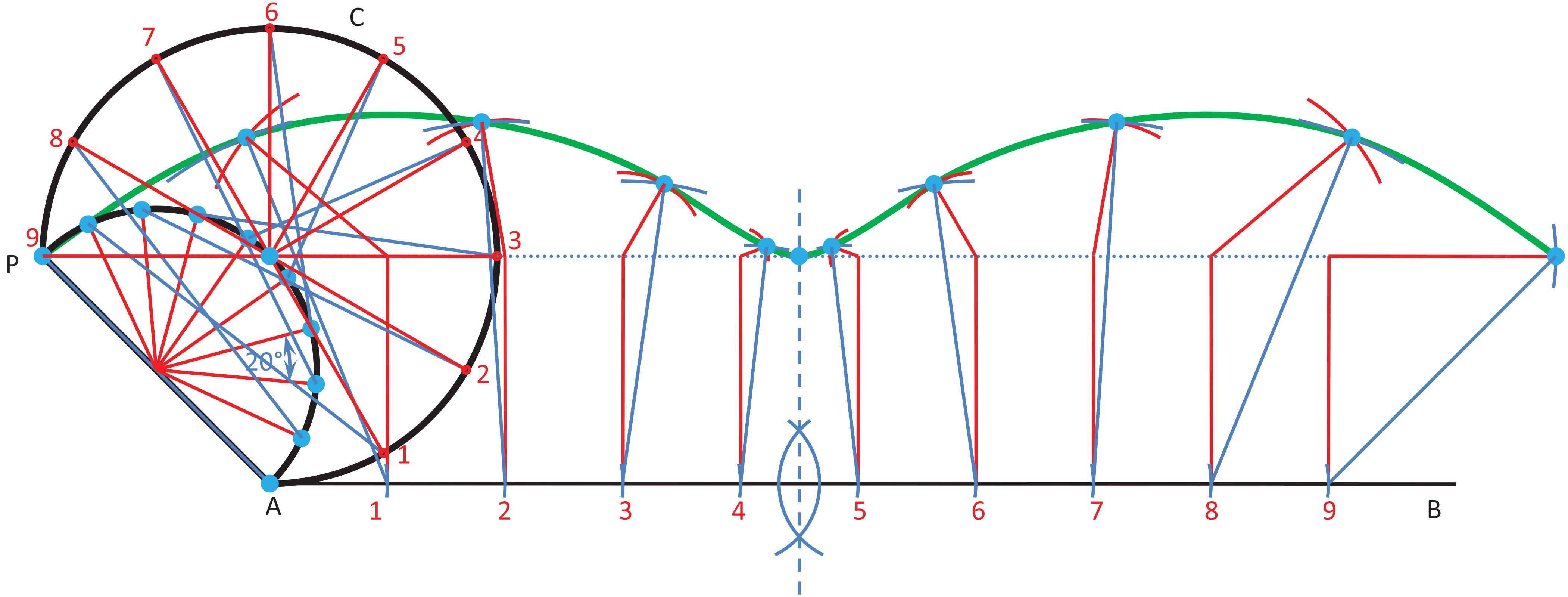
8
9
10
11
12

Combined Movement

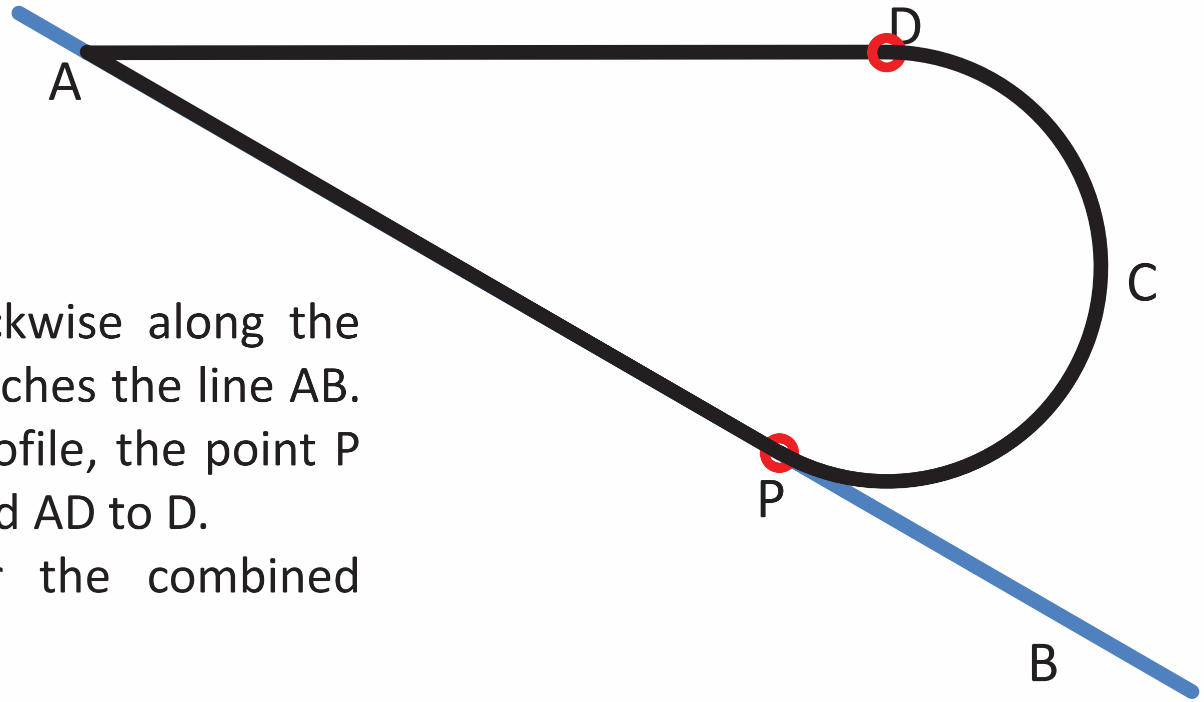
Shown is a circle C, which rolls clockwise along the line AB for three-quarters of a revolution. Also shown is the initial position of a point P on the circle. During the rolling of the circle, the point P moves along the semi-circle POA to A.

Draw the locus of P for the combined movement.

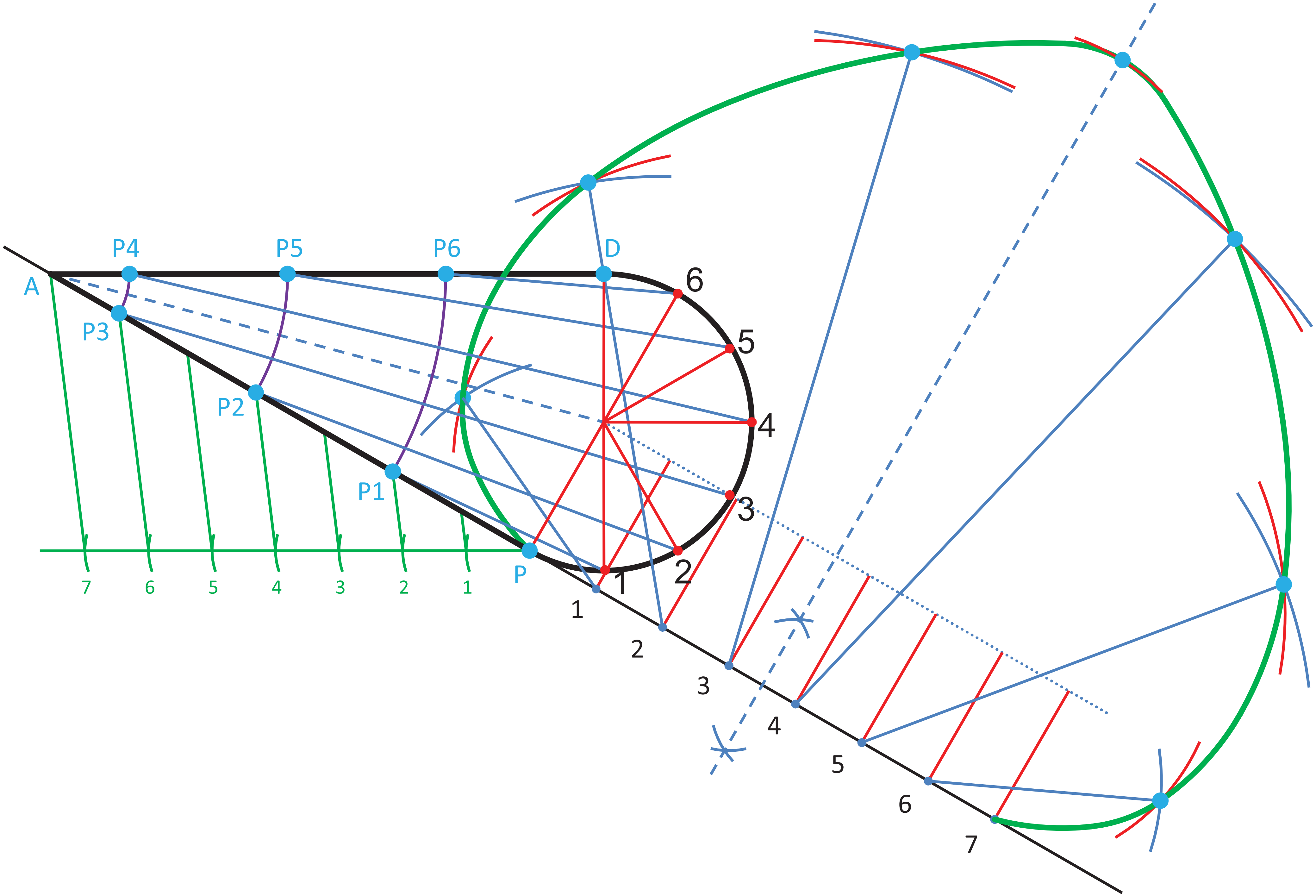



6
C
7
5
8
4
9
3
P
20°
2
1
A
1
2
3
4
5
6
7
8
9
B

Combined Movement

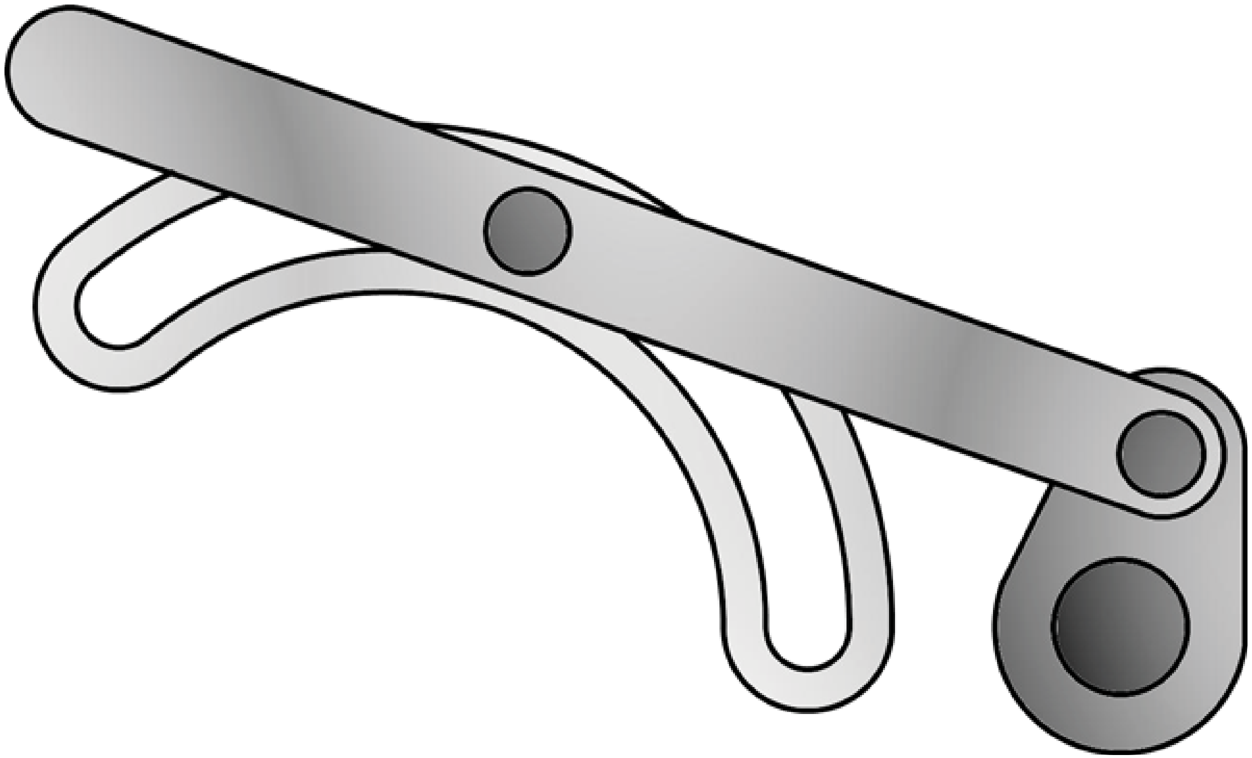


The profile PCDA rolls clockwise along the line AB until the point D reaches the line AB. During the rolling of the profile, the point P moves along the lines PA and AD to D.
Draw the locus of P for the combined movement.



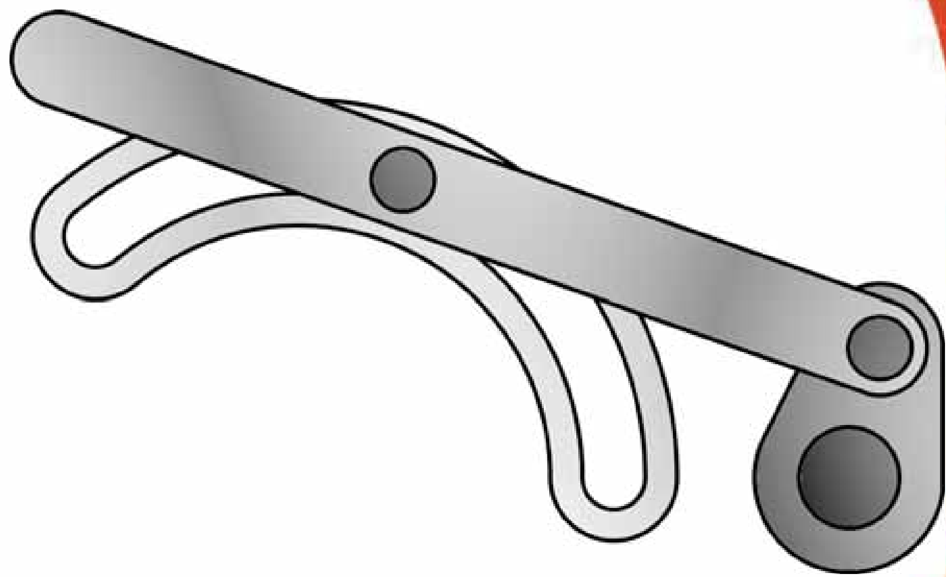
Design and Comm. Graphics

LINKAGES



Linkages

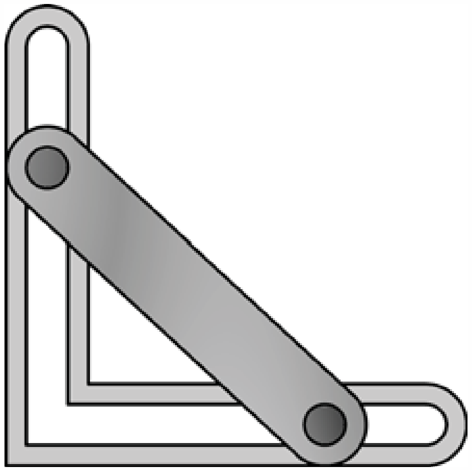
- Linkages are used to restrict the motion of objects so that they follow a regular path
- Linkages are commonly found in children's toys, windows, automotive parts, etc



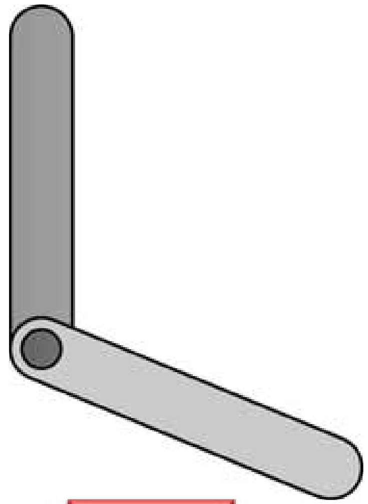
Linkages

- Linkages are used to redirect kinetic energy or other forces
- A scissors jack is a common example of a linkage

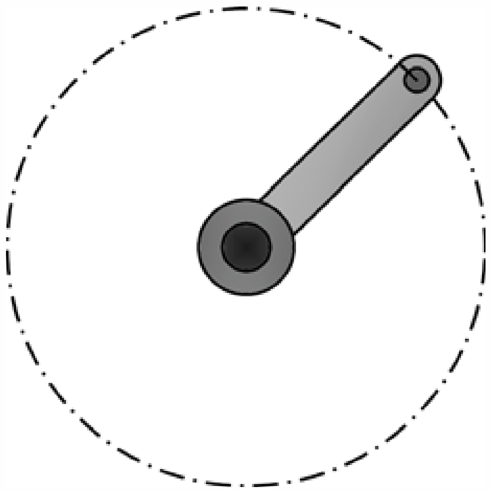
Basic Linkages



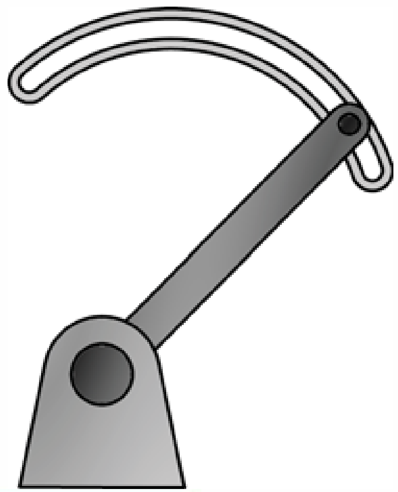
Sliding Link



Pivot



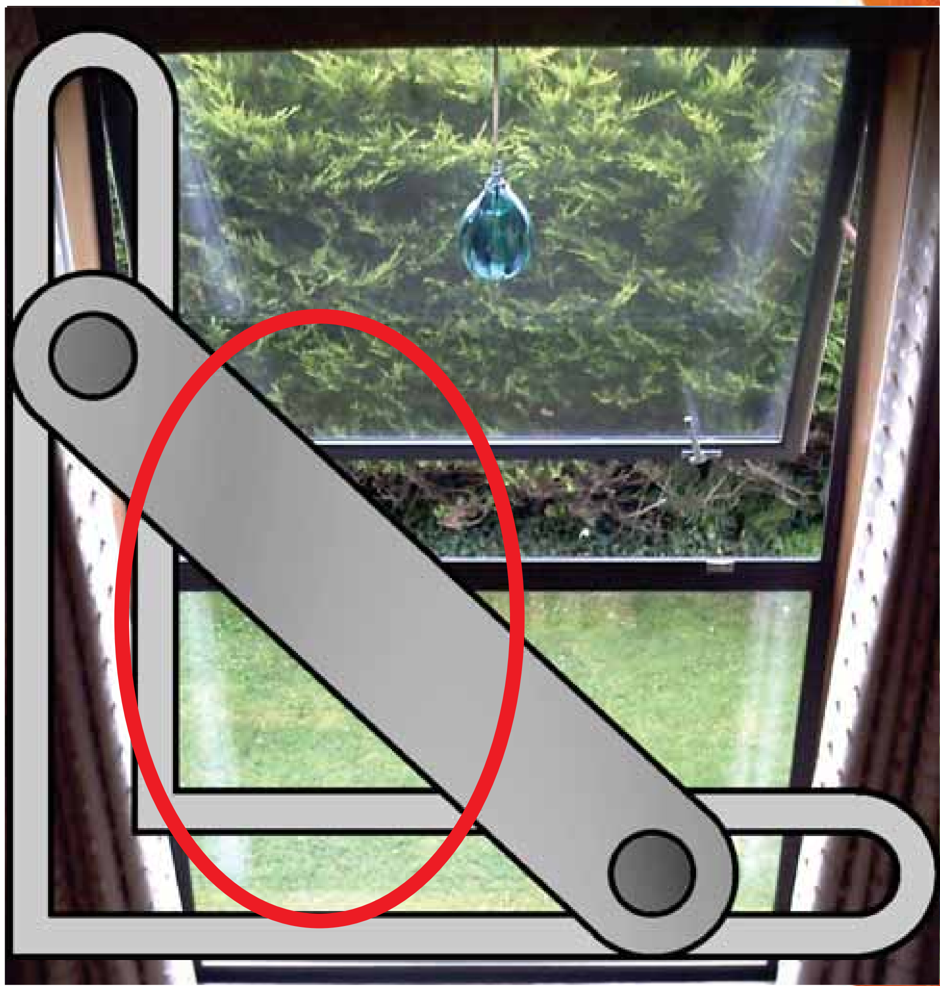
Crank



Rocker

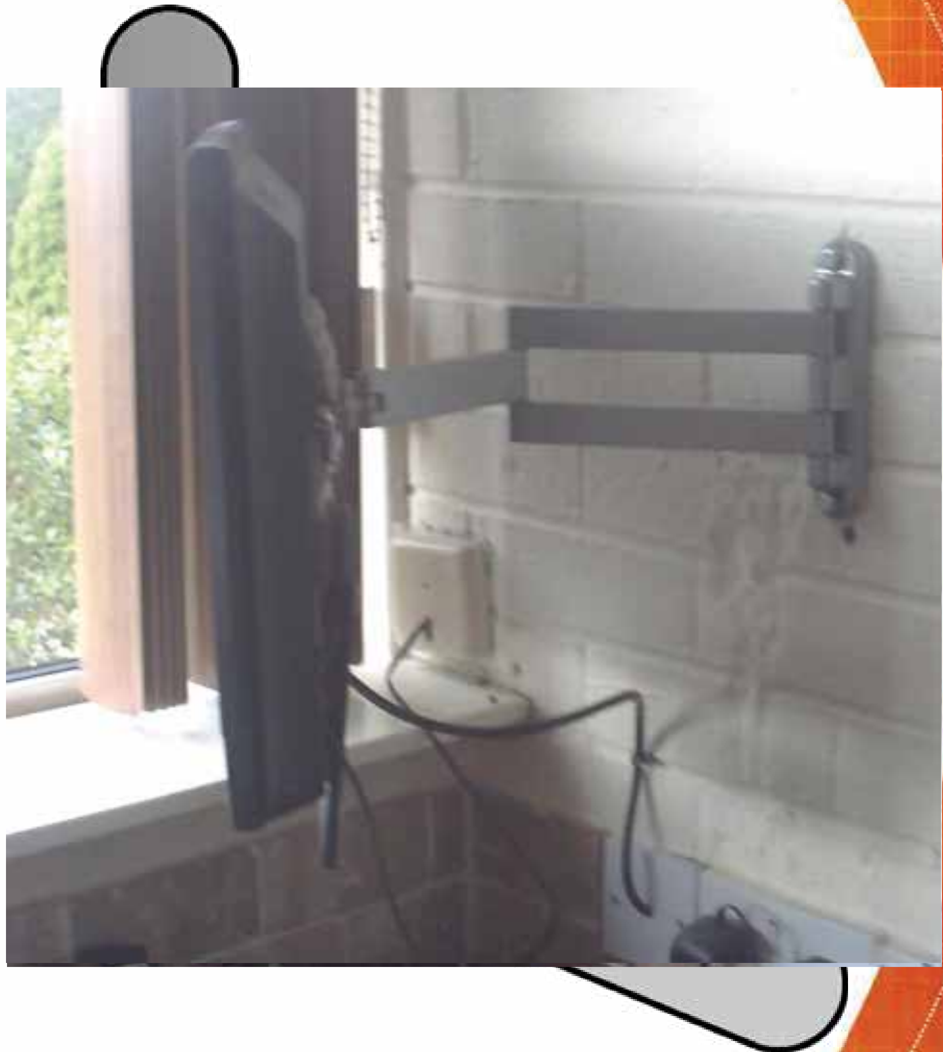
Sliding Link

- Sliding links are used to restrict the movement of a link to movement along an axis
- Sliding links have many applications and functions
- Sliding links restrict the opening in windows



Pivot

- A pivot joins two links together and allows 360° of freedom about the pivot
- A pivot acts like a hinge
- Pivots are found in many household items



Crank

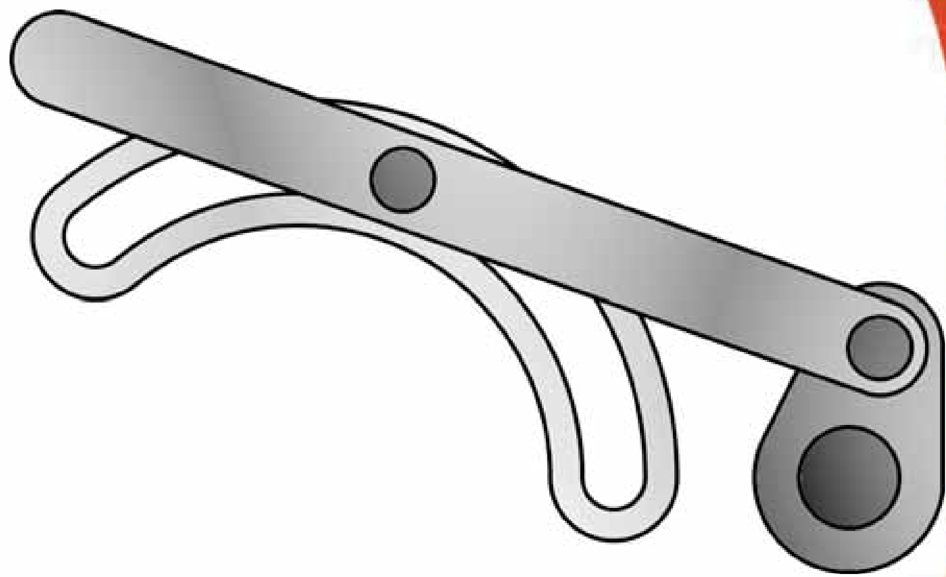
- Cranks are used to receive motion or to transfer rotary motion
- Cranks are often used with bevel gears or other variations of gears
- Common examples are in wheel braces, hand mixers etc

Rocker

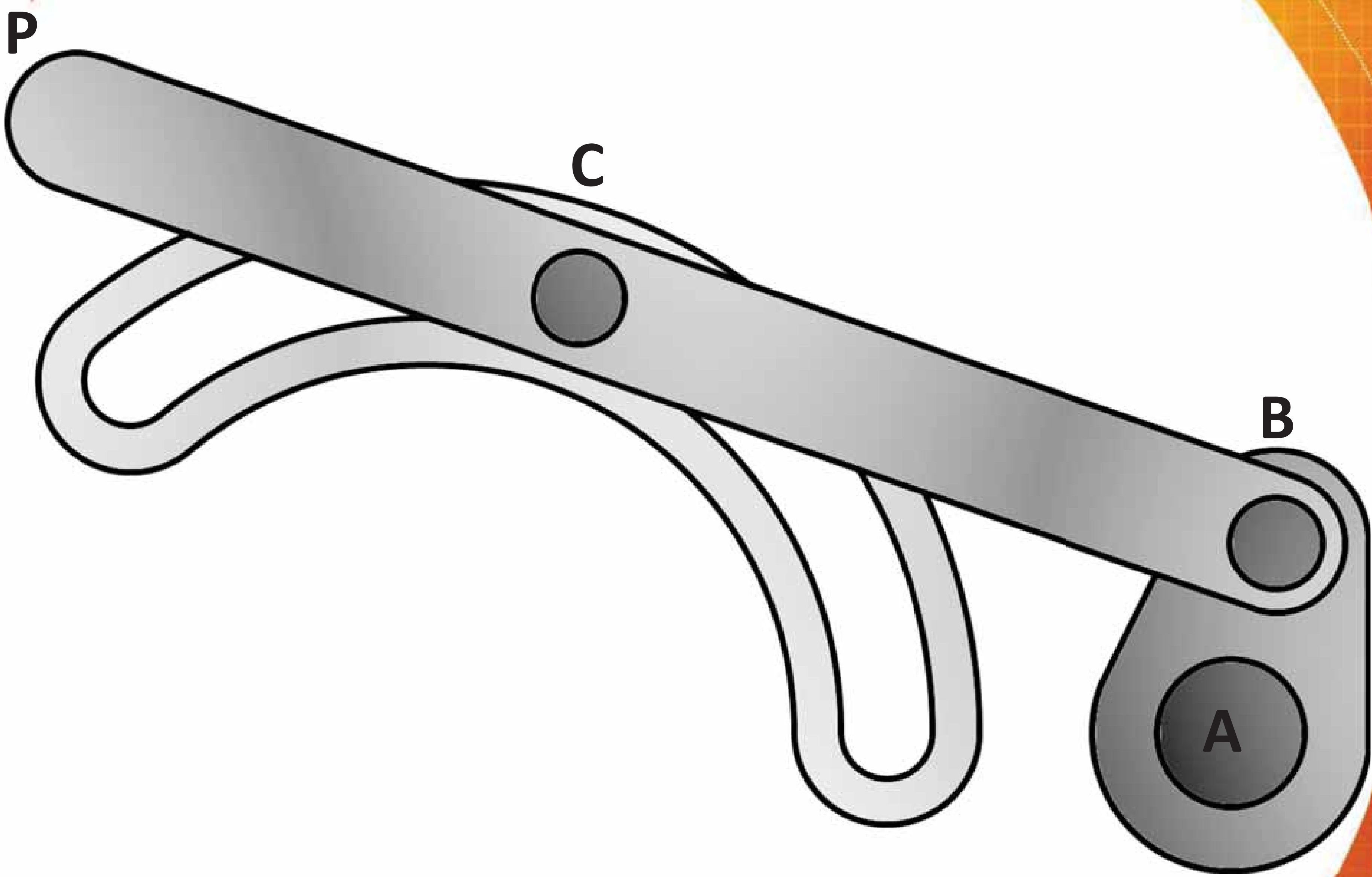
- A rocker mechanism restricts the swing of a linkage, to a known angle
- Rockers are very prominent in children's cradles, chairs and similar objects
- The locus of the rocker must be found to ensure the chair doesn't swing too far back

Linkages

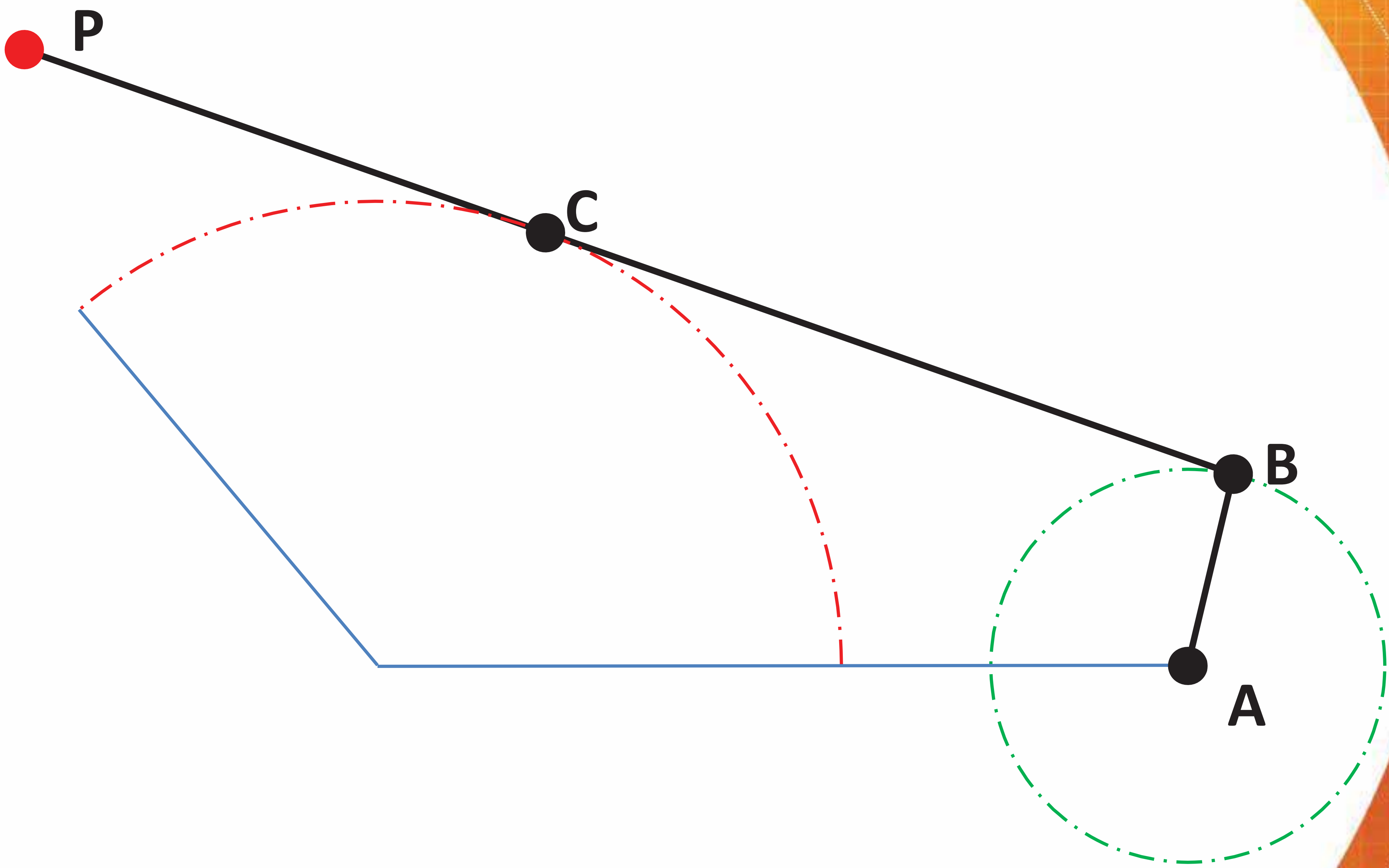
- Mechanisms like this are a common feature in machines
- In order for such mechansims to operate as desired it is necessary to plot the loci of the parts

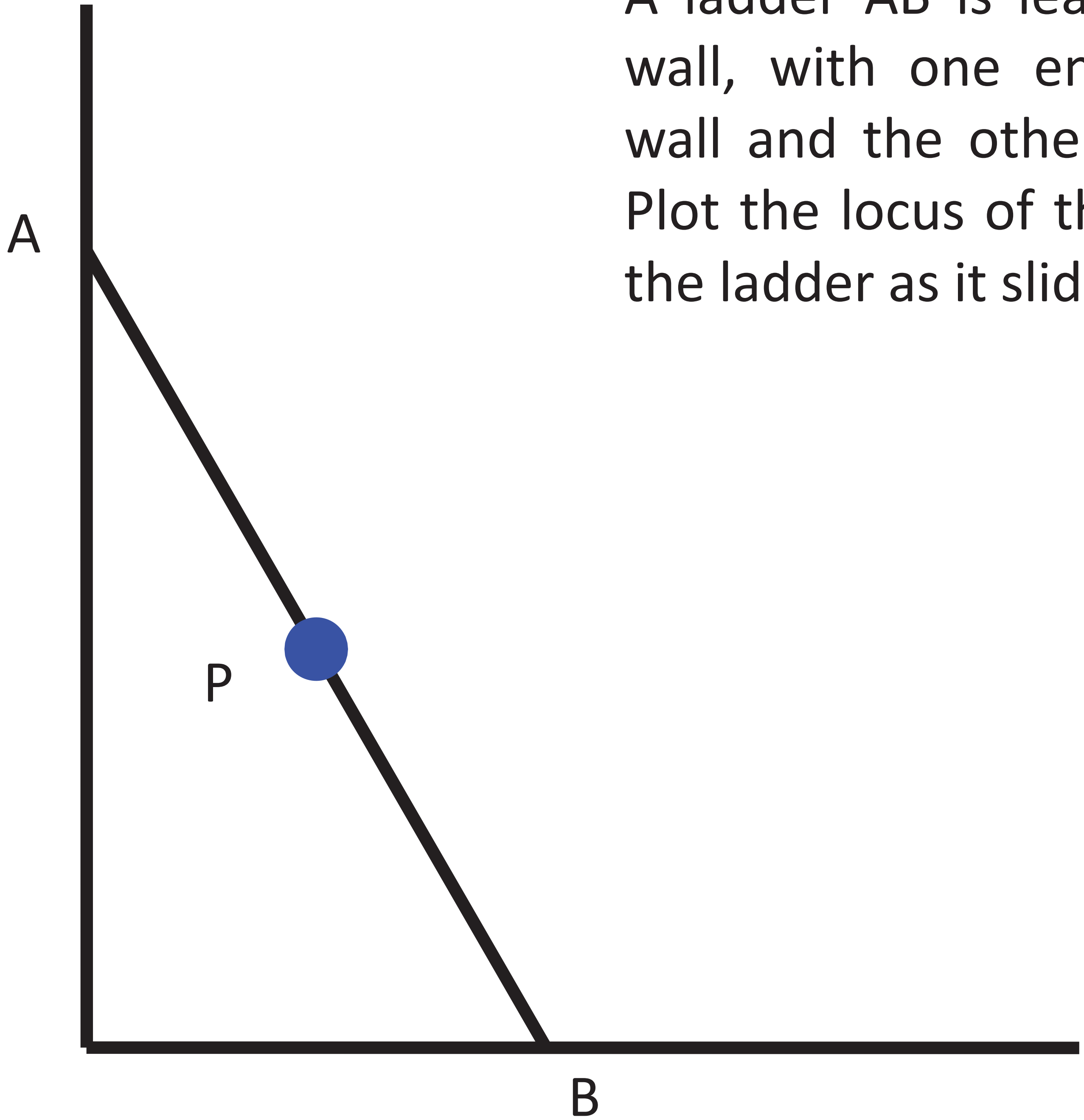


Converting Mechanisms to Line Drawings

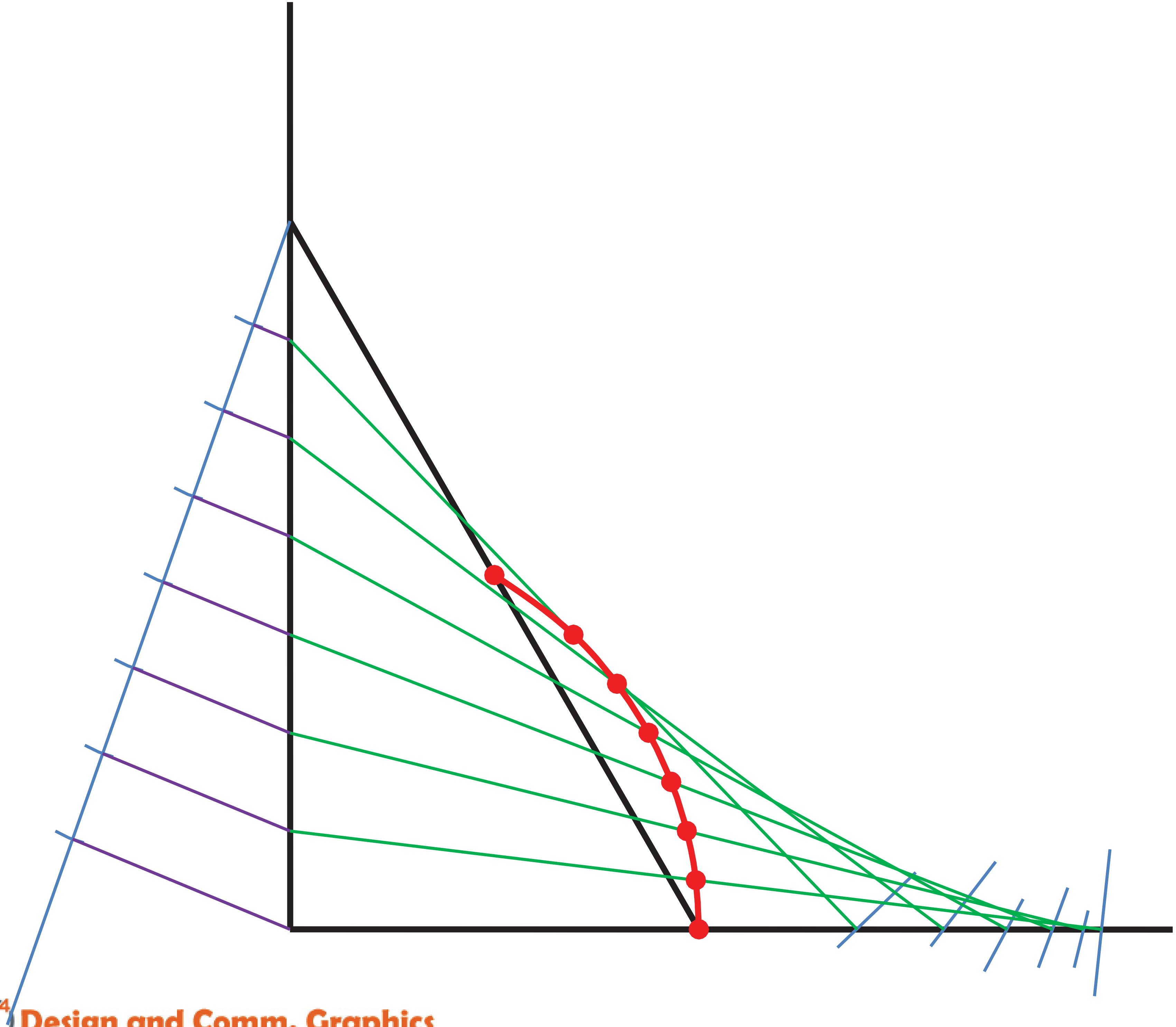


Converting Mechanisms to Line Drawings





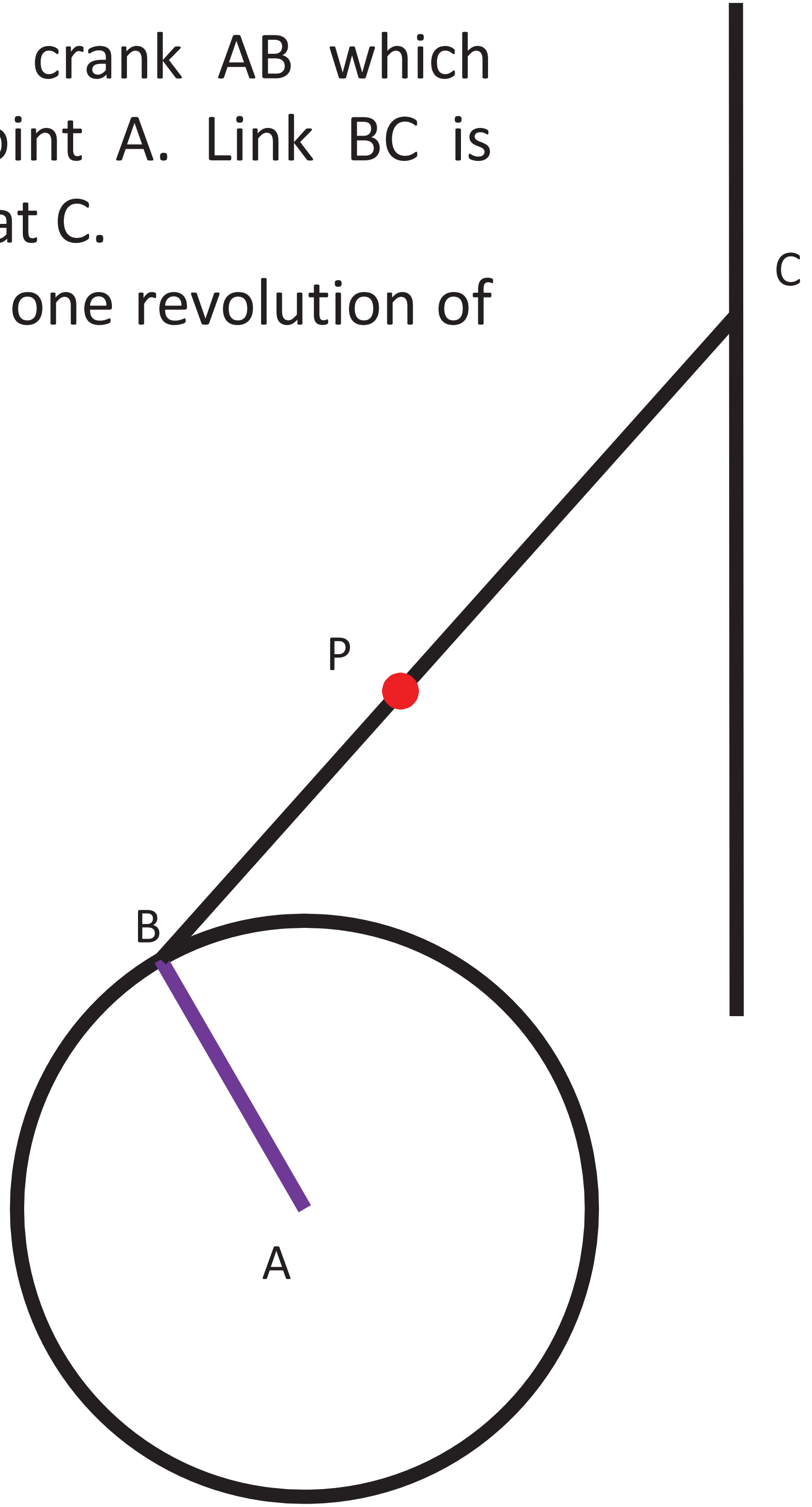
A ladder AB is leaning against a wall, with one end against the wall and the other on the floor. Plot the locus of the midpoint of the ladder as it slides to the floor

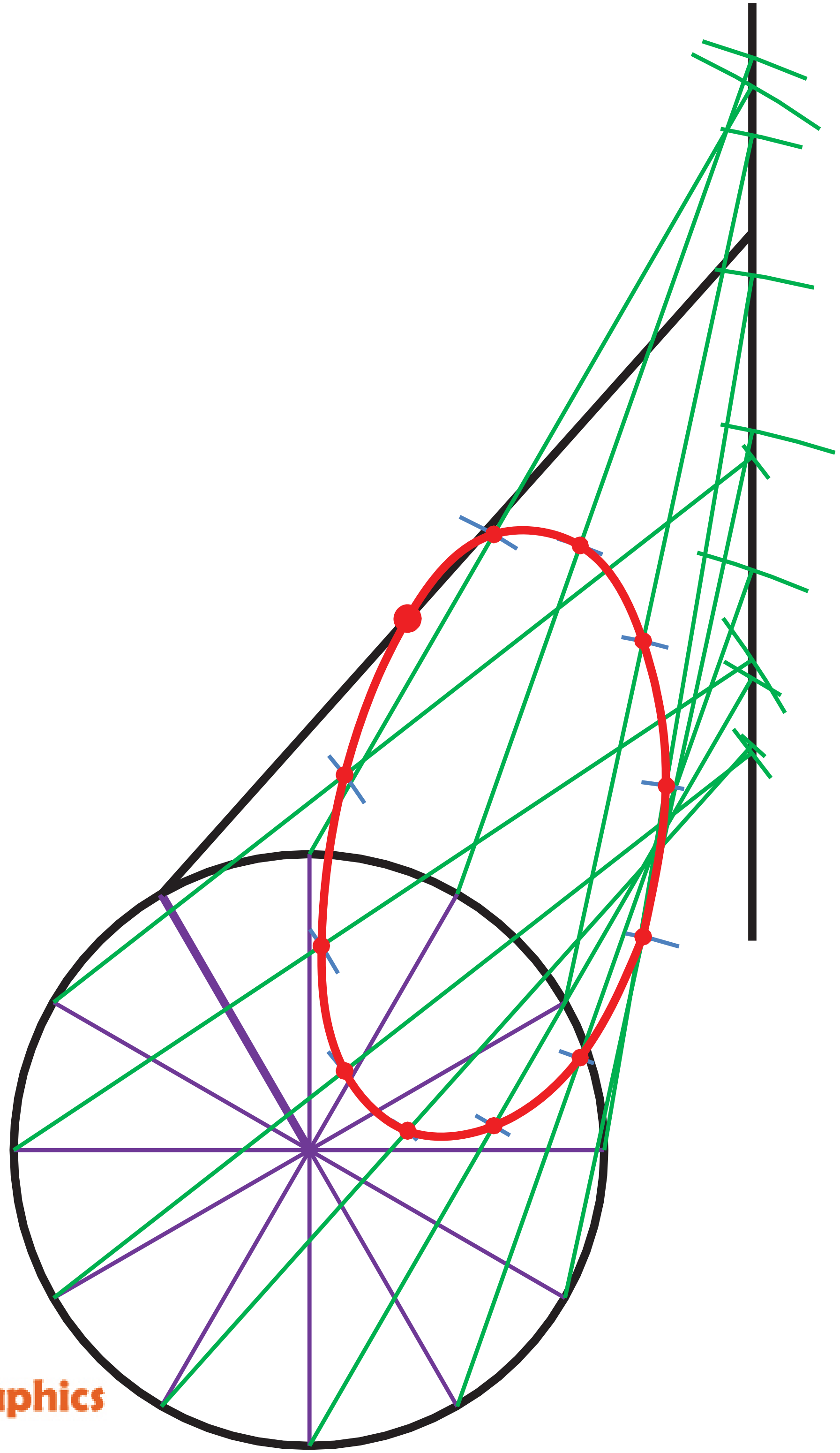


Design and Comm. Graphics

The figure below shows a crank AB which rotates clockwise about point A. Link BC is restricted to slide vertically at C.

Plot the locus of point P for one revolution of the crank.

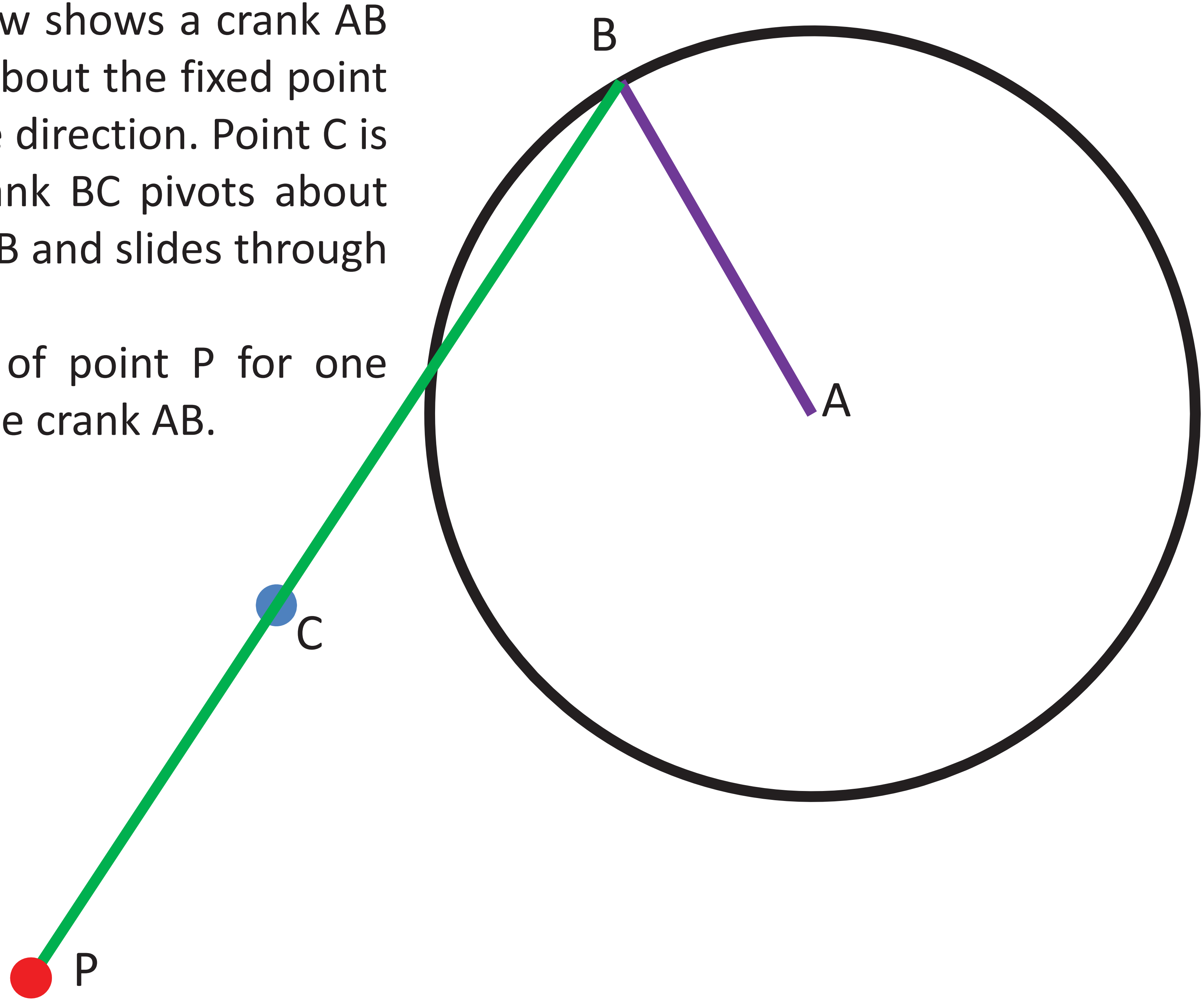


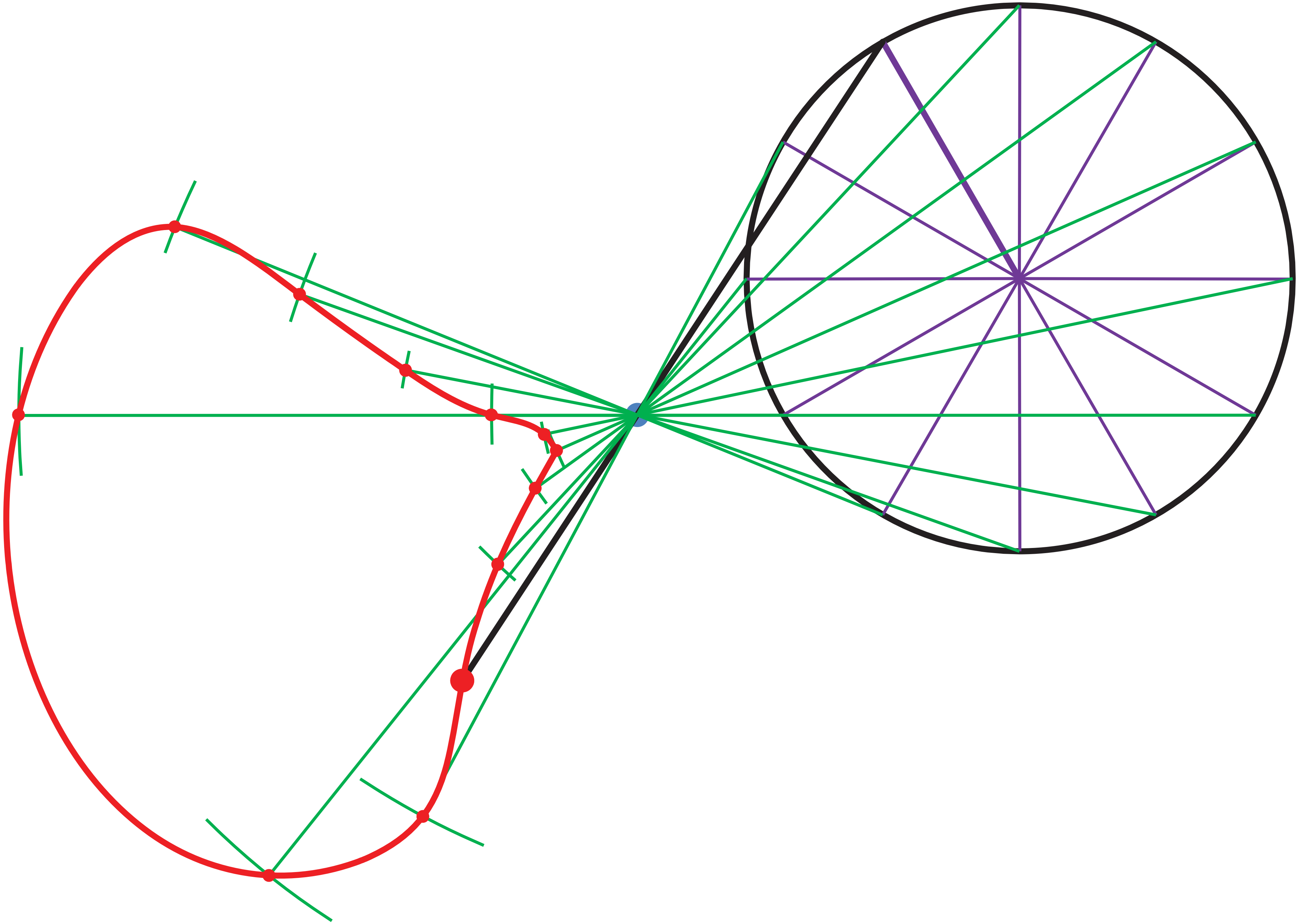


Design and Comm. Graphics

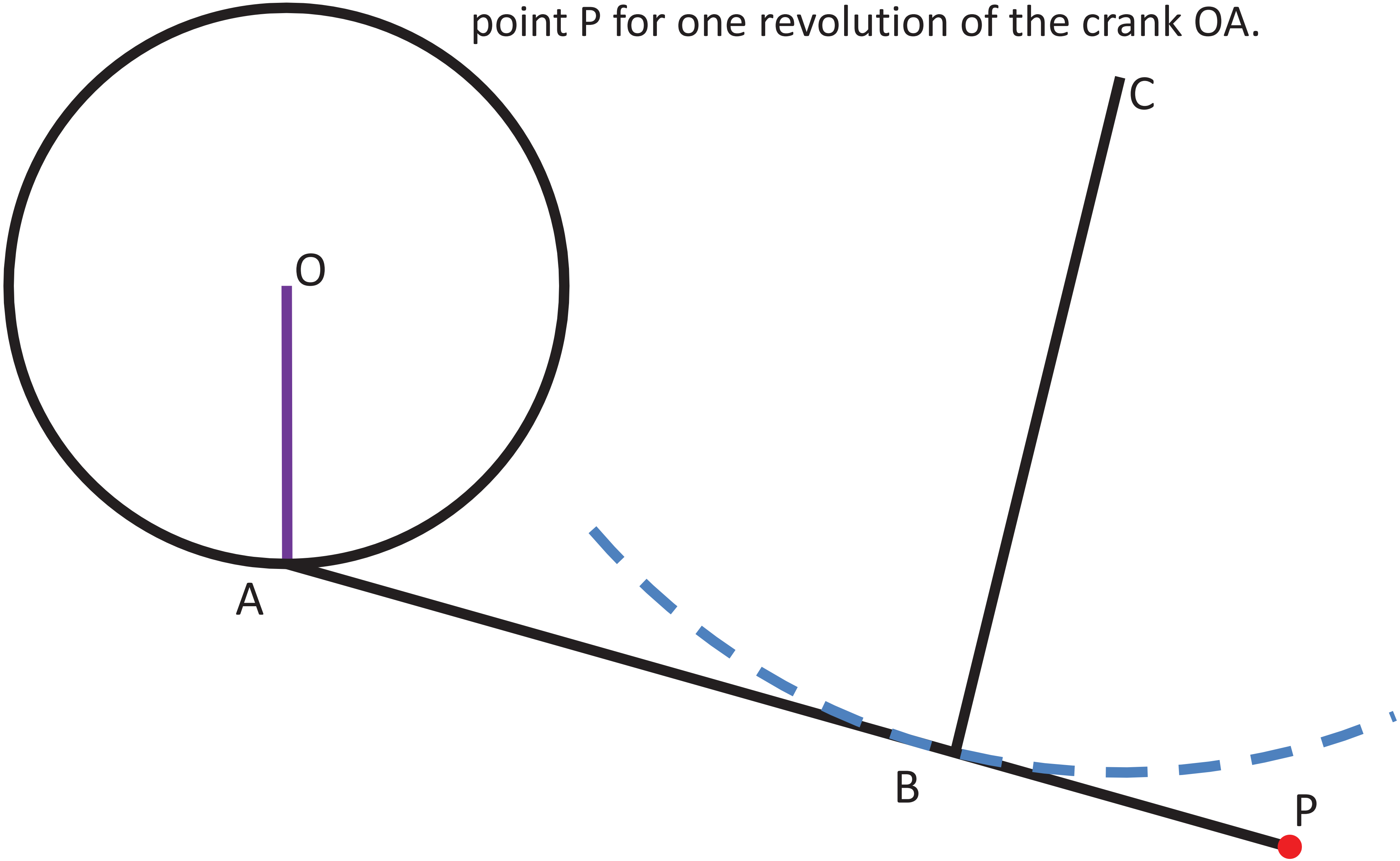
The figure below shows a crank AB which rotates about the fixed point B in a clockwise direction. Point C is a trunnion. Crank BC pivots about the fixed point B and slides through the trunnion.

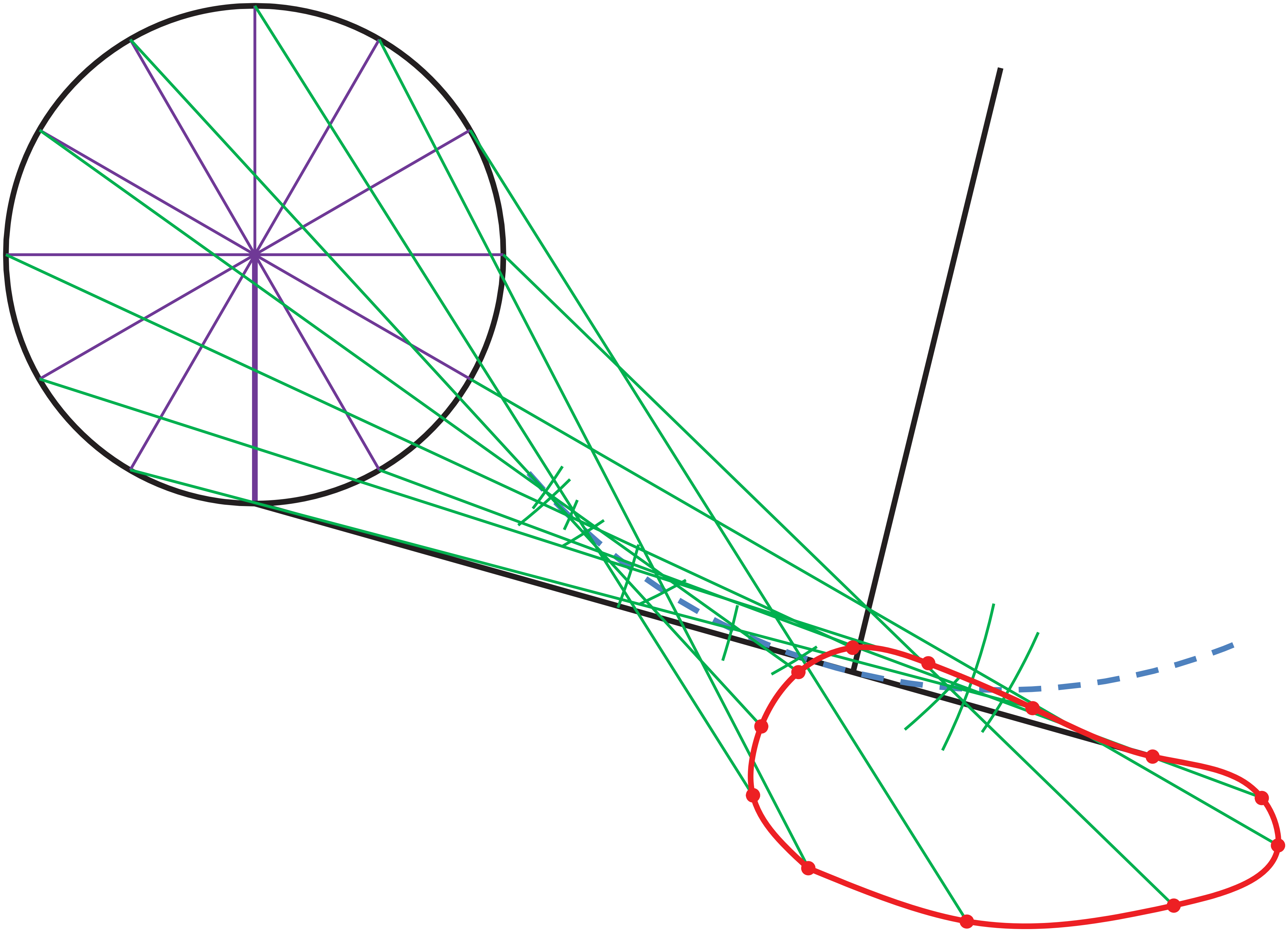
Plot the locus of point P for one revolution of the crank AB.





The figure below shows a crank OA which rotates about the fixed point O. A and B are pin joints. Crank BC pivots about the fixed point C. Plot the locus of point P for one revolution of the crank OA.

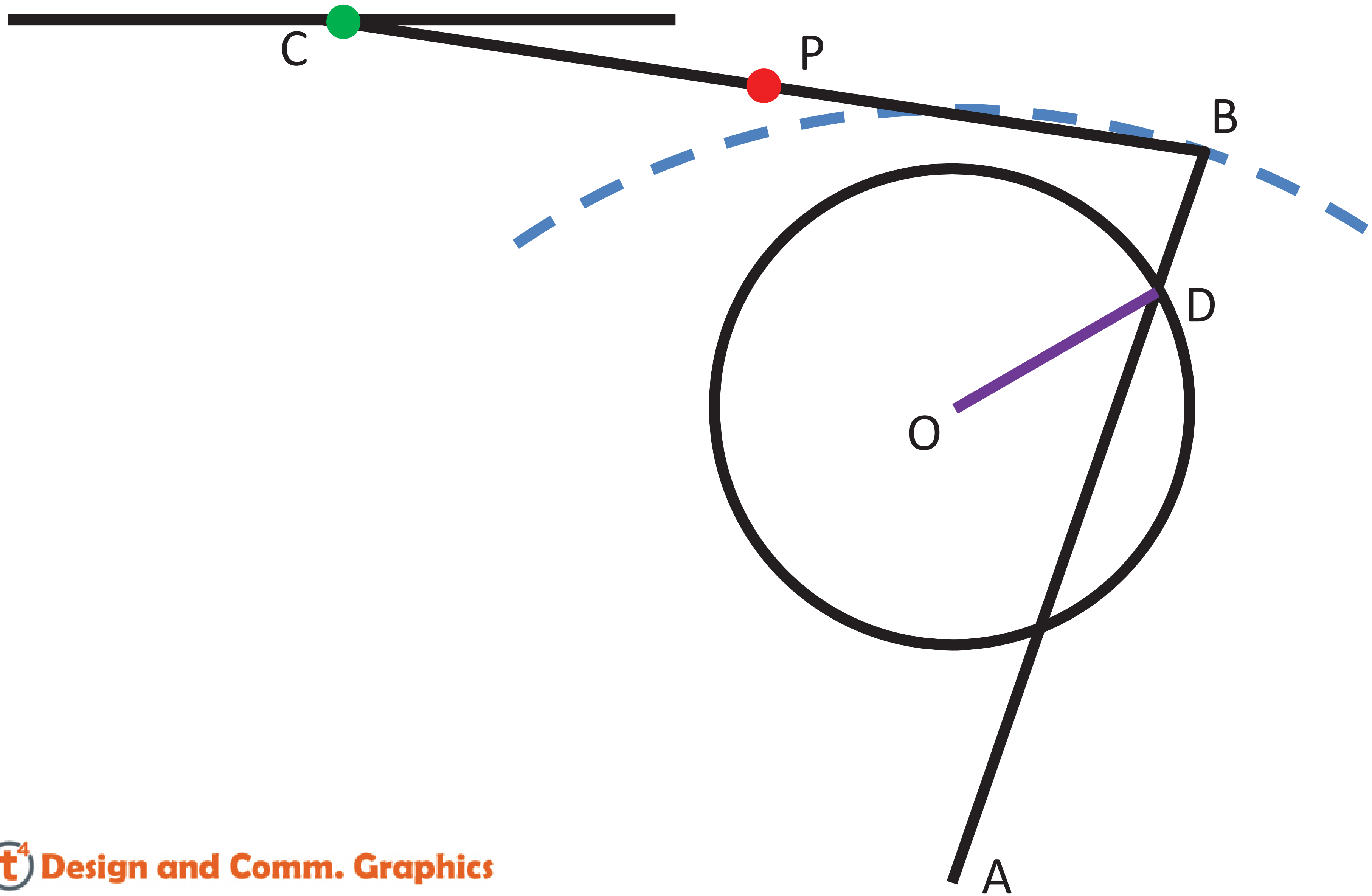


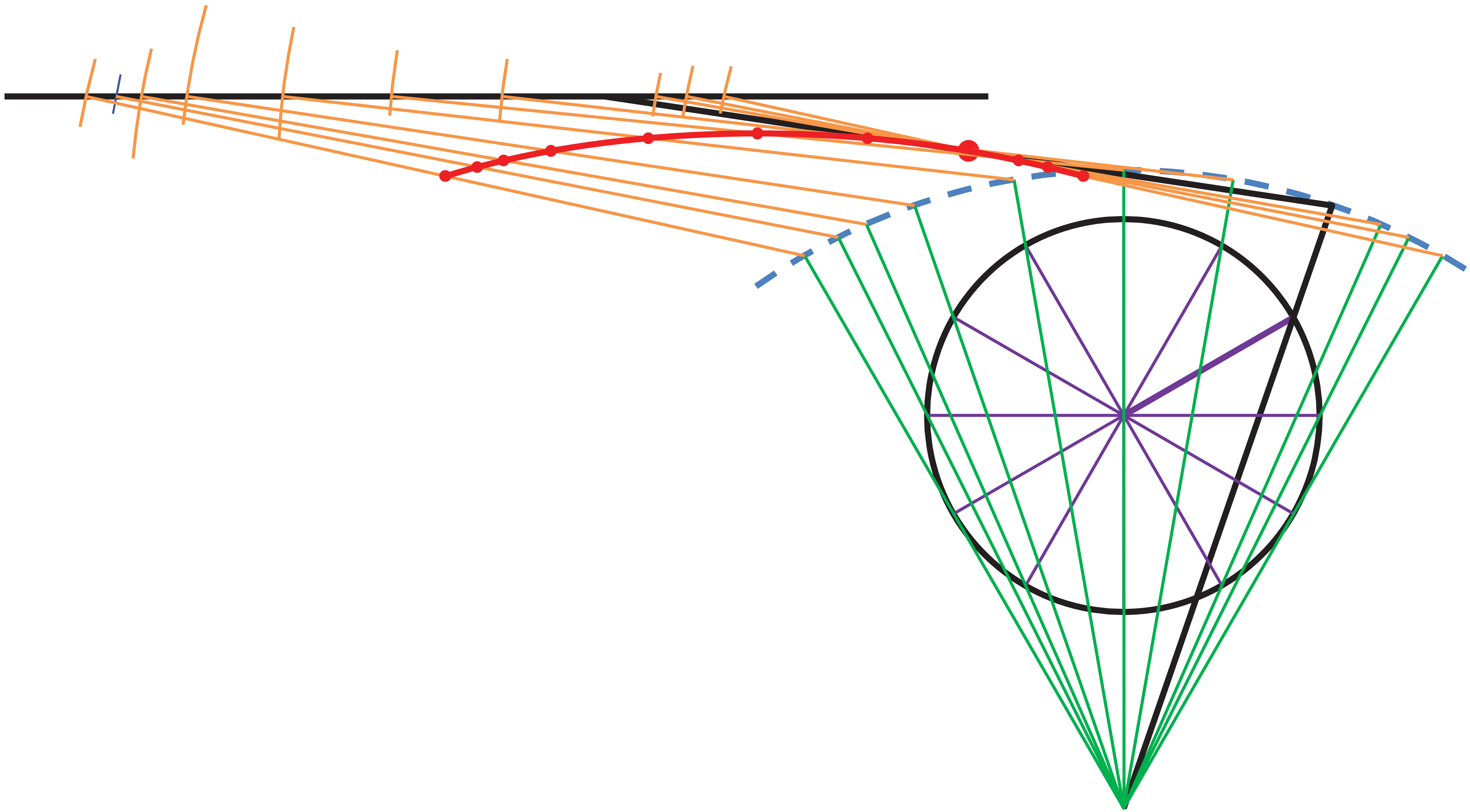


t4 Design and Comm. Graphics

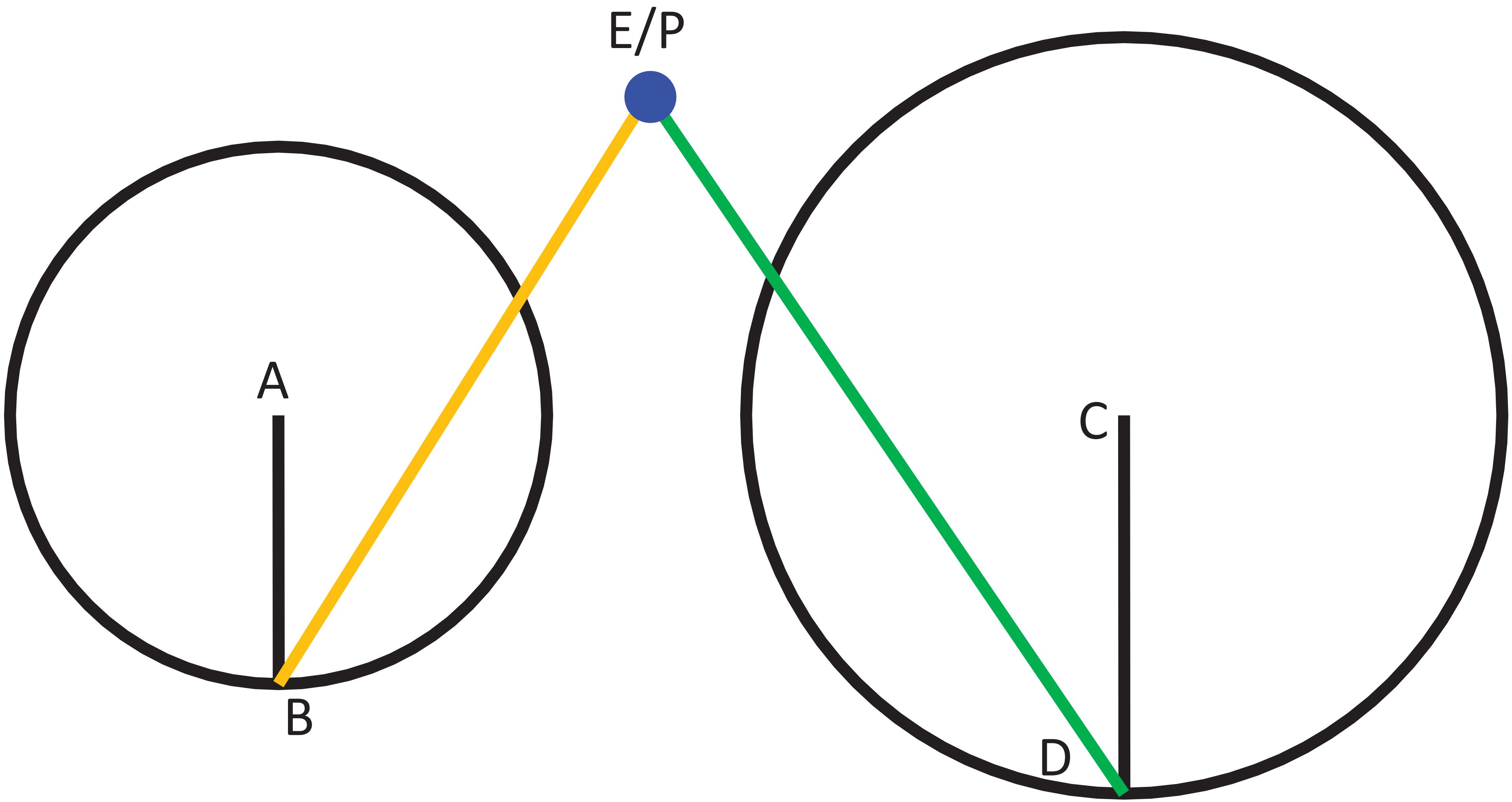
The crank OD rotates anticlockwise about a fixed point O. AB oscillates about the fixed point A. D is fixed to a block, which slides along AB. Point C is constrained to slide horizontally.

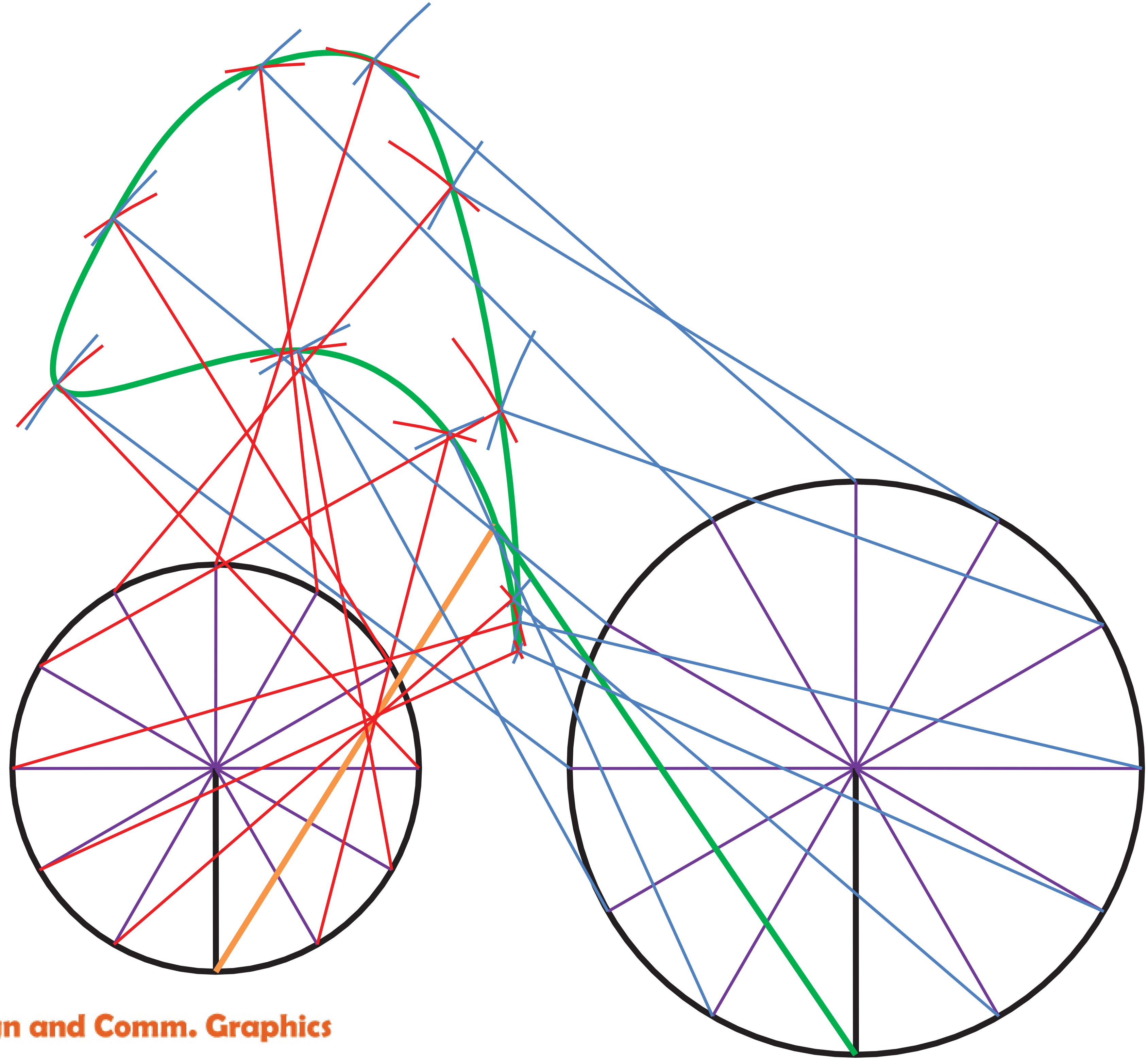
Plot the locus of point P during one complete revolution of the crank OD



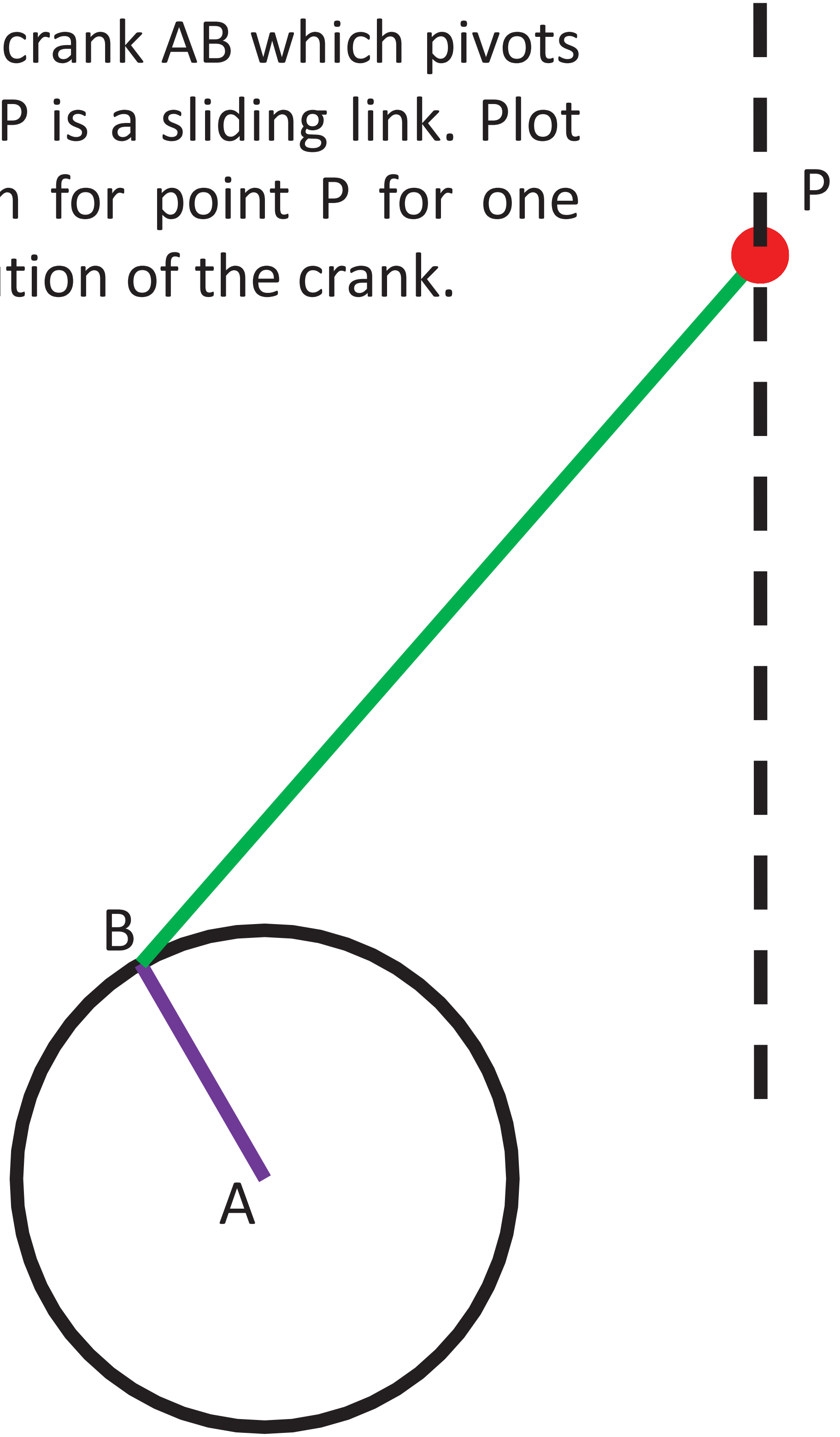


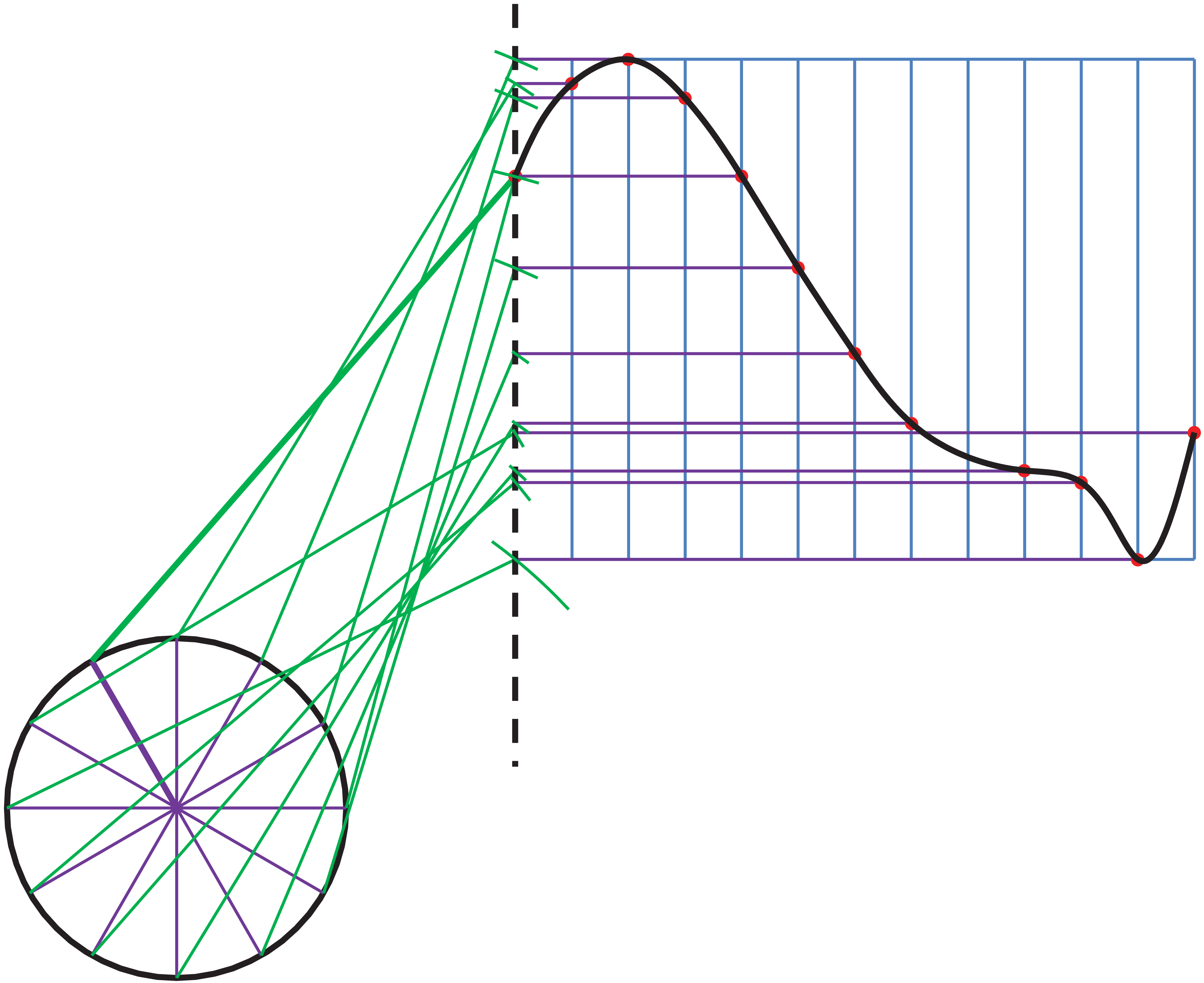
The figure below shows a crank Ab which rotates anti-clockwise about pivot C. Another crank CD rotates clockwise about Pivot C. Link BE pivots at B and E. Link DEP pivots at D and E. Plot the locus of point P for one revolution of the cranks. (Both cranks rotates at the same rate)



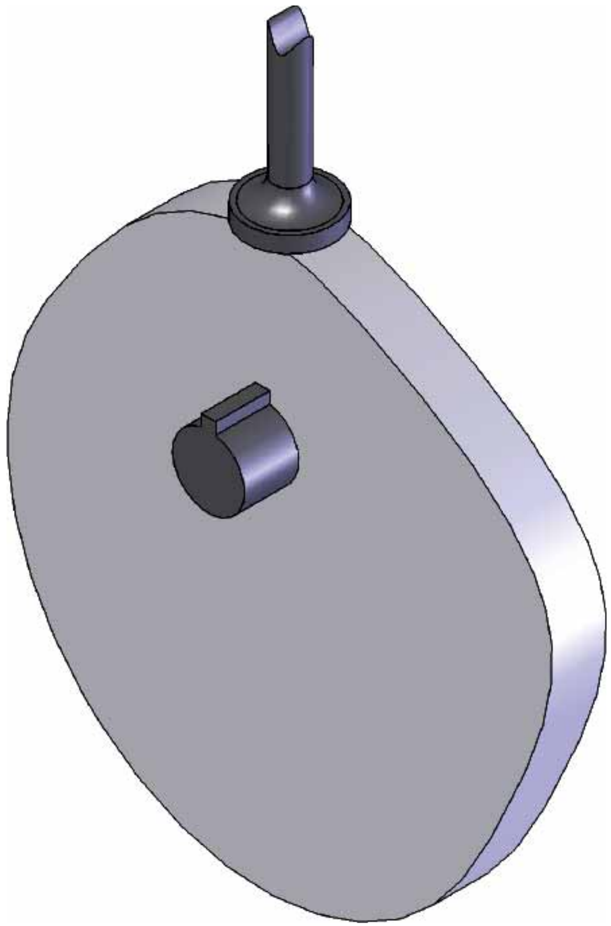


The figure below shows a crank AB which pivots about A. B is a pivot and P is a sliding link. Plot the displacement diagram for point P for one complete clockwise revolution of the crank.



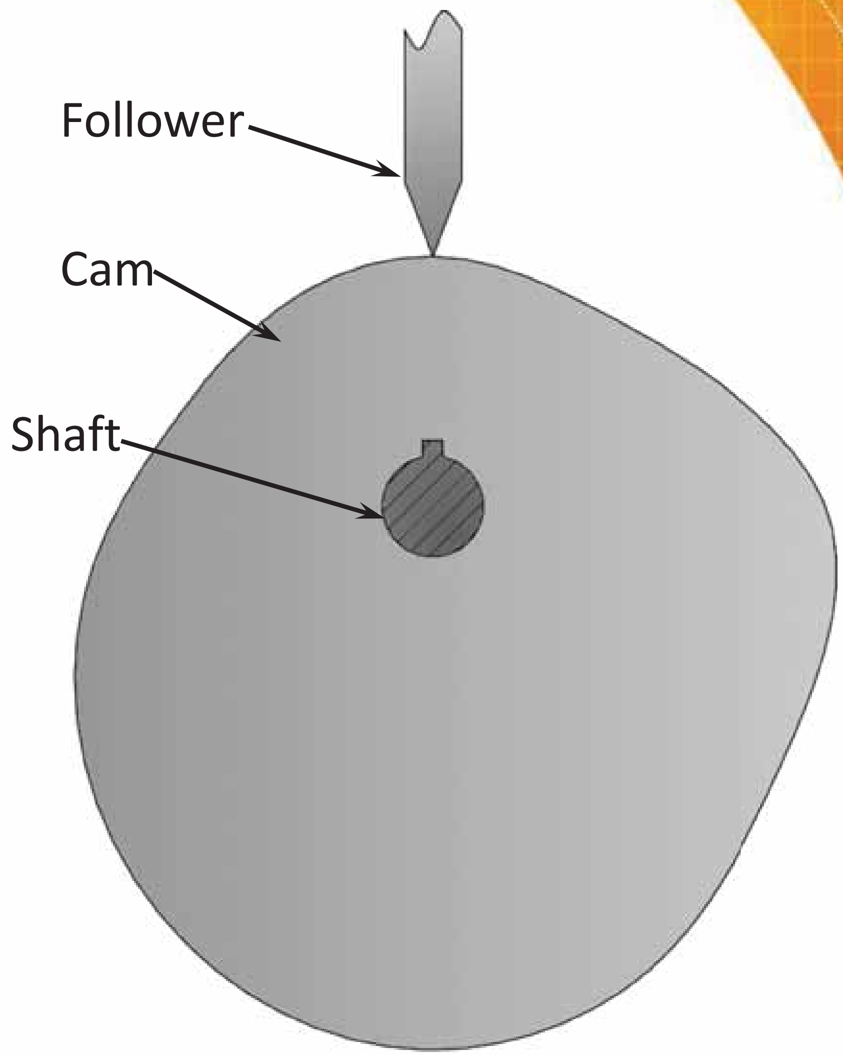


CAMS



Cams

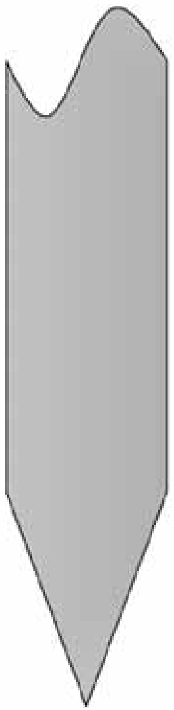
- A cam is a machine part for transferring rotary motion to linear motion
- In a radial plate cam, the cam is mounted on a rotating shaft
- The motion is recieved by a follower
- To see a cam in operation click on the link



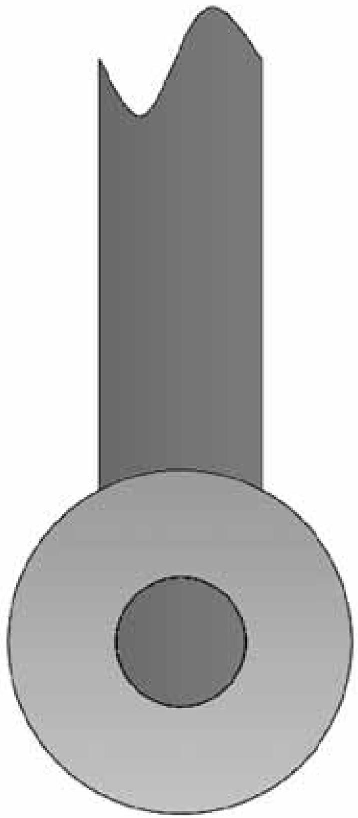
http://www.engr.colostate.edu/~dga/video_demos/mechanisms/IC_engine_cam_crank_animation.gif

Follower Types

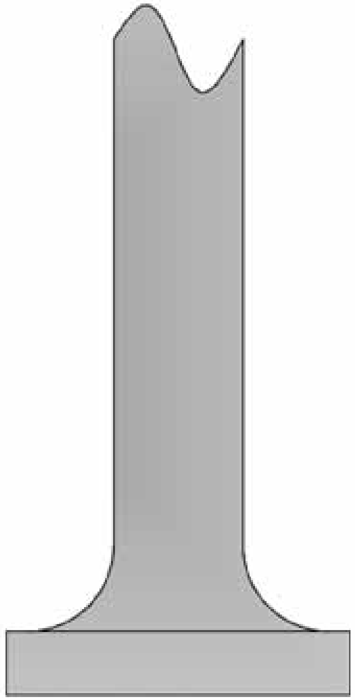
- Followers can be knife edged, rollers or flat footed



Knife Edged

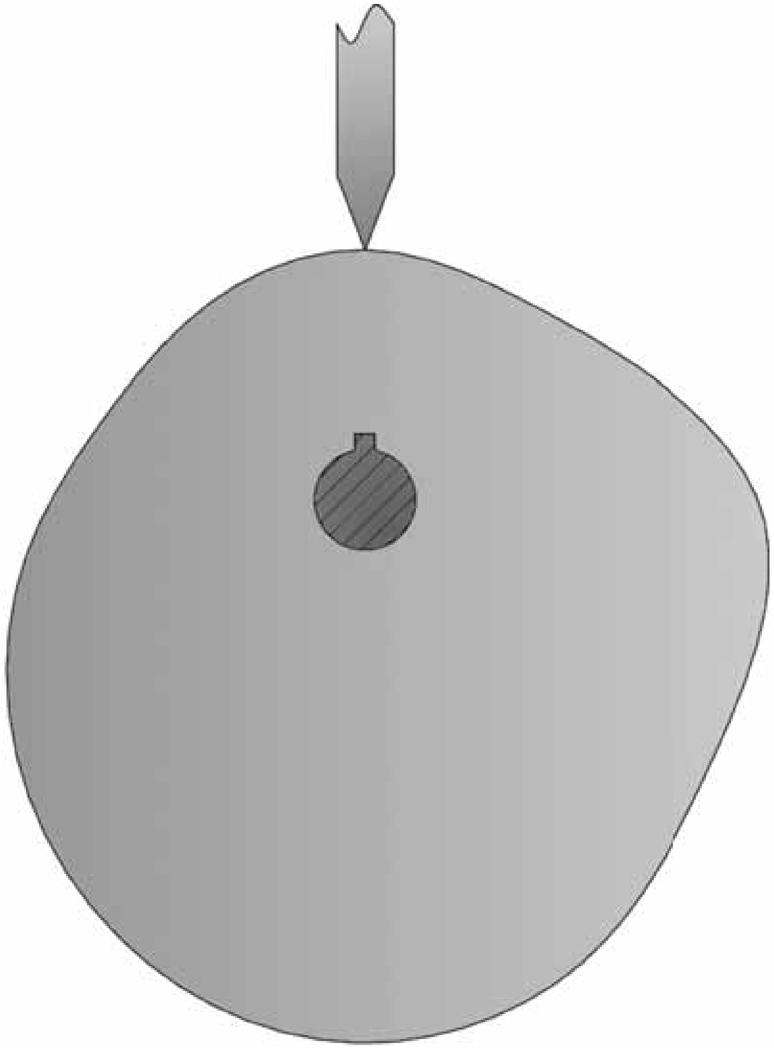


Roller



Flat Footed

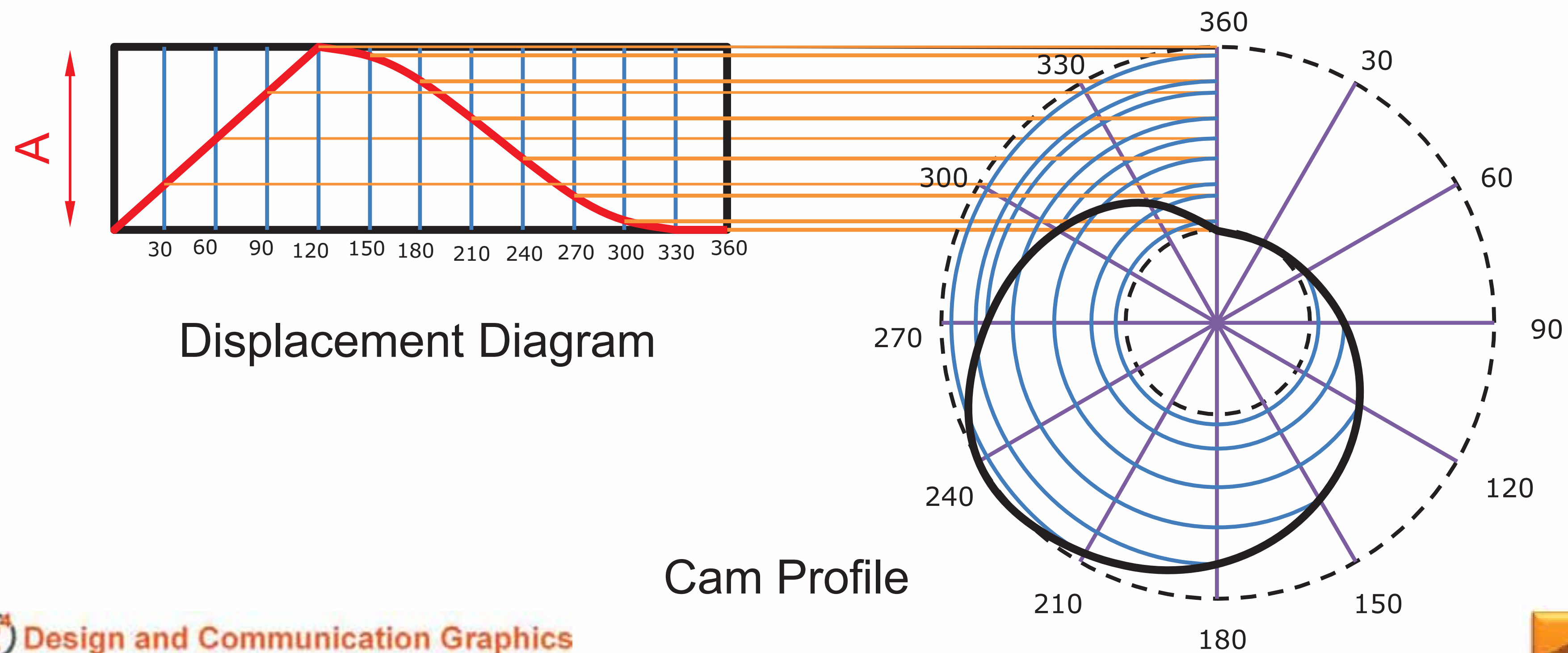
Knife Edged Follower



- The point of the follower can follow very complicated cam profiles
- Wears Rapidly
- Must be used at low speeds

Displacement Diagrams

- In order to determine the shape of a cam, a displacement diagram is drawn first
- The height of the diagram (A) is equal to the total displacement of the follower ie. the difference between the highest and lowest points
- The width of the displacement diagram does not matter but it is divided into regular divisions representing angular increments (on the cam)
- 30° increments are generally used

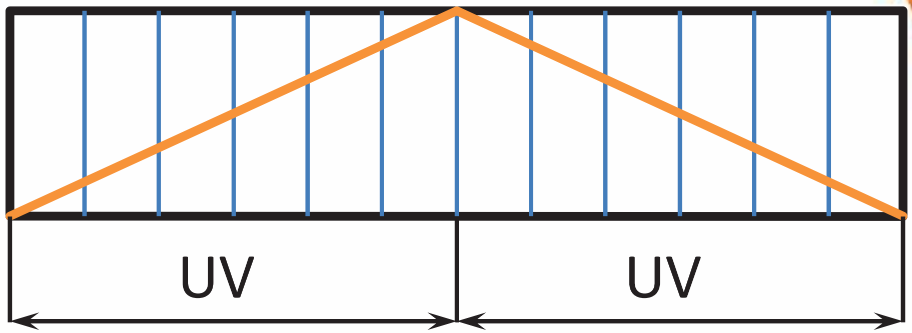


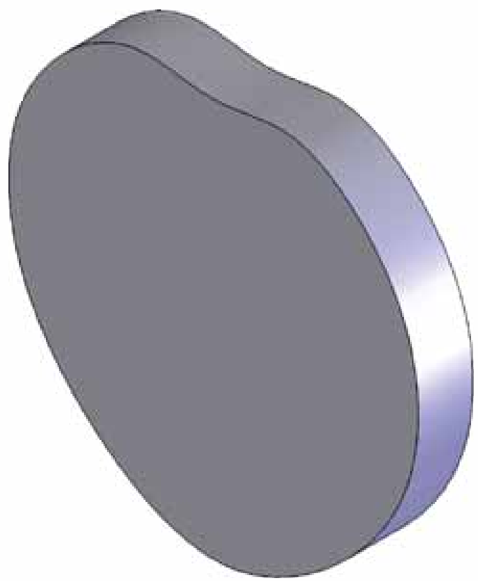
Displacement Diagram

Cam Profile

Uniform Velocity (UV)

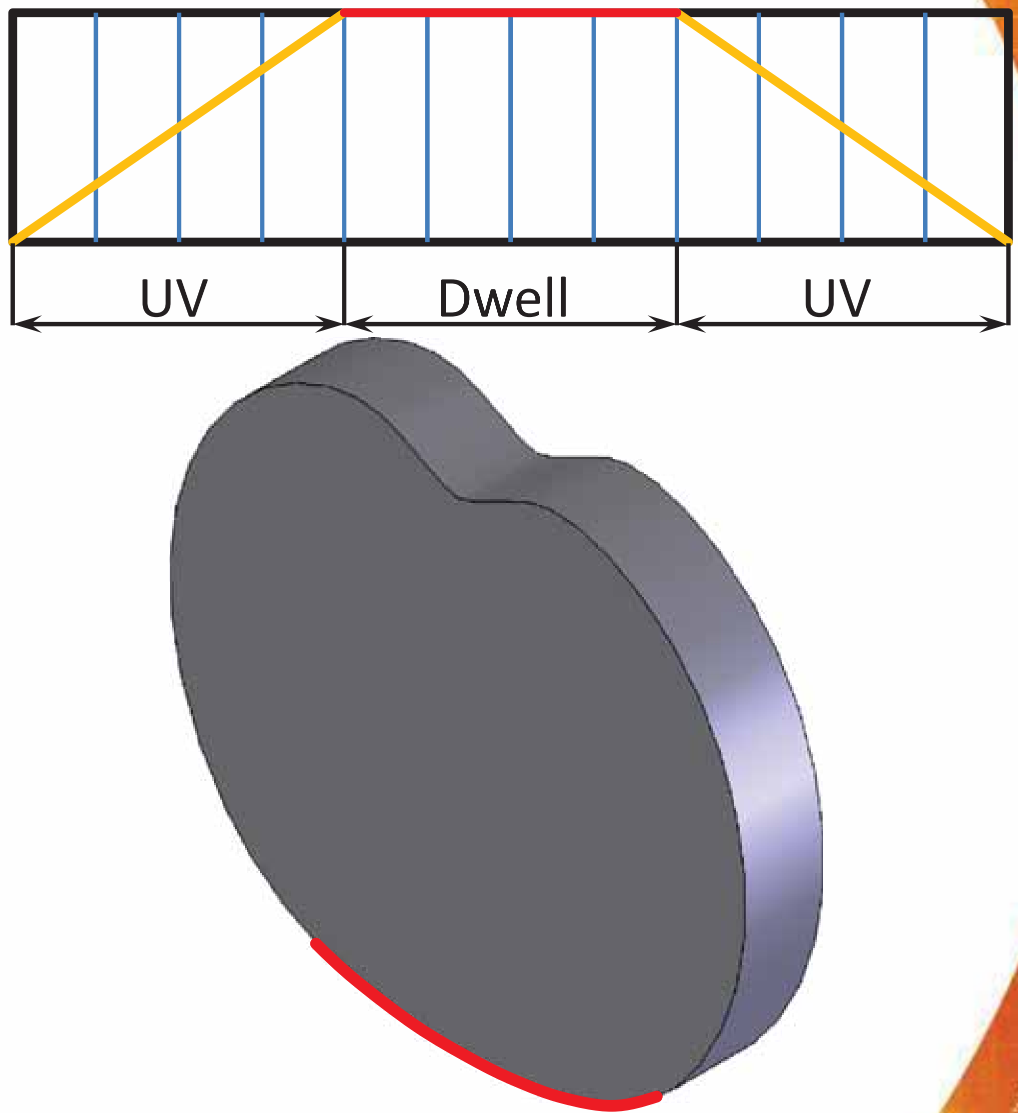
- A cam that imparts uniform velocity (UV) has the following displacement diagram
- The cam shown has a rise at uniform velocity, followed by a fall at uniform velocity
  - The follower rises and falls at a constant speed
- Shown over is the cam profile with uniform velocity rise and uniform velocity fall
- The disadvantage of uniform velocity is abrupt changes of movement of the follower





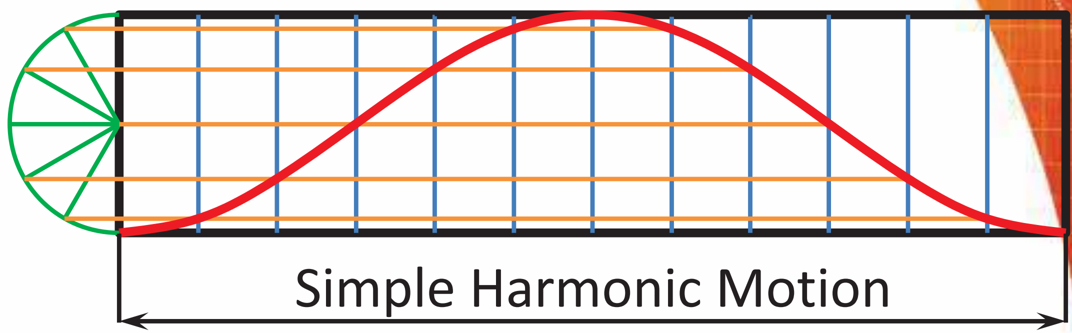
Dwell

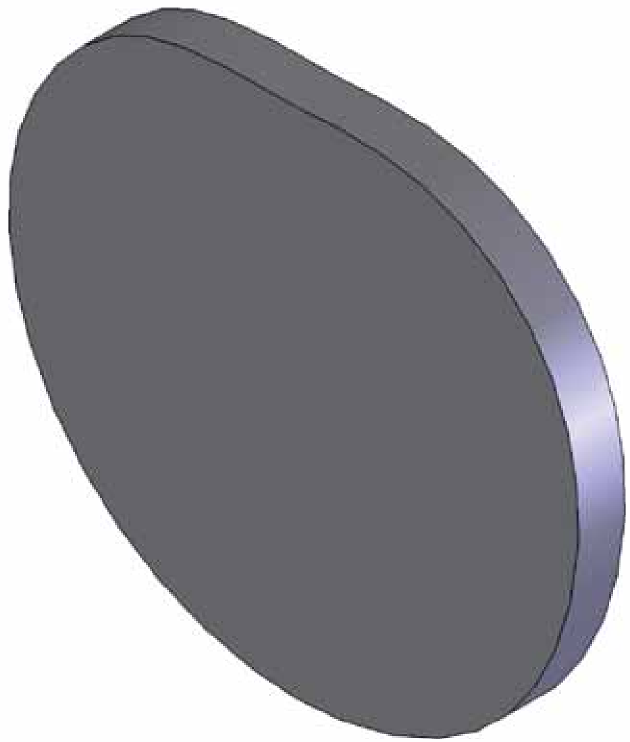
- A dwell is a period when there is no displacement of the follower
  - Cam radius remains constant
- A cam will have a circular profile for periods of dwell
- Note the circular segment on the cam



Simple Harmonic Motion (SHM)

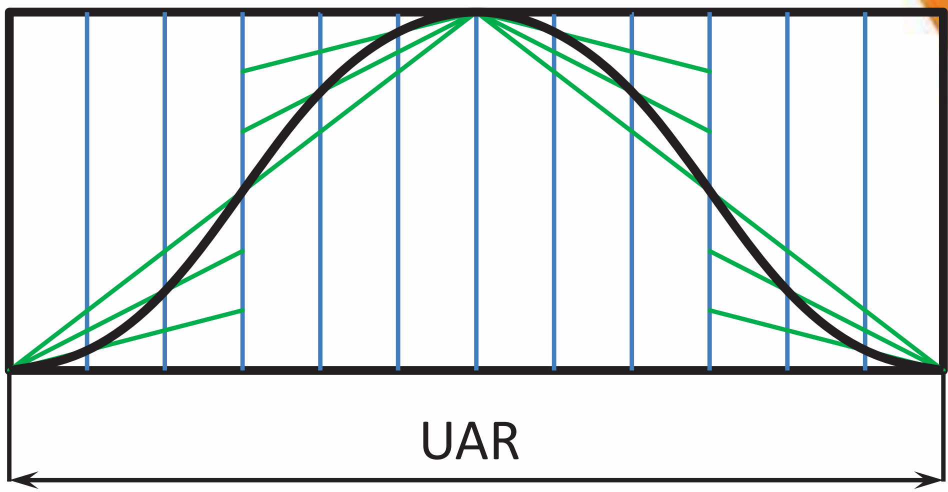
- Simple Harmonic Motion (SHM) is the gentle acceleration and deceleration of the end view of a point as it rotates at constant speed around the diameter of a circle
- Simple harmonic motion produces a sine curve
- Shown over is the outline of a cam with SHM rise and SHM fall

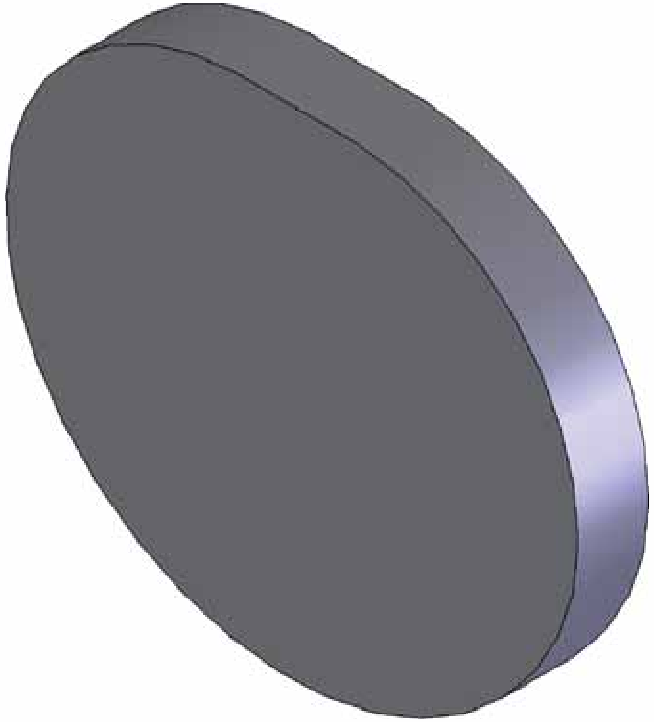




Uniform Acceleration and Retardation (UAR)

- A follower with Uniform Acceleration and Retardation (UAR) will accelerate and decellerate at the same rate
- The path of UAR is parabolic and can be drawn using the rectangle method
- Shown over is the outline of a cam with UAR rise and UAR fall





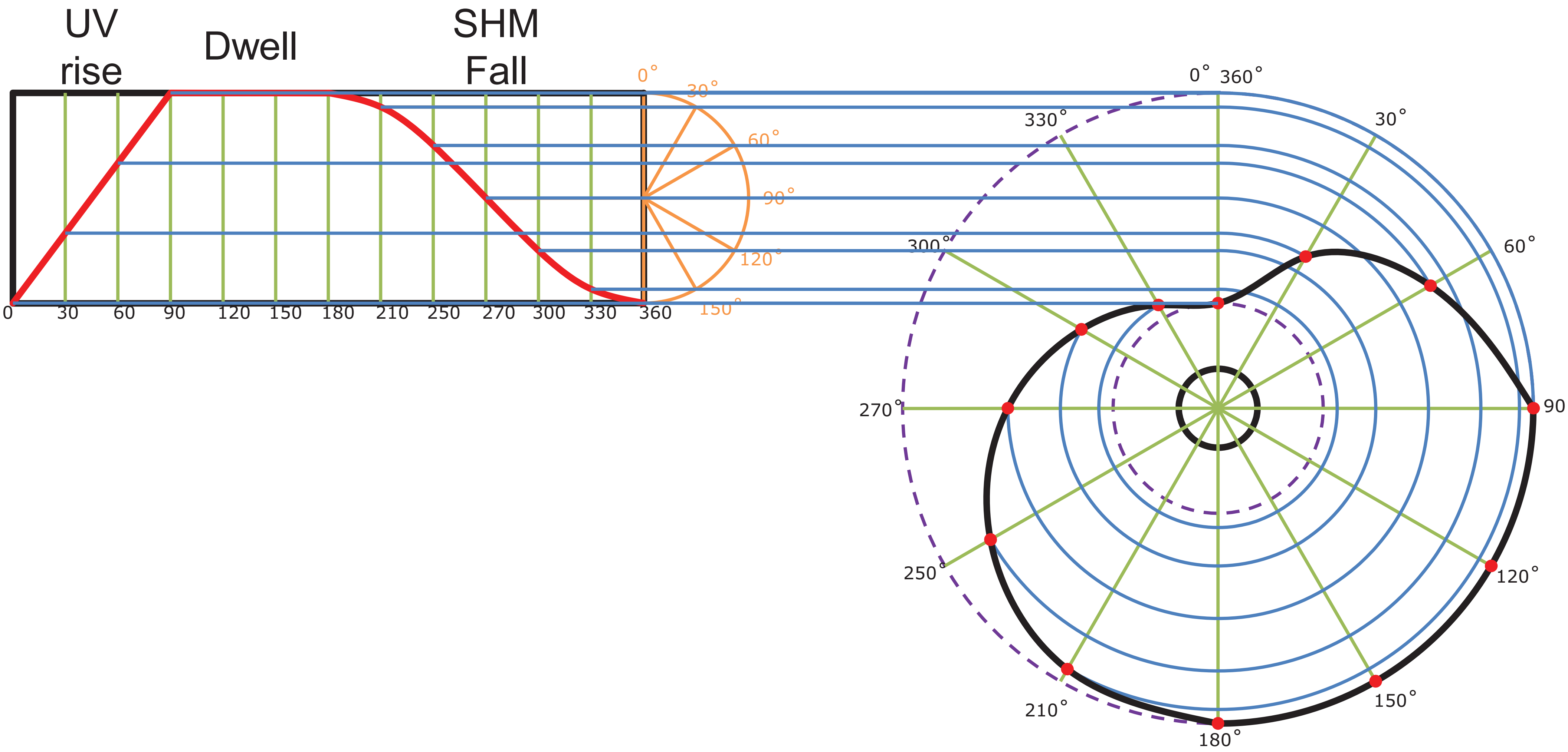
Cams

- Draw the displacement diagram for a plate cam rotating in an anticlockwise direction imparting the following motion to the inline knife edge follower:
  - UV rise 0˚-90˚ of 40mm
  - Dwell 90˚-180˚
  - SHM fall 180˚-360˚ of 40mm
- The nearest approach of the follower to the cam shaft centre is 20mm
- The can shaft diameter is 15mm

Total rise 40

Nearest approach of follower 20mm

Ø15 mm shaft



Design and Comm. Graphics

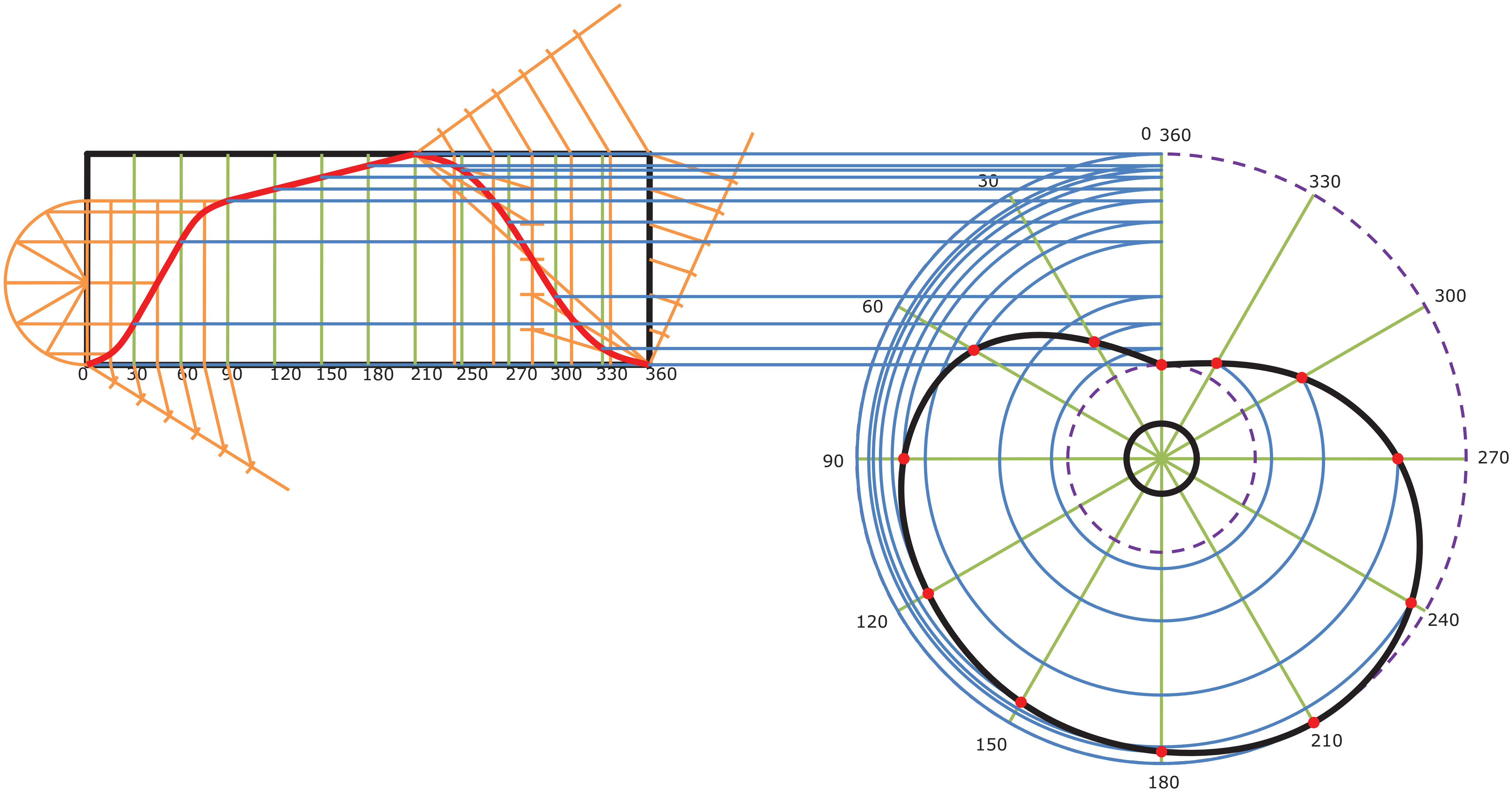
Cams

- Draw the displacement diagram for a plate cam rotating in a clockwise direction imparting the following motion to the inline knife edge follower:
  - SHM rise 0° -90° of 35mm
  - UV rise 90° -210° of 10mm
  - UAR fall 210° -360° of 45mm
- The nearest approach of the follower to the cam shaft centre is 20mm
- The cam shaft diameter is 15mm

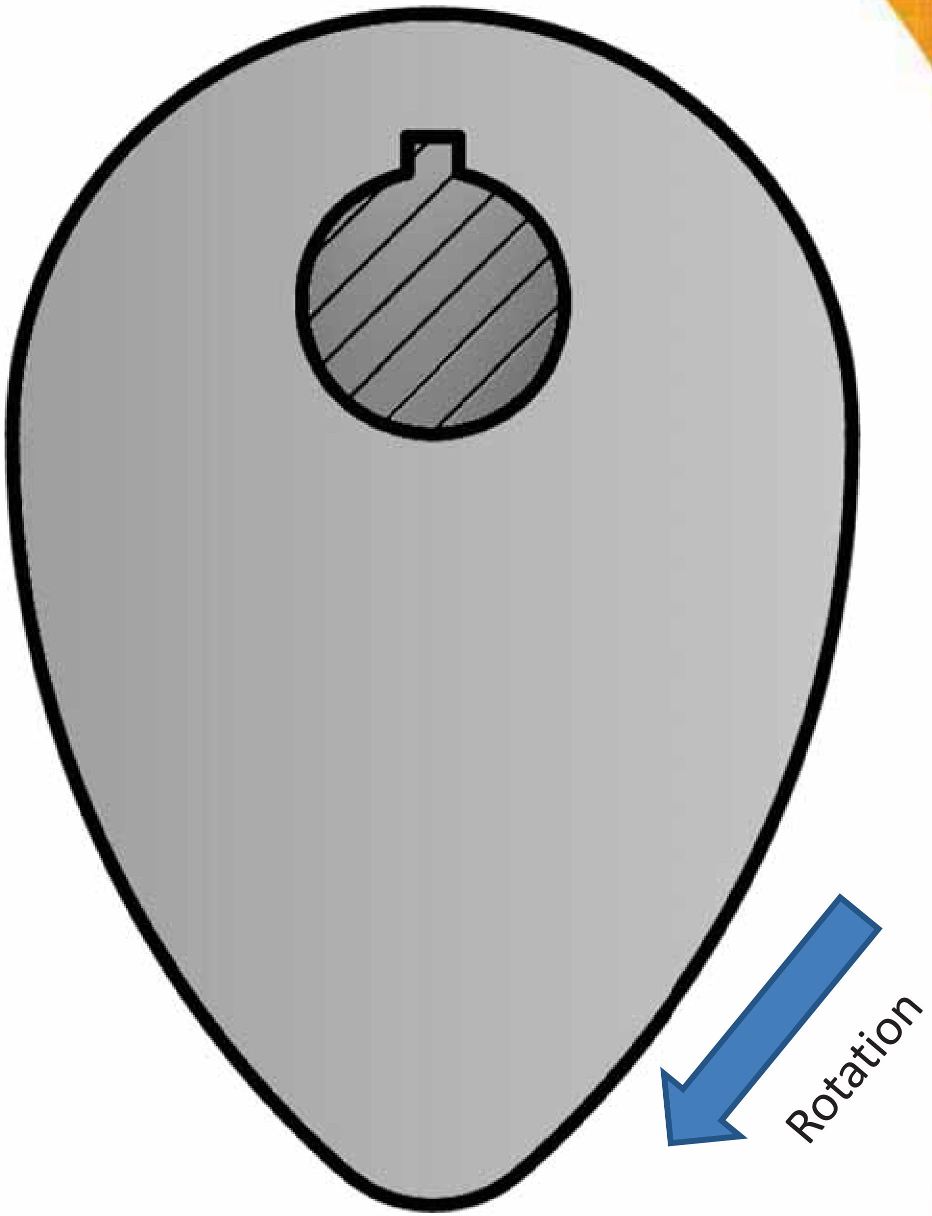
Nearest approach of follower 20mm

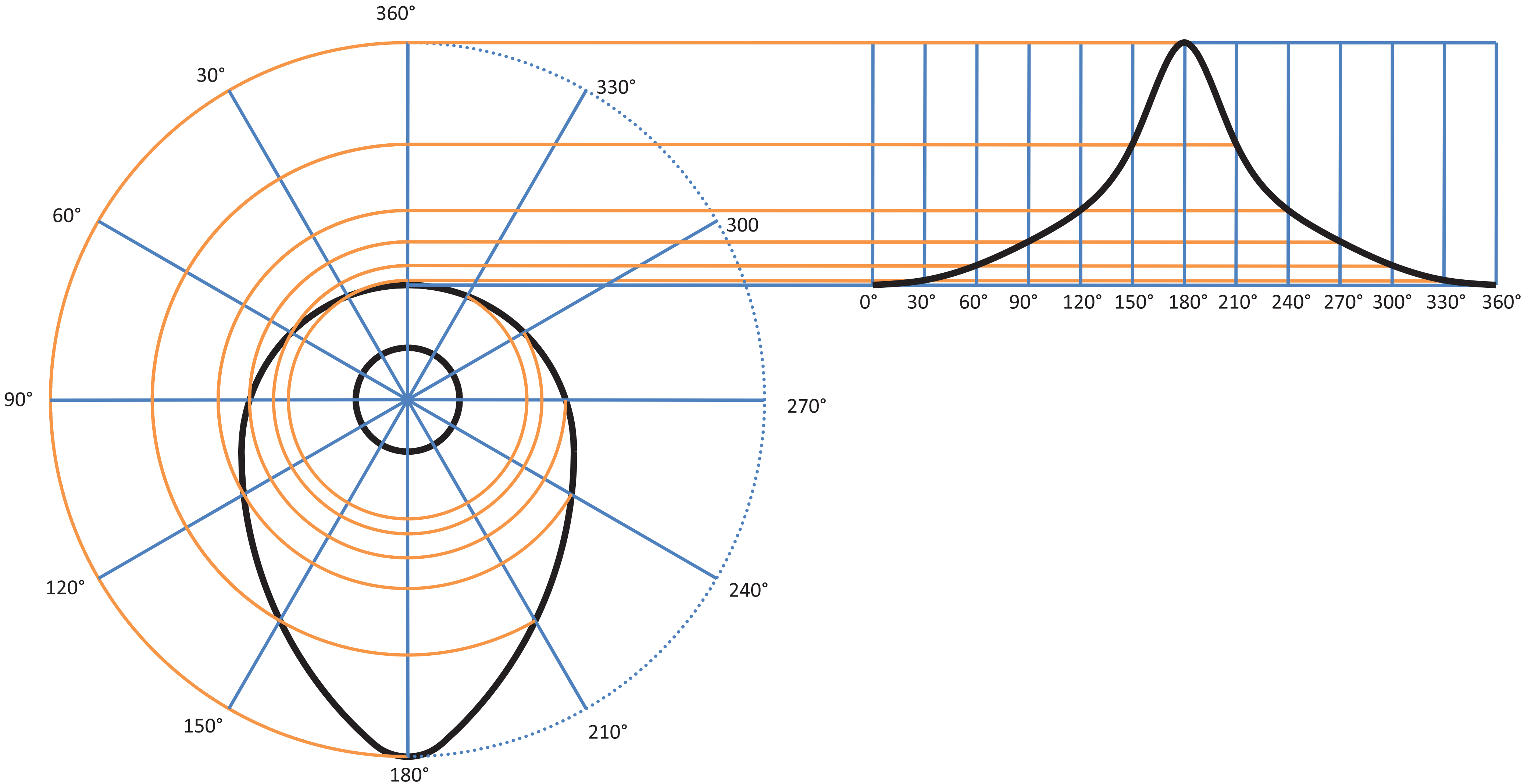
Total rise 45

Ø15 mm shaft



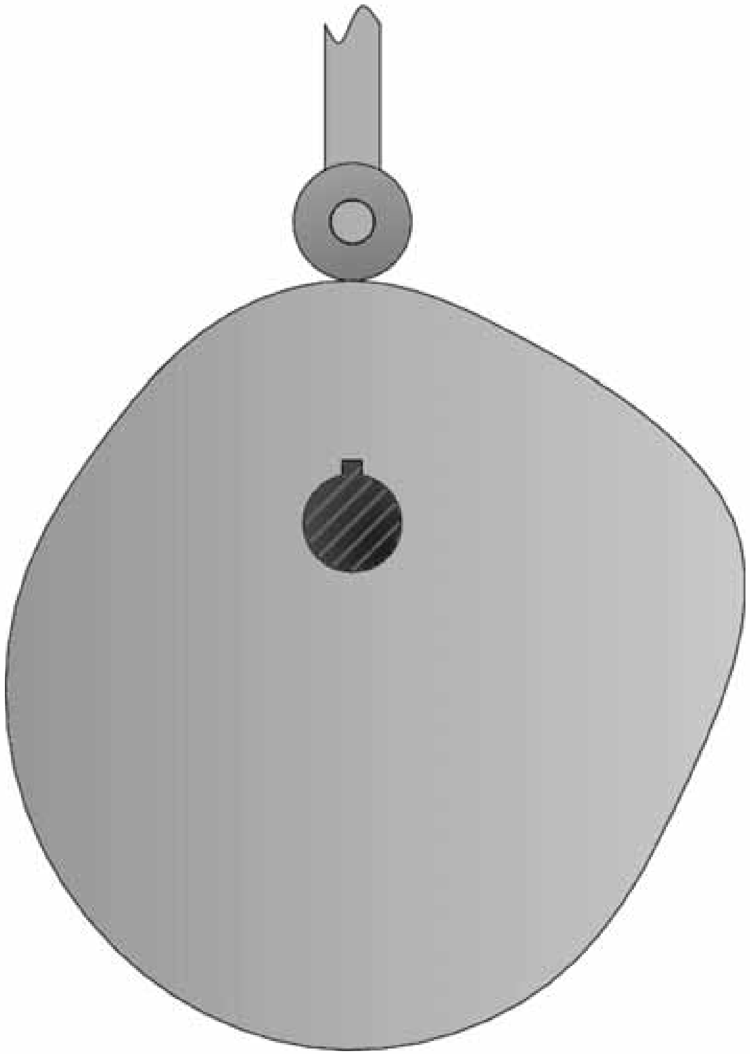
Plot the follower displacement diagram for an in-line knife-edge follower in contact with the cam profile shown below





360°
30°
330°
60°
300
90°
270°
120°
240°
150°
210°
180°
0°
30°
60°
90°
120°
150°
180°
210°
240°
270°
300°
330°
360°

Roller Followers



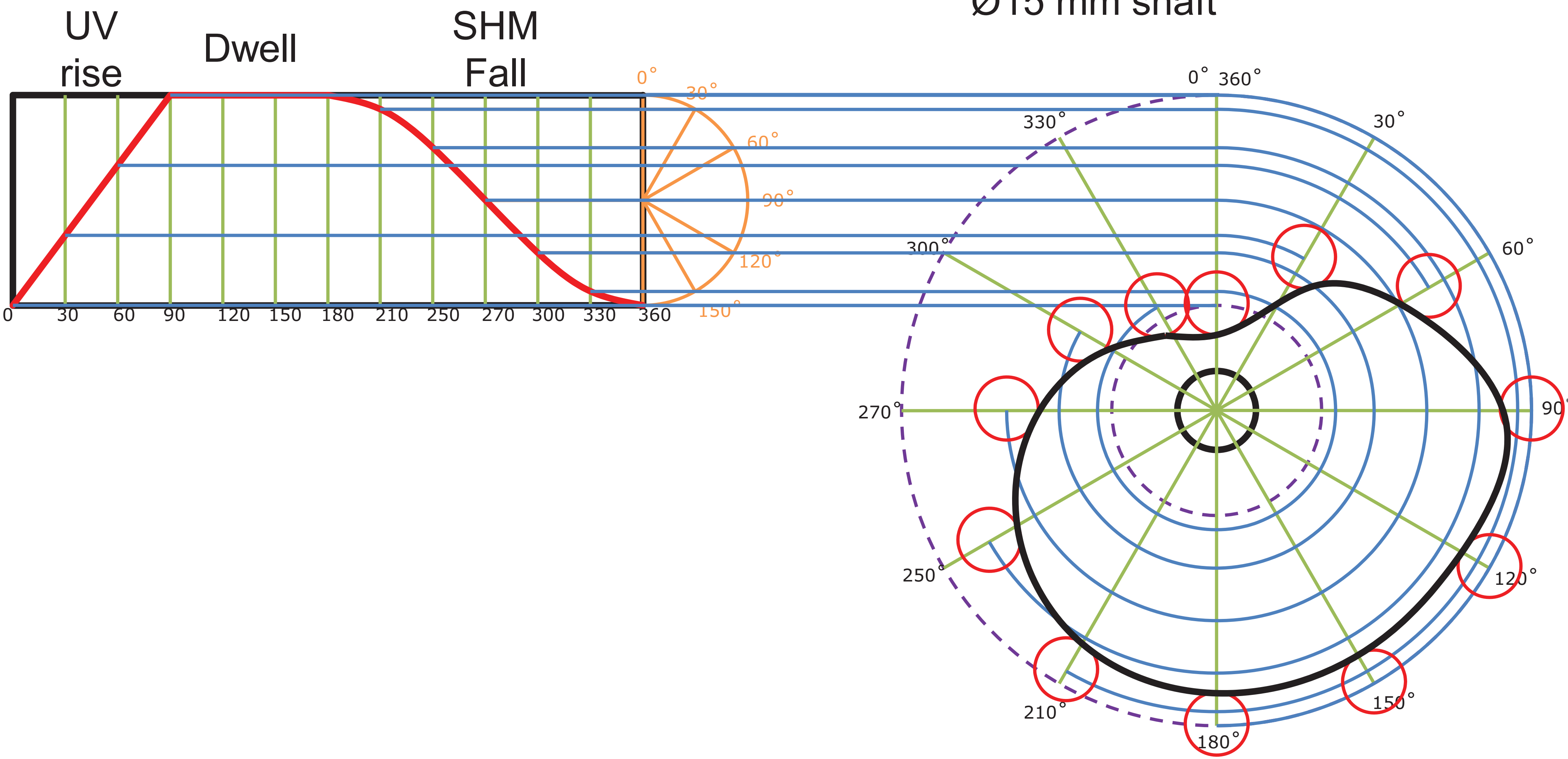
- Are used because they give a smoother movement and they wear more evenly

Cams

- Draw the displacement diagram for a plate cam rotating in an anticlockwise direction imparting the following motion to the roller follower:
  - UV rise 0˚ -90˚ of 40mm
  - Dwell 90˚ -180˚
  - SHM fall 180˚ -360˚ of 40mm
- The roller follower has a diameter of 12mm
- The nearest approach of the roller centre to the cam shaft centre is 20mm
- The cam shaft diameter is 15mm

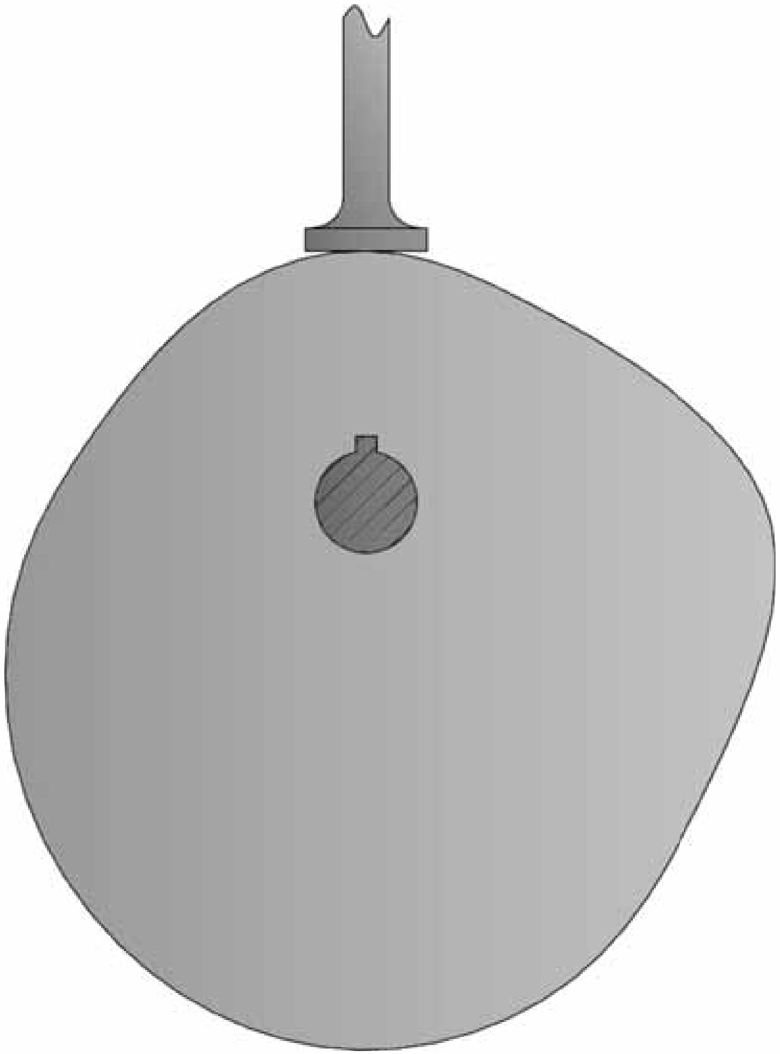
Nearest approach of follower 20mm
Roller Ø12
Ø15 mm shaft

Total rise 40



Design and Comm. Graphics

Flat Footed Follower



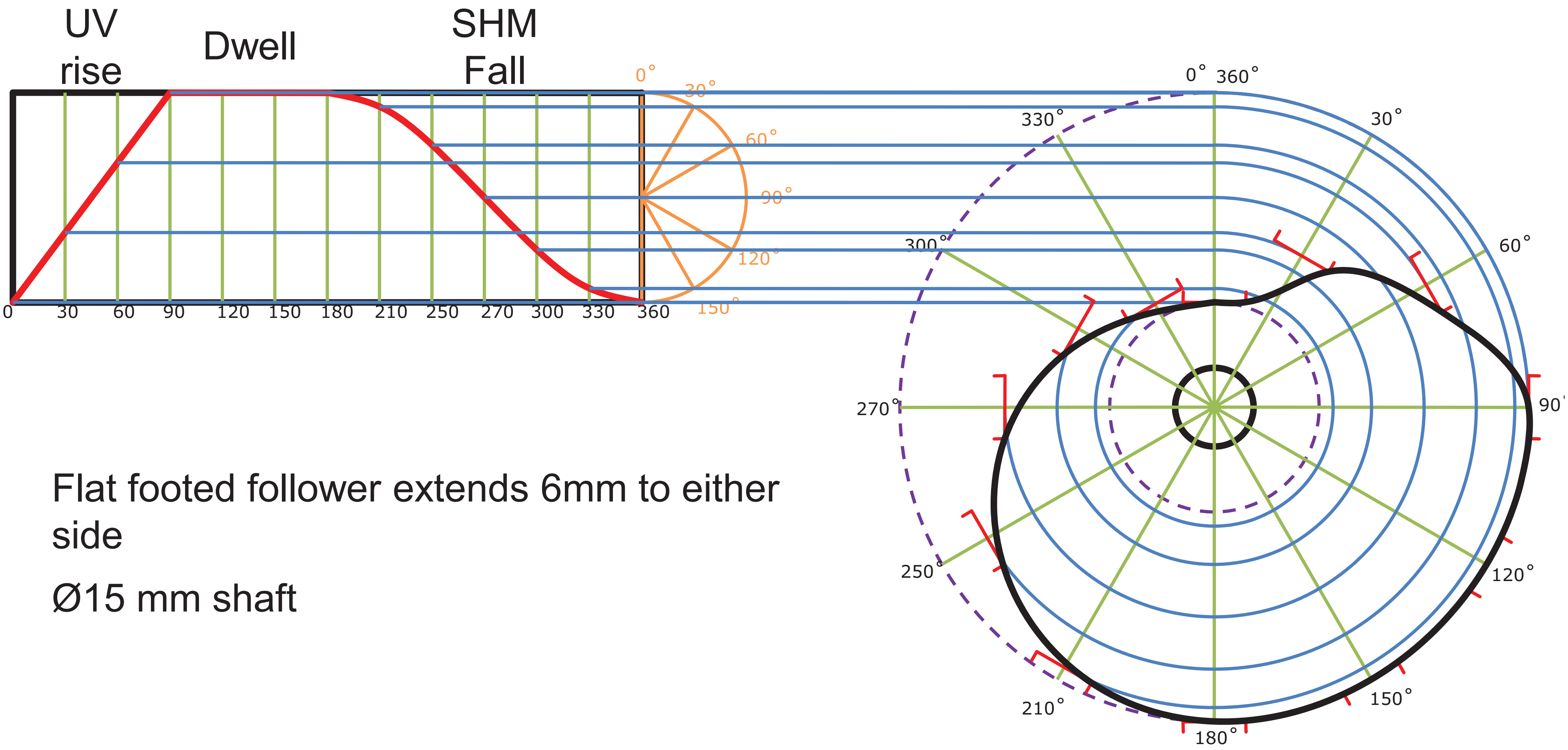
- Wears slower than a knife edge follower
- May bridge over hollows

Cams

- Draw the displacement diagram for a plate cam rotating in an anticlockwise direction imparting the following motion to the flat follower:
  - UV rise 0˚ -90˚ of 40mm
  - Dwell 90˚ -180˚
  - SHM fall 180˚ -360˚ of 40mm
- The follower extends 6mm to either side
- The nearest approach of the follower to the cam shaft centre is 20mm
- The can shaft diameter is 15mm

Nearest approach of follower
20mm

Total rise 40

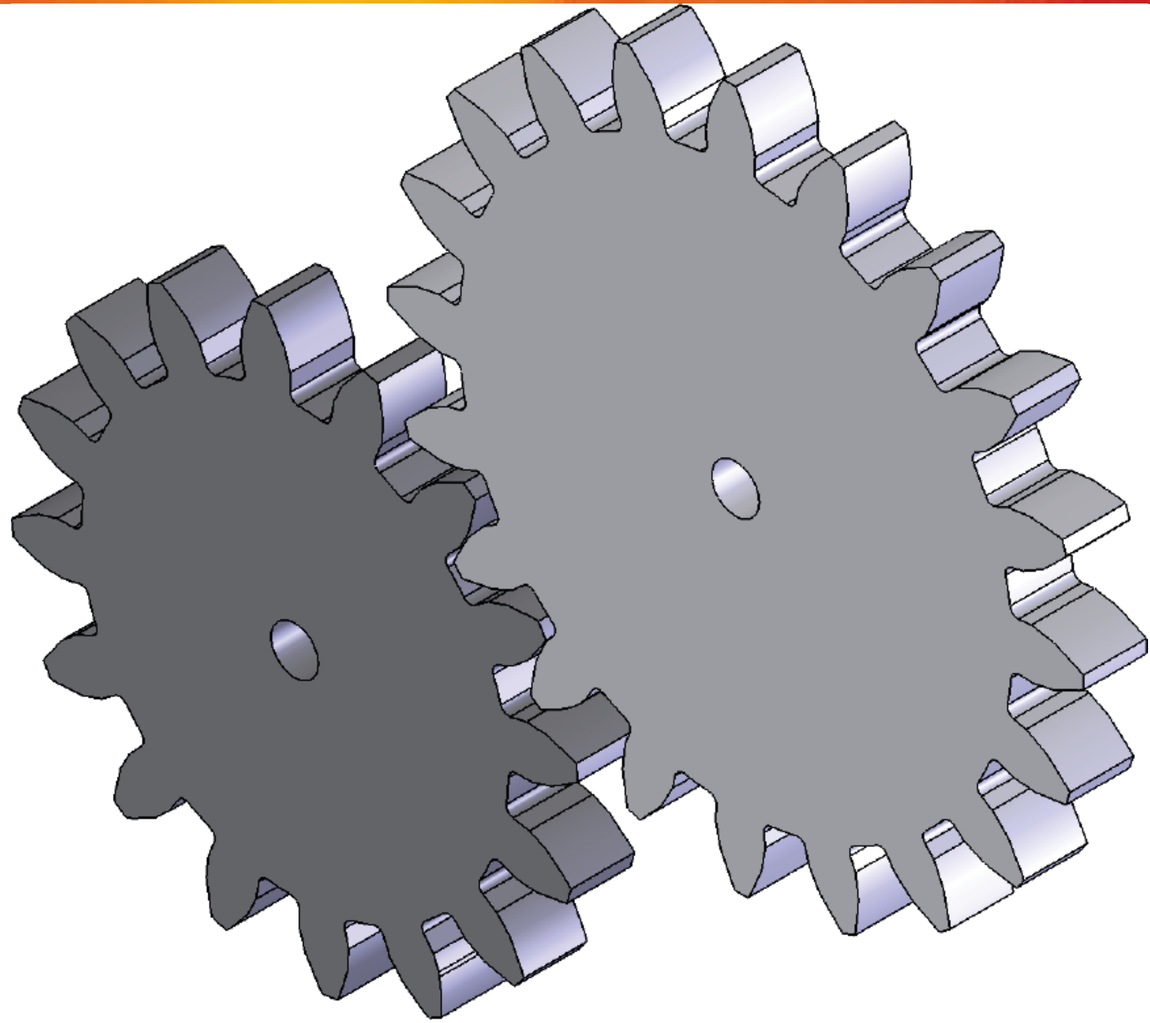


Flat footed follower extends 6mm to either side

Ø15 mm shaft

TECHNOLOGY SUBJECTS SUPPORT SERVICE

GEARS



Gears

- Gears are toothed wheels
- Gears are used to transmit motion
- Gears are also used to convert rotary to linear motion or visa versa
- Gears can be used to reduce or increase the torque on an object
- Gears are found in watches, engines and toys

Types of Gears

- There are many different types of gears, each of which are designed for their specific purpose
- Different types of gears are used in the following machines:
- Drills: Bevel Gears
- Car engines: Helical Gears
- Watches: Epicycloidal Gears
- Power transmission: Involute Spur Gears

Gear animations

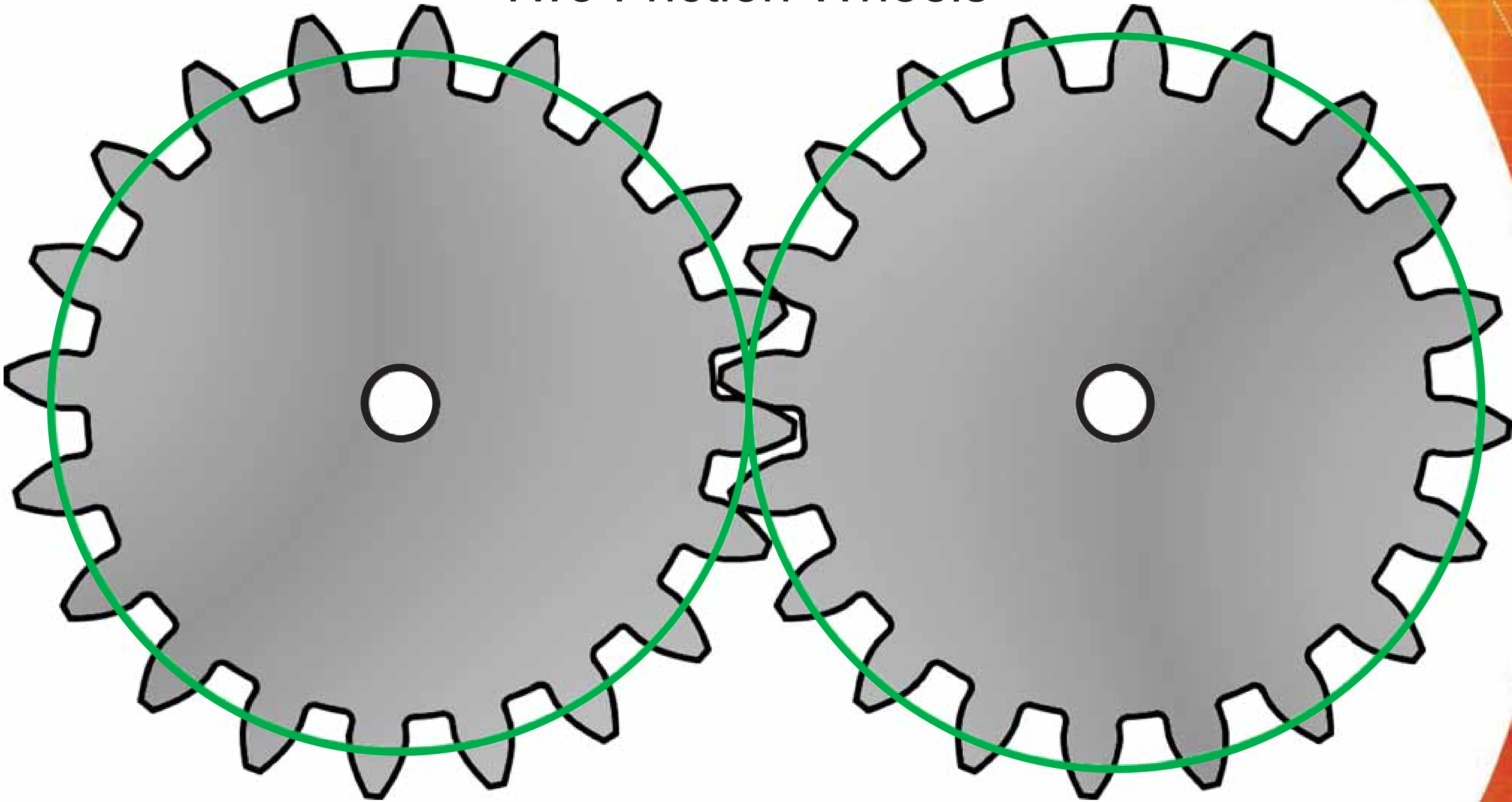
- http://www.mekanizmalar.com/involute1.shtml

Why gears?

- Imagine two disks in contact at their circumference (friction wheels)
- These two disks meet at one point
- If one disk rotates it imparts motion to the other disk
- However these disks are prone to slipping
- Large pressure must be exerted between the disks in order to create a sufficient frictional force between them
- Friction wheels will only be used where low power is required
- Introducing teeth will eliminate slipping occurring

Why gears?

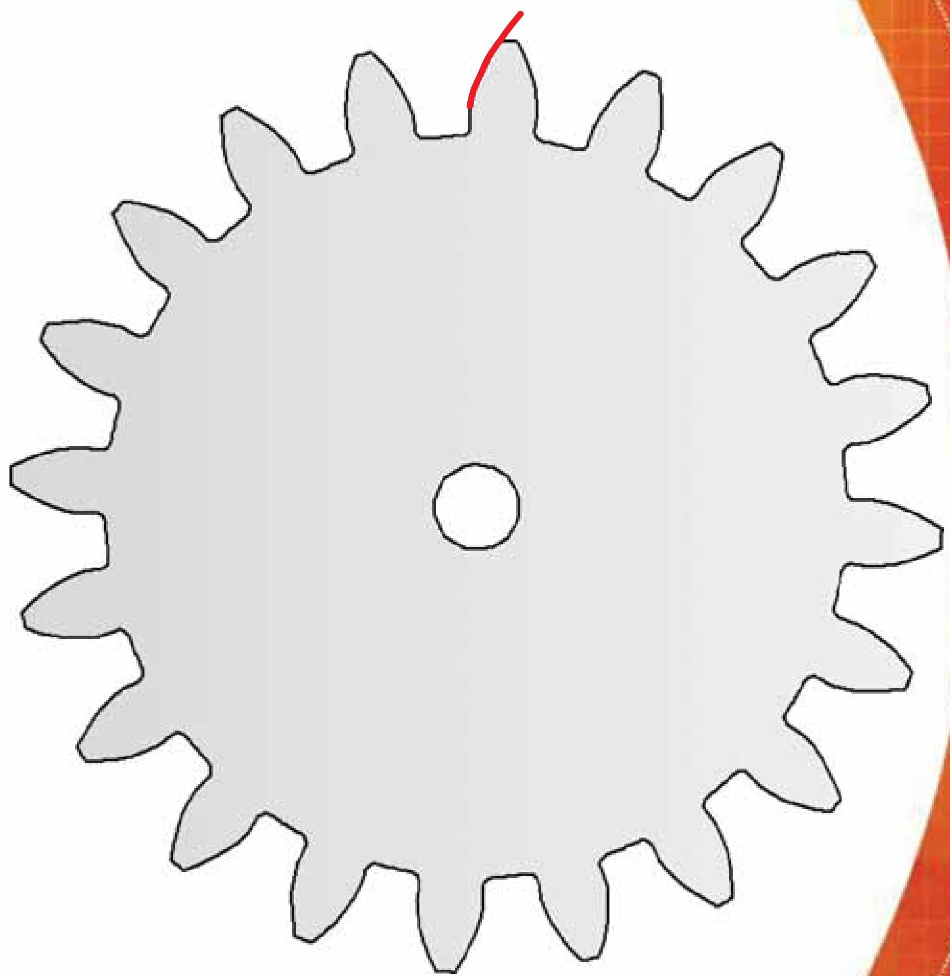
Two Friction Wheels



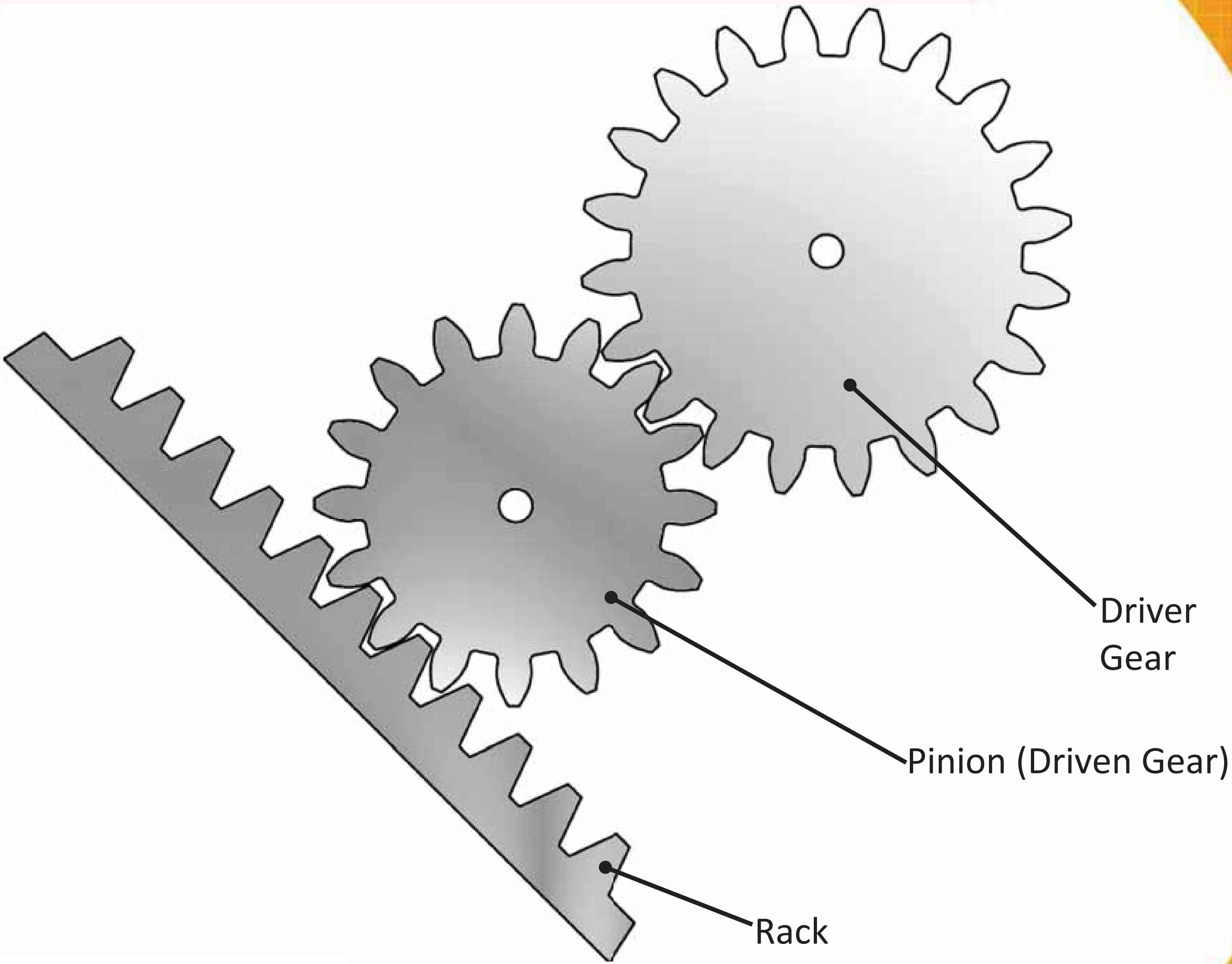
Two Toothed Wheels

Spur Gears

- A spur gear is a toothed wheel
- The shape of the teeth is derived from either an involute curve or an epicycloidal curve
- The involute is the most commonly used curve



Gear Terminology



Gear Terms

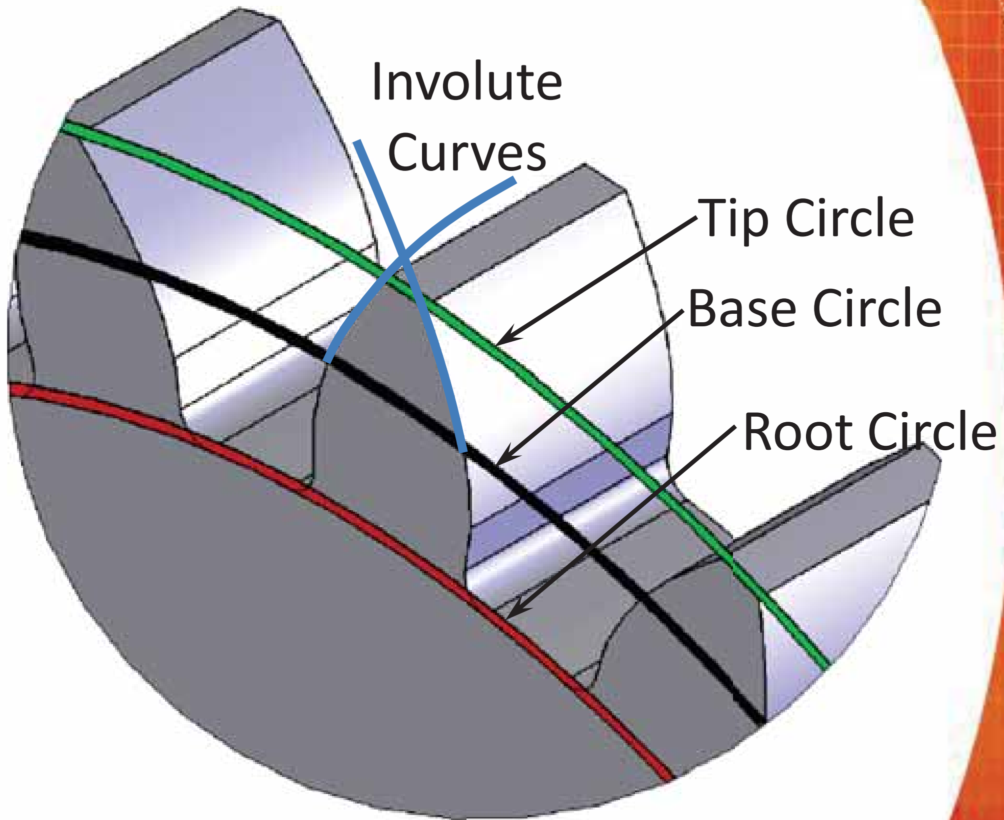
Term	Definition
Driver Gear	When two gears are in mesh the gear with the power (connected to the shaft) is called the driver
Pinion	When two gears are in mesh the smaller gear is called the pinion, and the gear which power is transmitted to is called the driven gear
Rack	It is a spur gear whose radius is at infinity)

Gear ratio

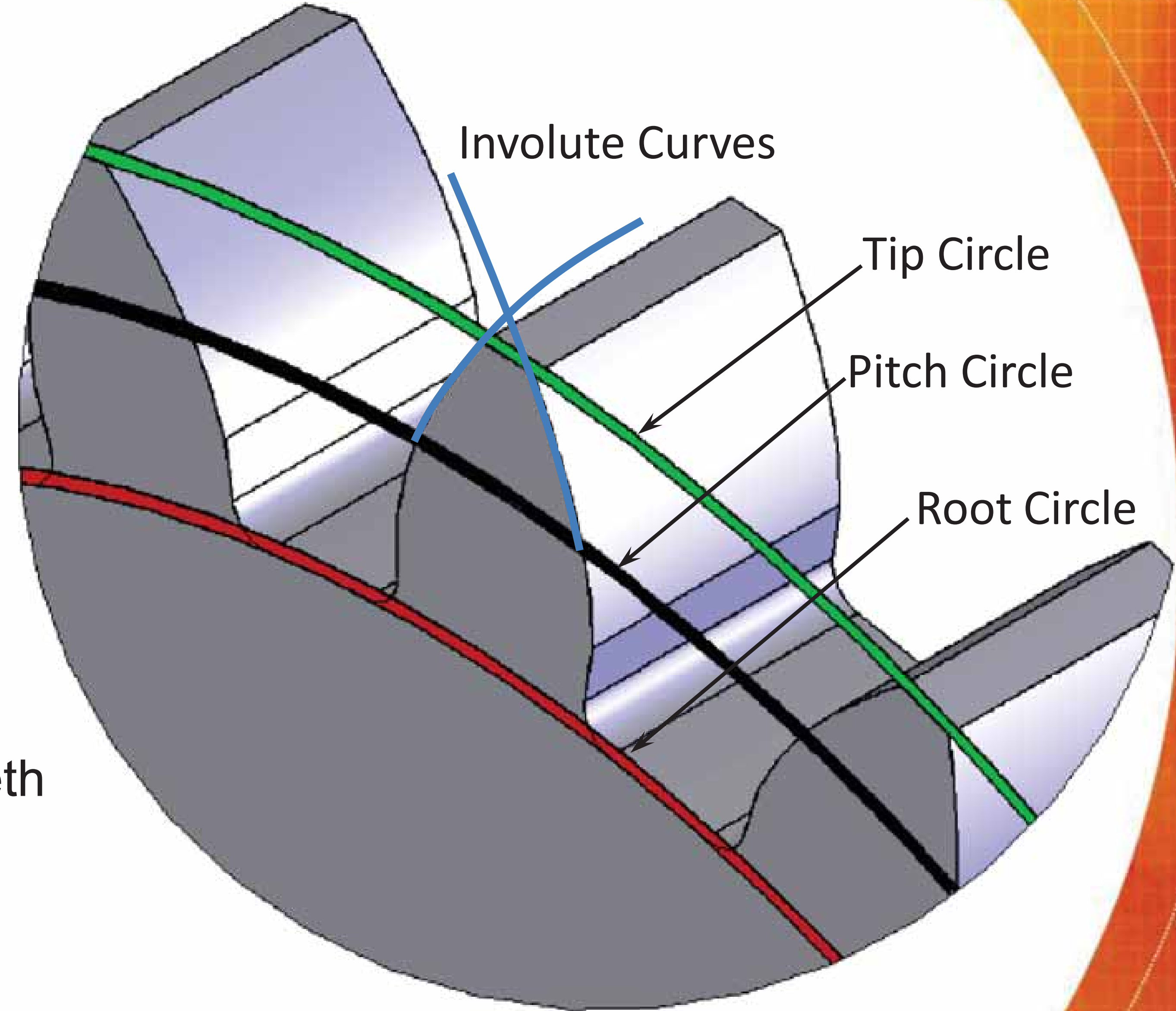
- The gear ratio is the ratio at which one gear rotates relative to the other and it is directly proportional to the diameter of the gears
- Diameter of driver gear = D
- Diameter of driven gear = d
- Gear ratio= D:d
- If the driver gear has a diameter of 200 and the driven gear has a diameter of 100 then the gear ratio will be 2:1
  - The driven gear rotates twice as fast as the driver gear

Gear teeth

- The aim of designing gear teeth is that the faces of the teeth will roll across each other, minimalising the sliding friction
- Two types of curves are commonly used:
  - Involute
  - Epicycloidal

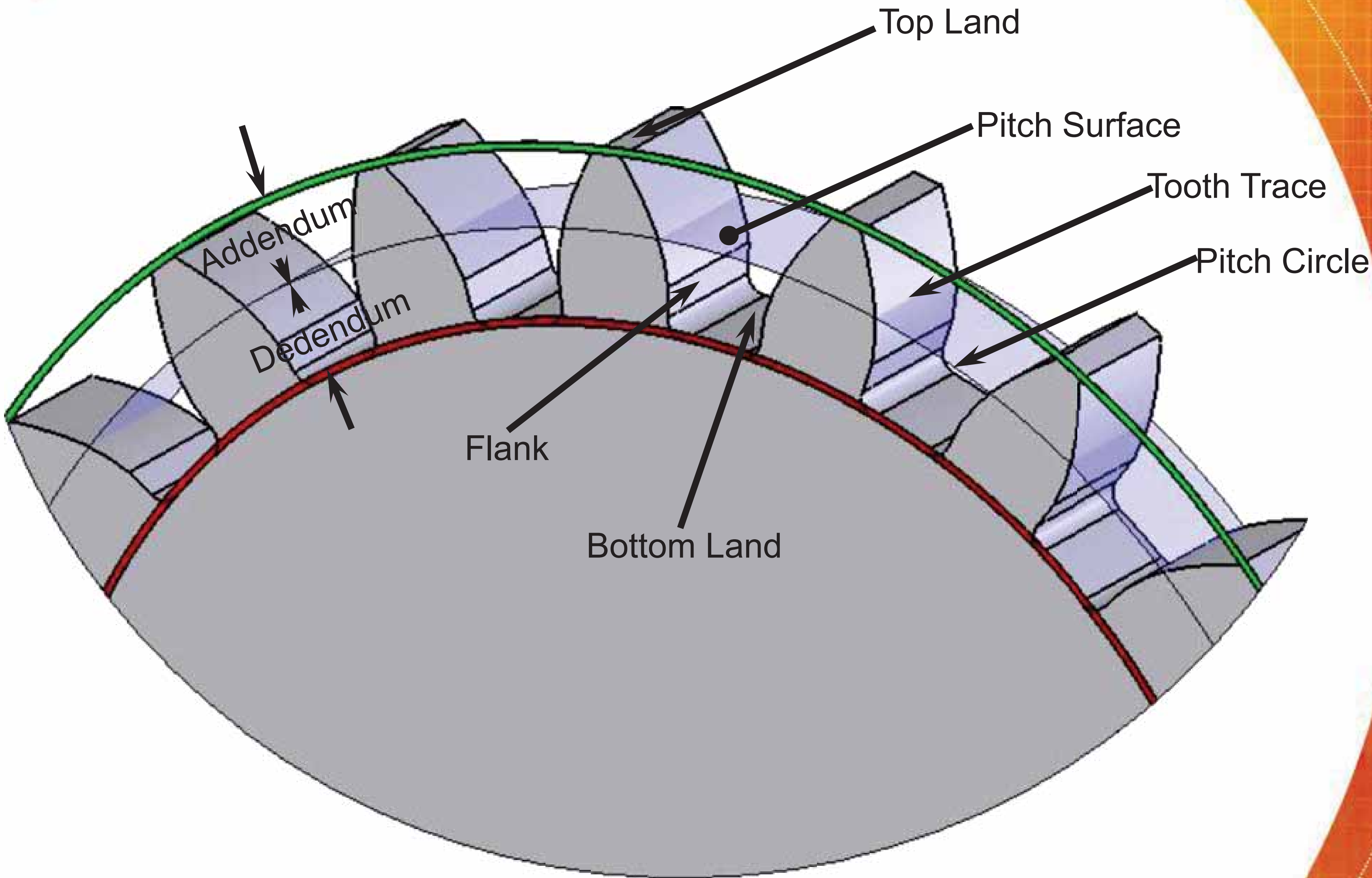


Involute Curves

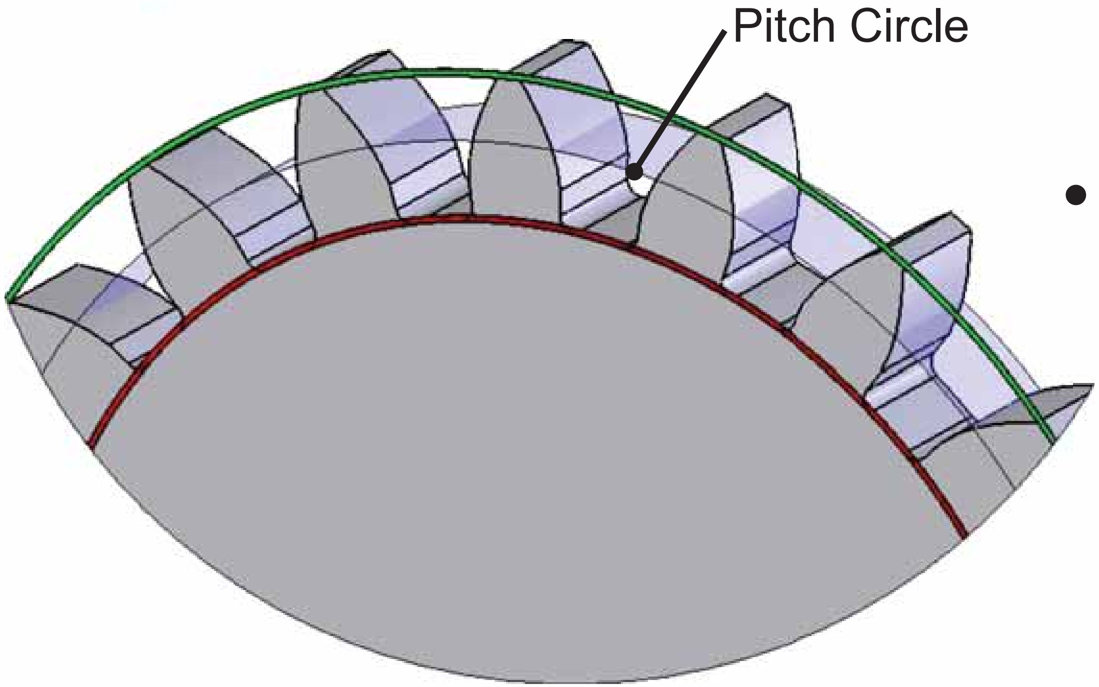


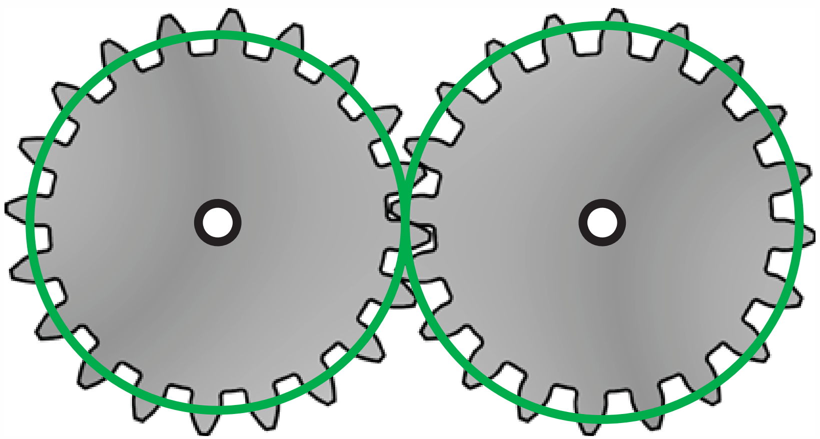
The involute profile provides continuous contact between the teeth while also providing a smooth rolling motion

Parts of a Gear Tooth



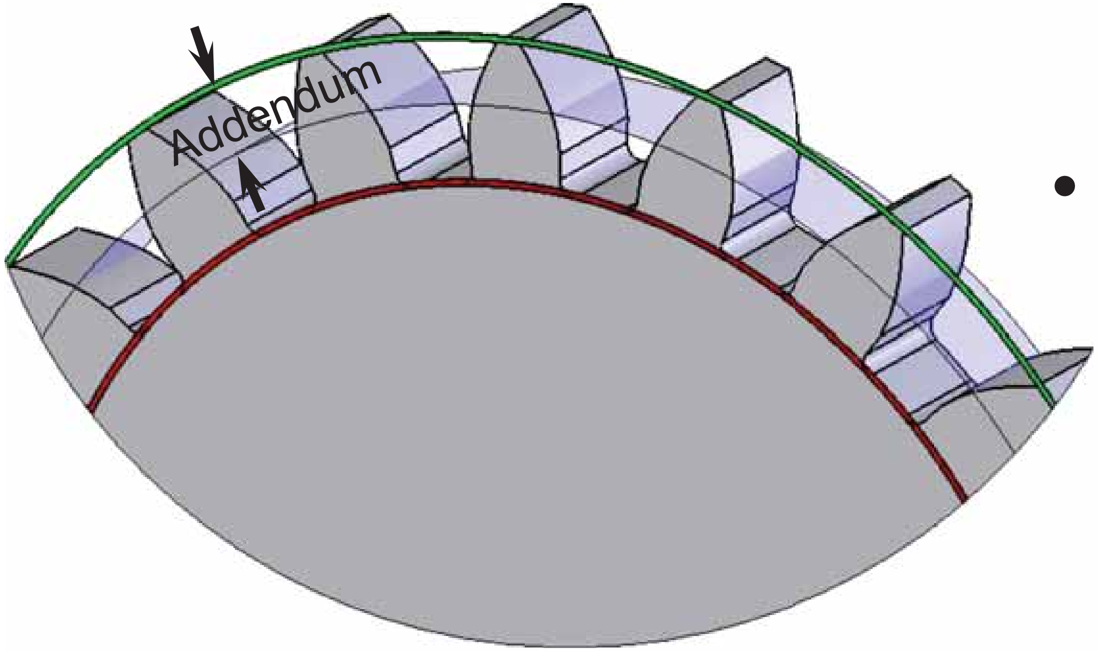
Parts of a Gear Tooth





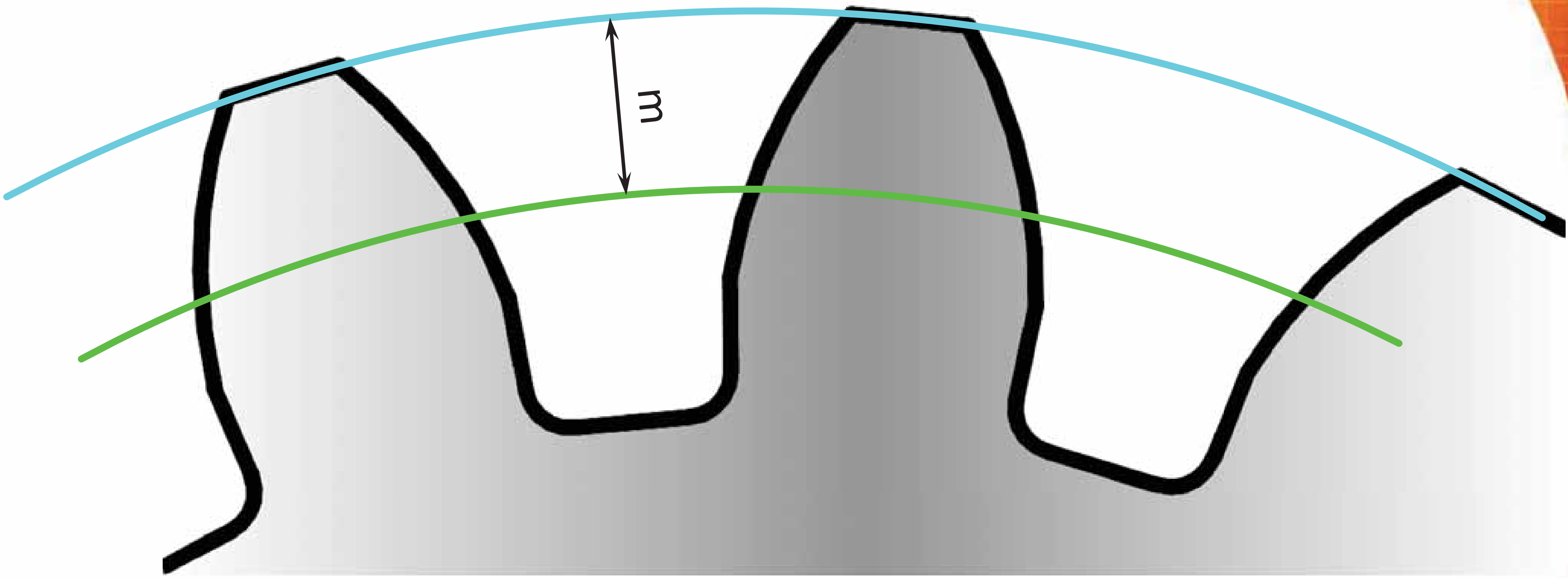
- An imaginary circle which corresponds to the outside diameter of the friction rollers from which the spur gears are derived.
- Formula =
  - Module × Number of teeth

Parts of a Gear Tooth

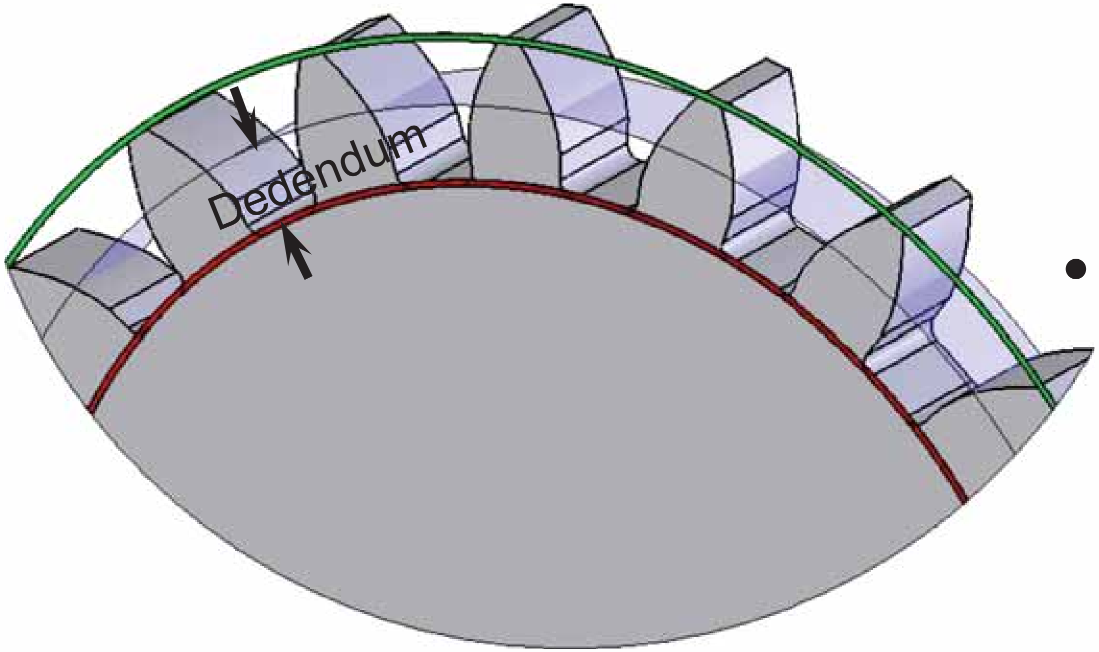


- Radial distance from the pitch circle to the top of the tooth
- Formula:
  - Addendum = module

Addendum

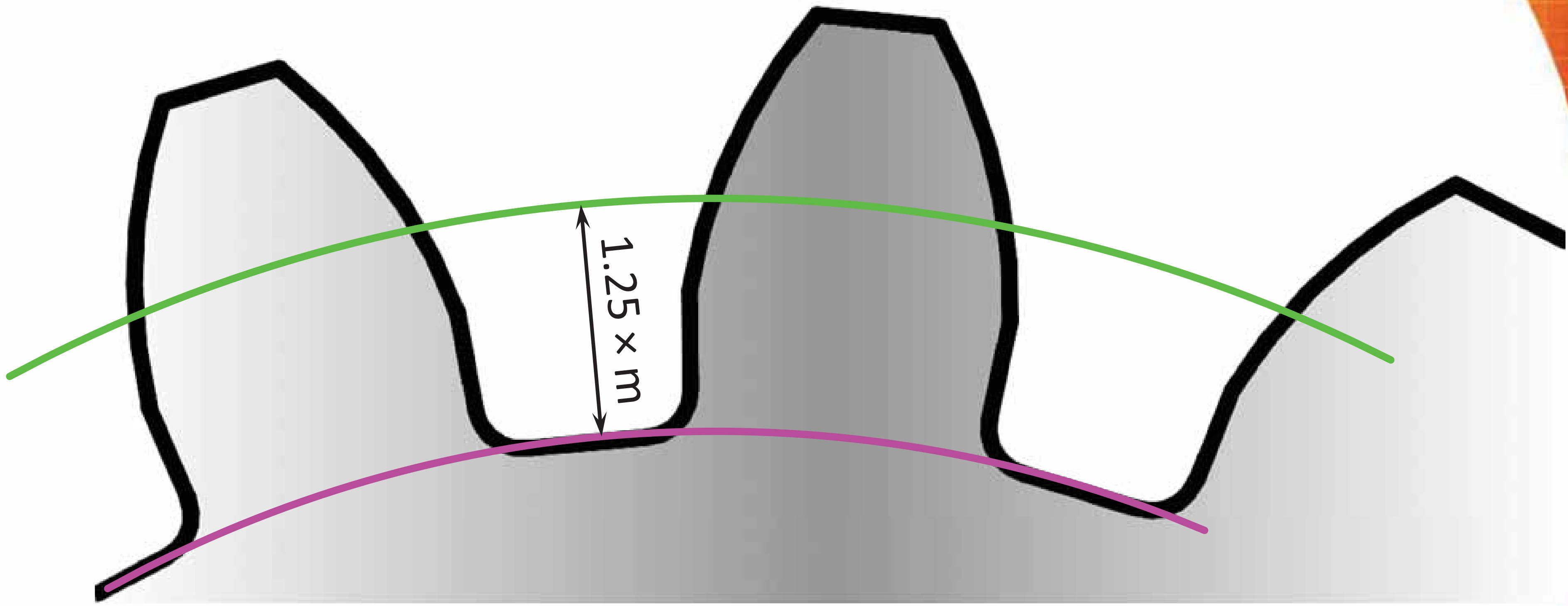


Parts of a Gear Tooth

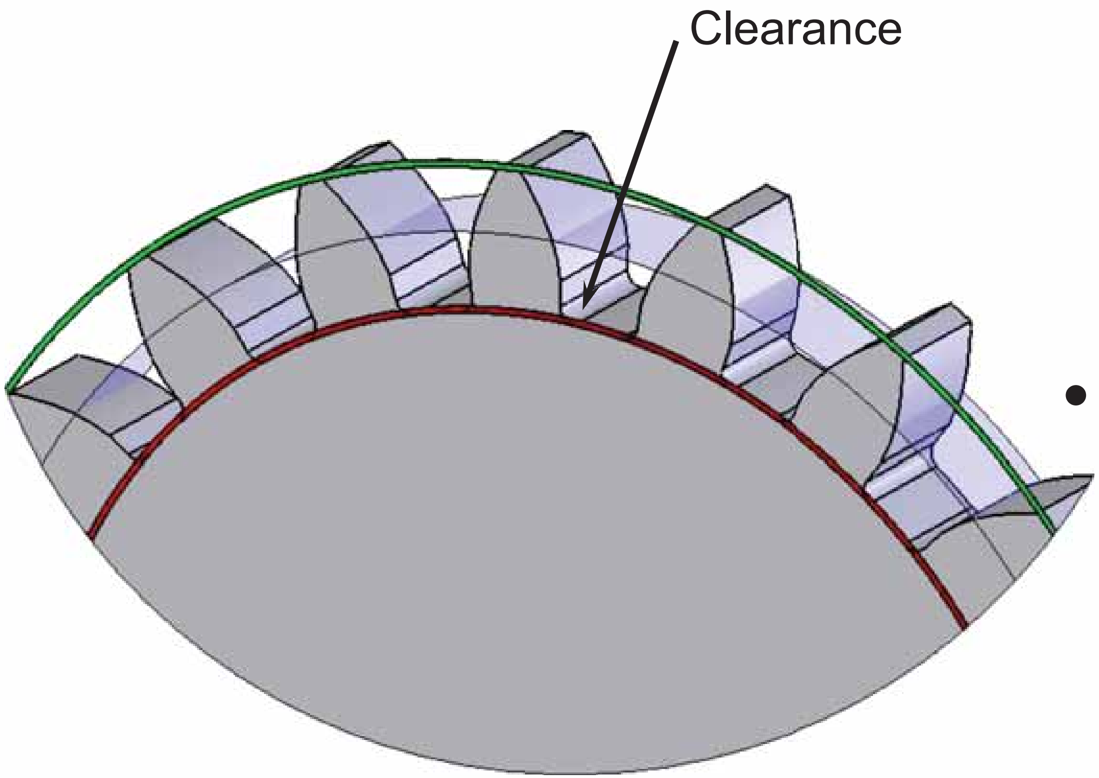


- Radial distance from the pitch circle to the bottom of the tooth space
- Formula:
  - Dedendum = 1.25 × module

Dedendum

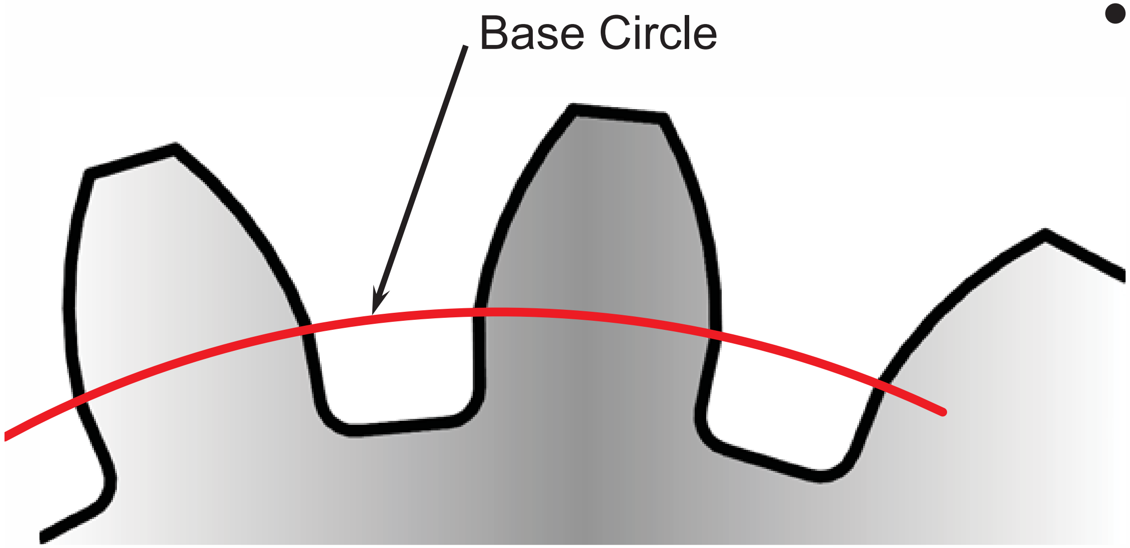


Parts of a Gear Tooth



- Distance between the top of a tooth and the bottom of the mating space
- Formula:
  - Dedendum - Addendum

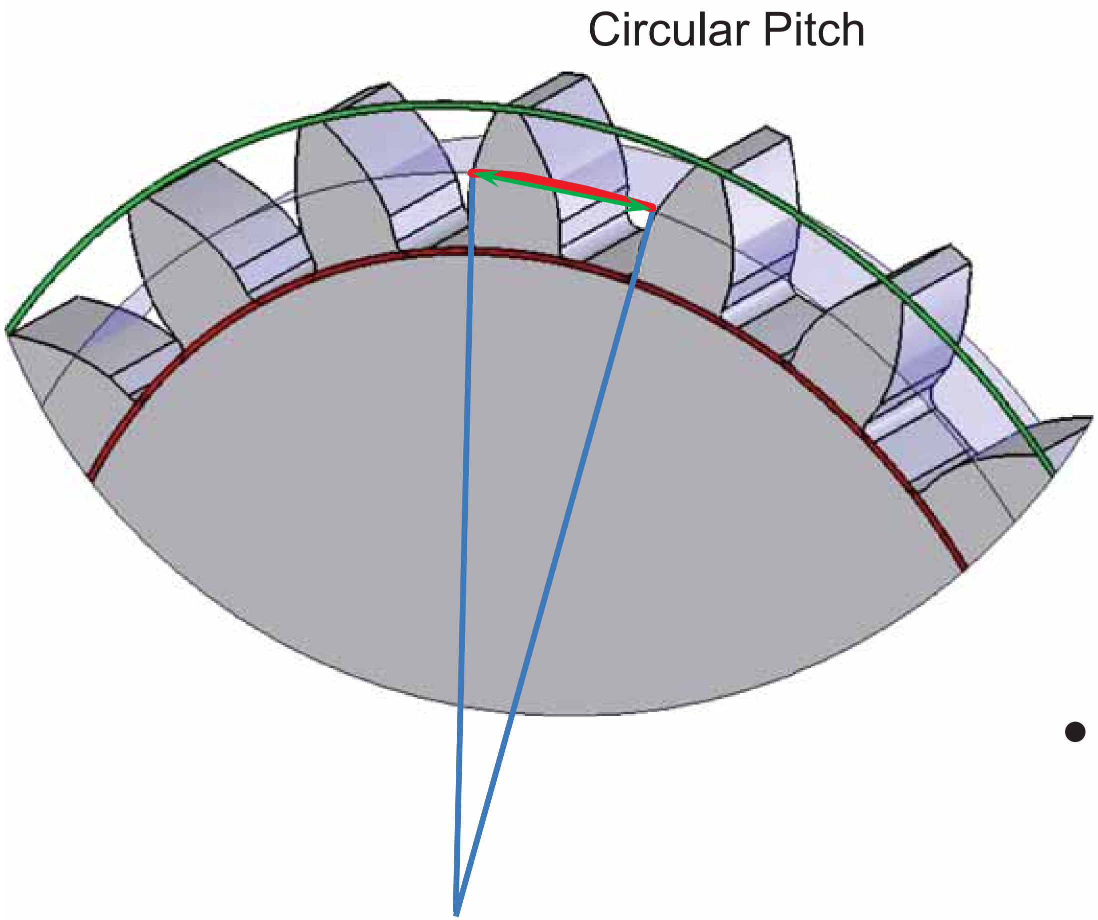
Parts of a Gear Tooth



- The base circle is the imaginary circle from which the involute is created

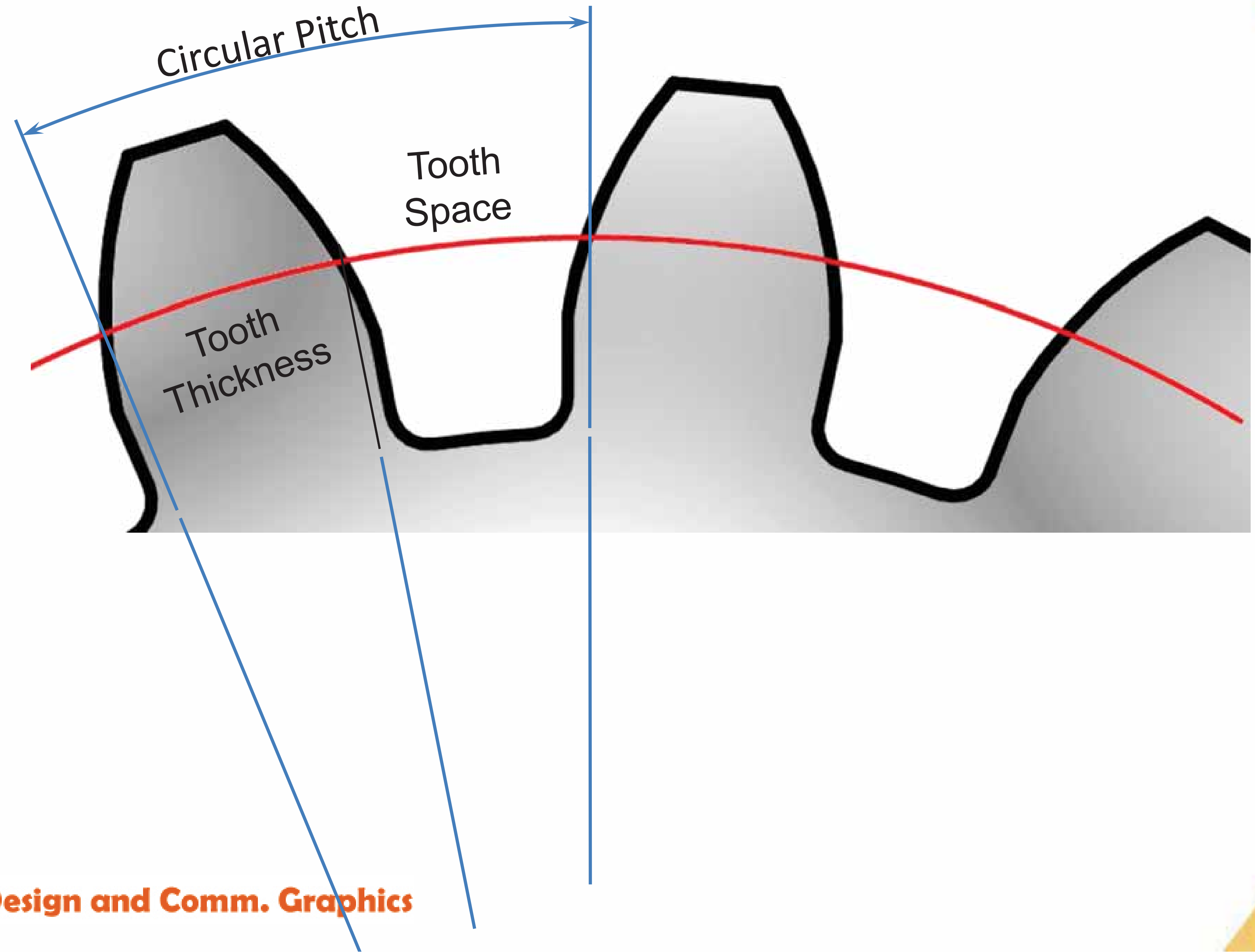
  Formula:
  - Base Circle Diameter = Pitch Circle Diameter × cos (pressure angle)

Parts of a Gear Tooth

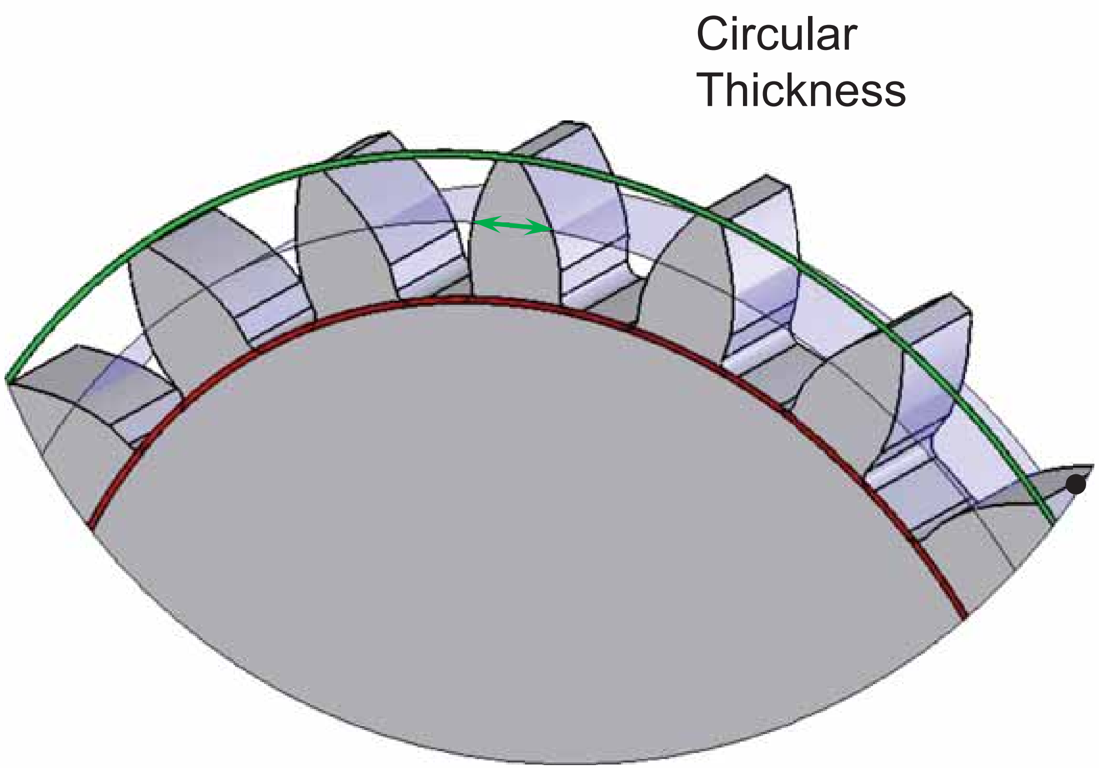


- Distance measured along the pitch circle from a point on one tooth to a corresponding point on the next tooth. This includes one tooth and one space.
- Formula:
  - {πd(circumference)} ÷ n

Circular Pitch

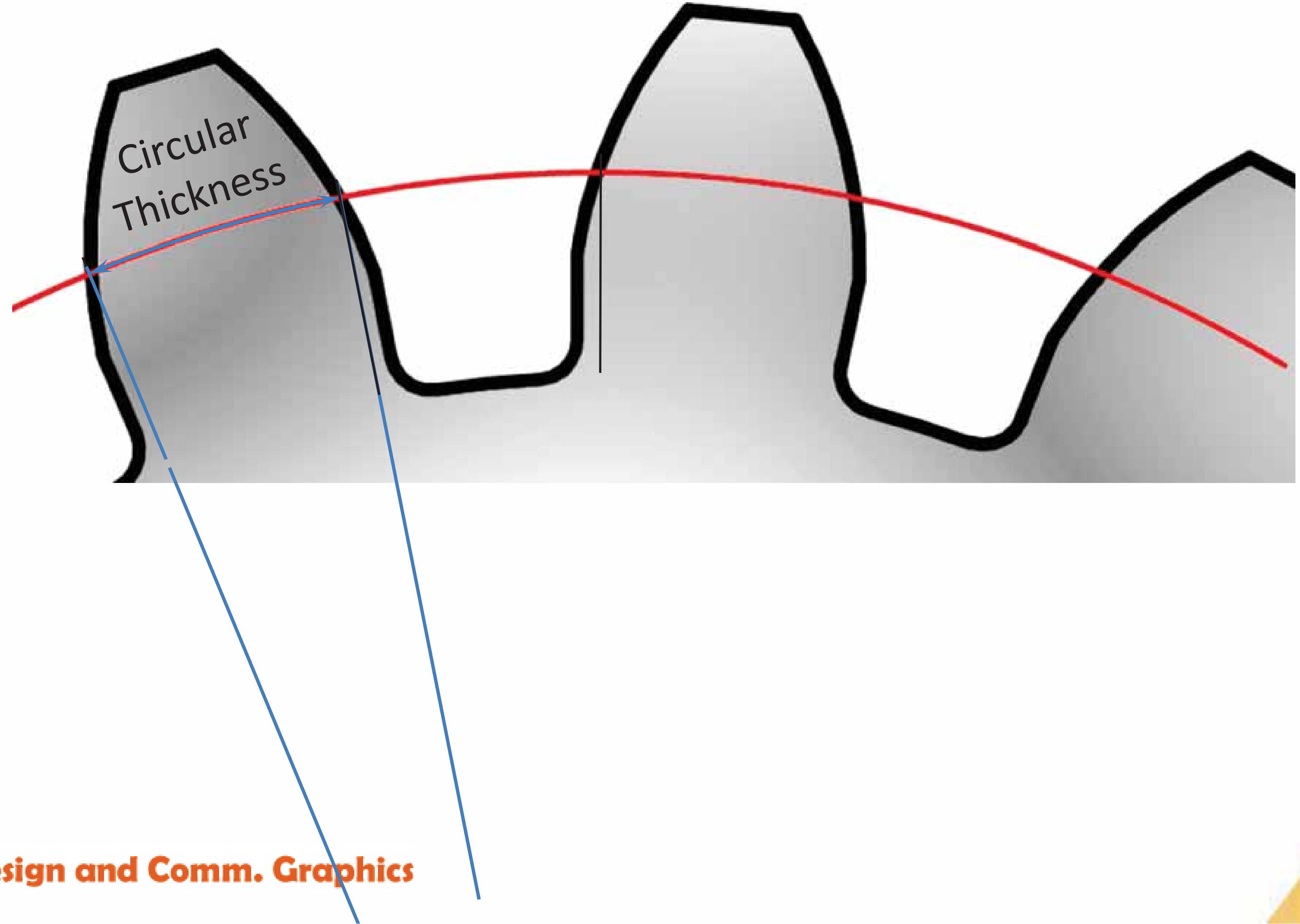


Parts of a Gear Tooth

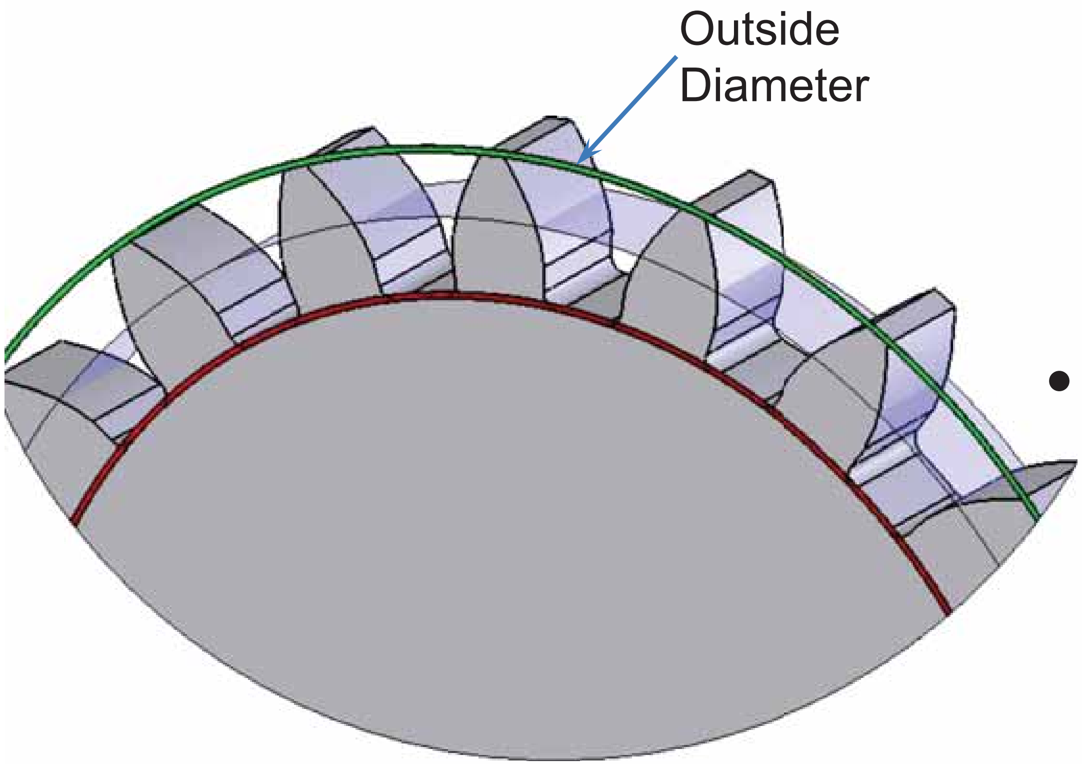


- Circular thickness: Thickness of one tooth measured along the pitch circle, equal to ½ the circular pitch.
- Formula:
  - Circular pitch ÷ 2

Circular Thickness

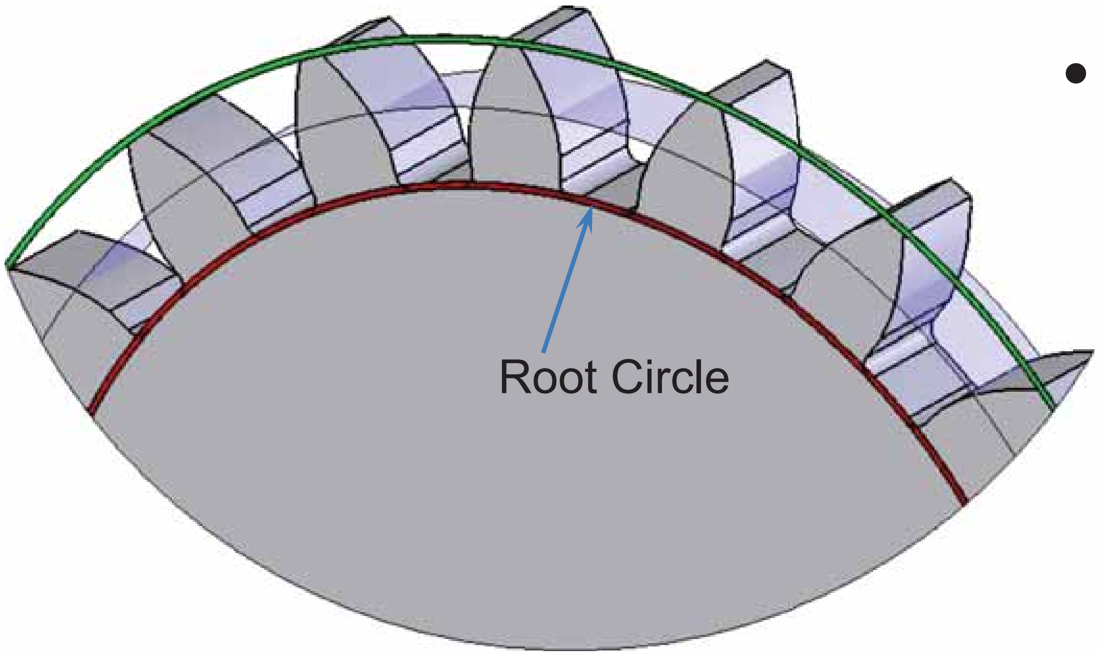


Parts of a Gear Tooth



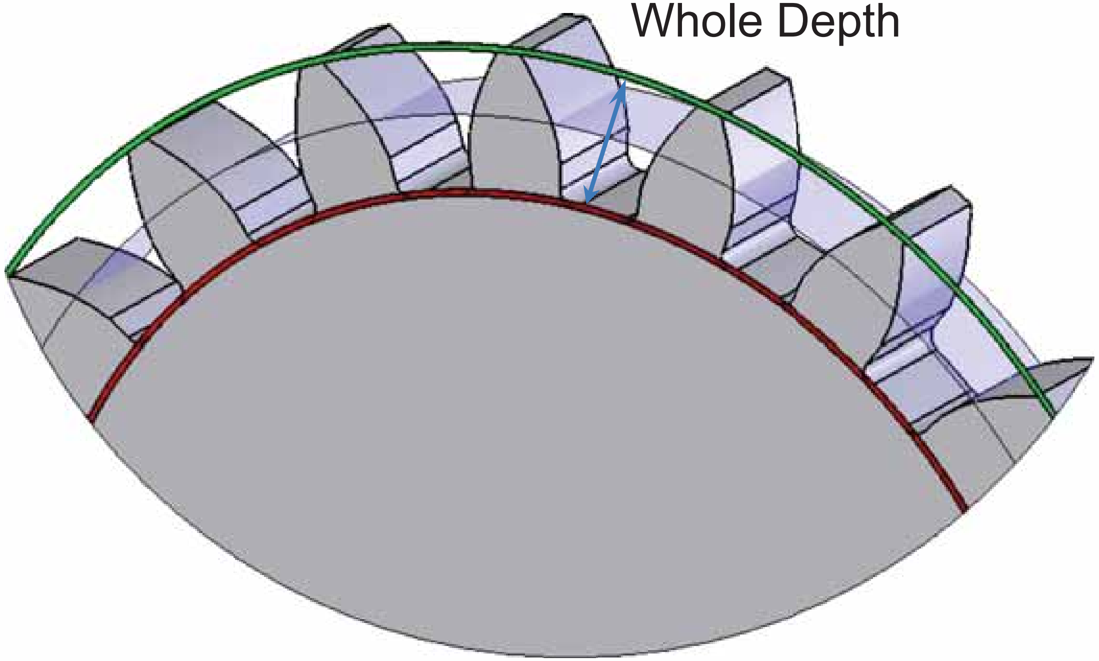
- Outside Diameter: is the diameter of the circle that contains the top of the teeth =
- Formula:
  - Pitch Circle Diameter + 2 addendum

Parts of a Gear Tooth



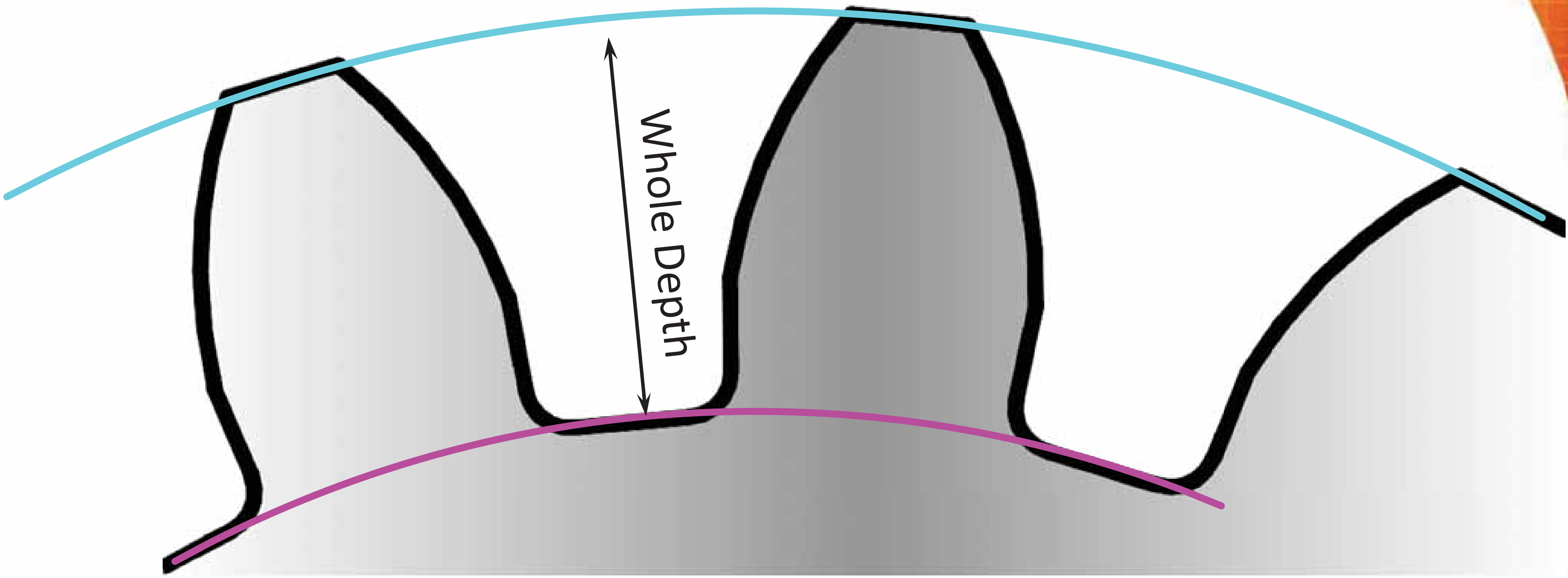
- Diameter of the root circle
- Formula:
  – Pitch Circle Diameter - 2 dedendum

Parts of a Gear Tooth

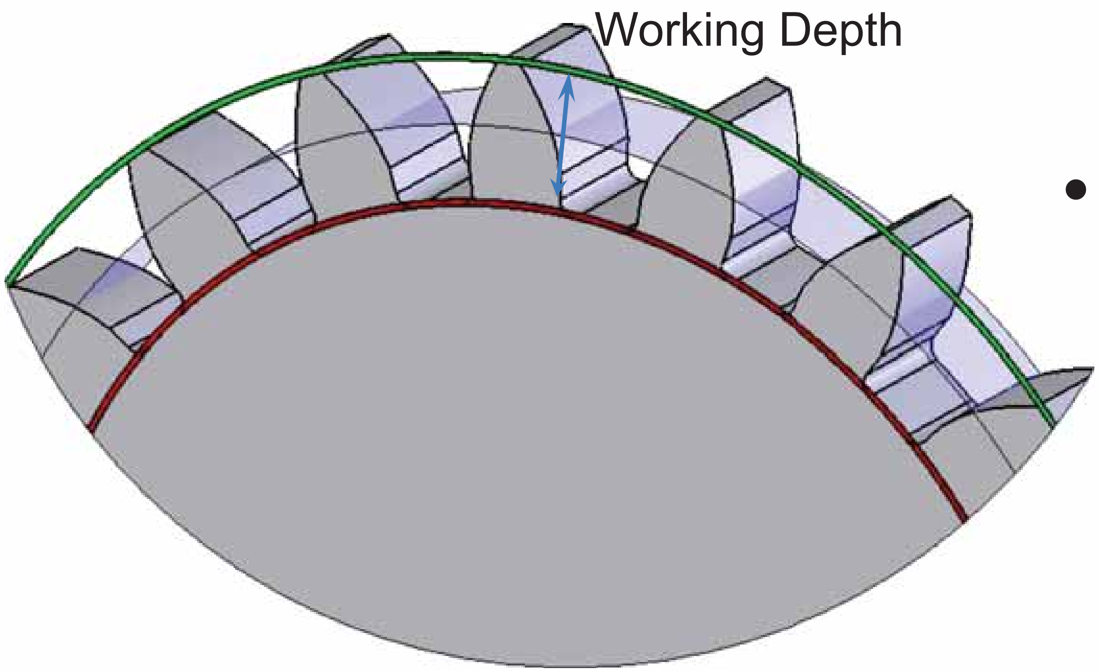


- Full height of the tooth
- Formula:
  - Addendum + Dedendum

Whole Depth

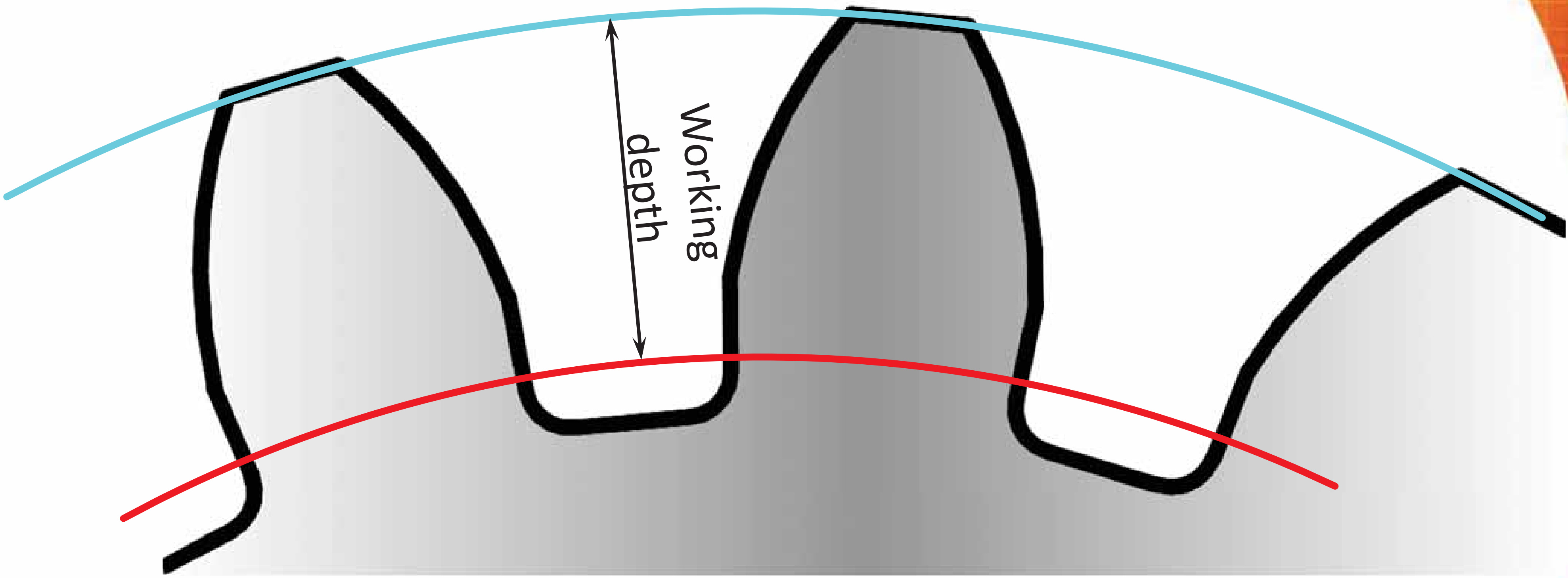


Parts of a Gear Tooth

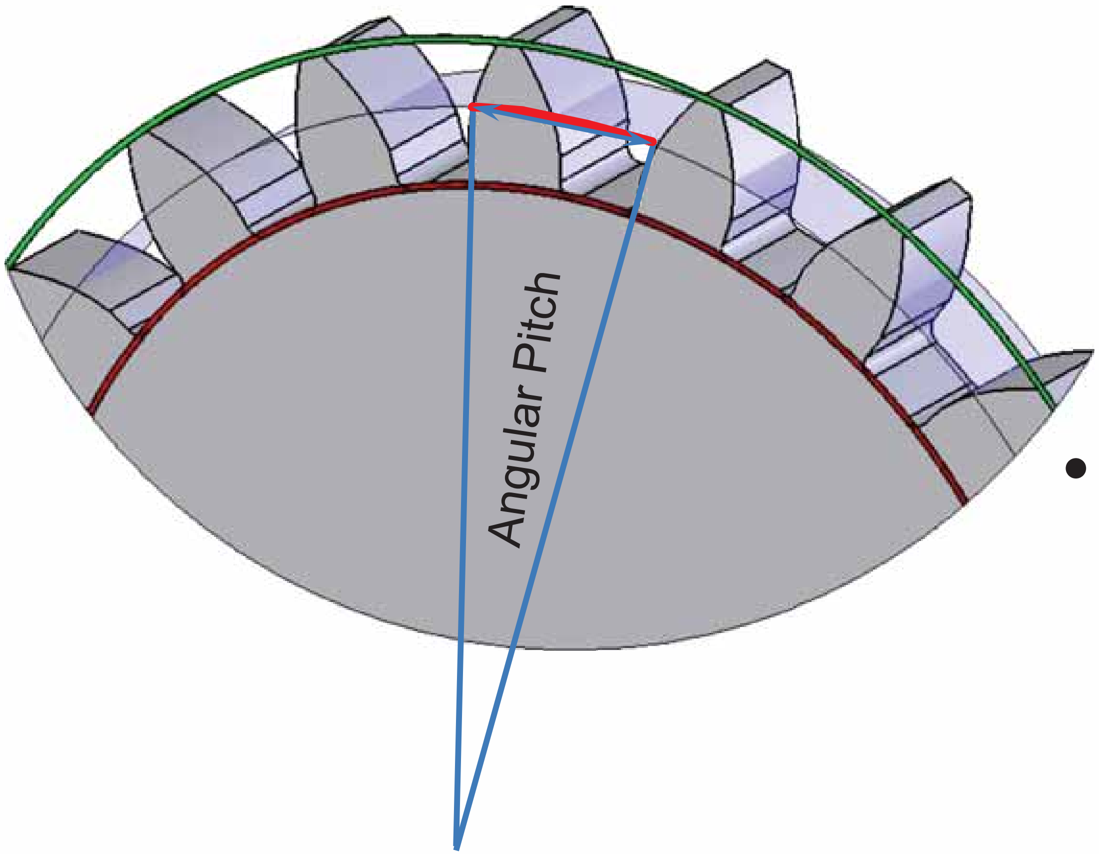


- Distance a tooth projects into the mating space
- Formula:
  - 2 × Addendum

Working Depth

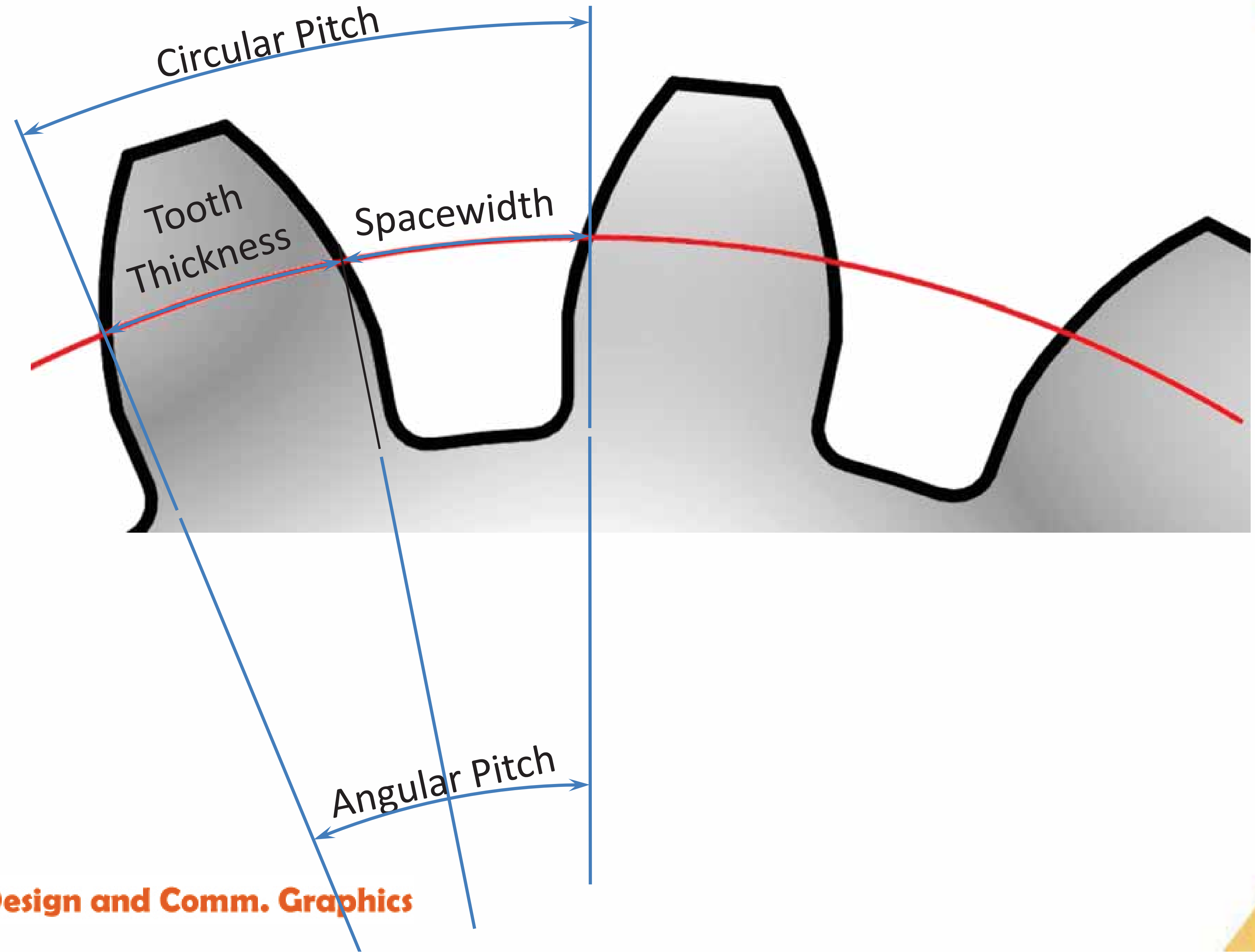


Parts of a Gear Tooth



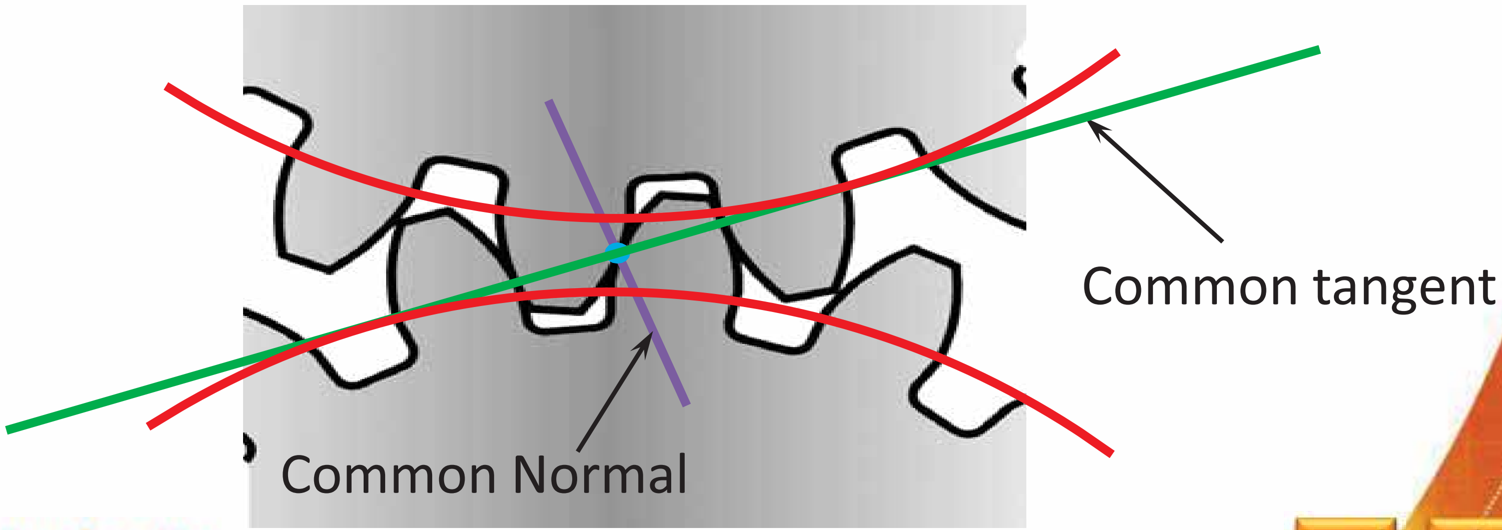
- Angle created at the centre of the gear between a point on one tooth on the PCD, and the corresponding point on an adjacent tooth
- Formula:
  - 360 ÷ number of teeth

Angular Pitch

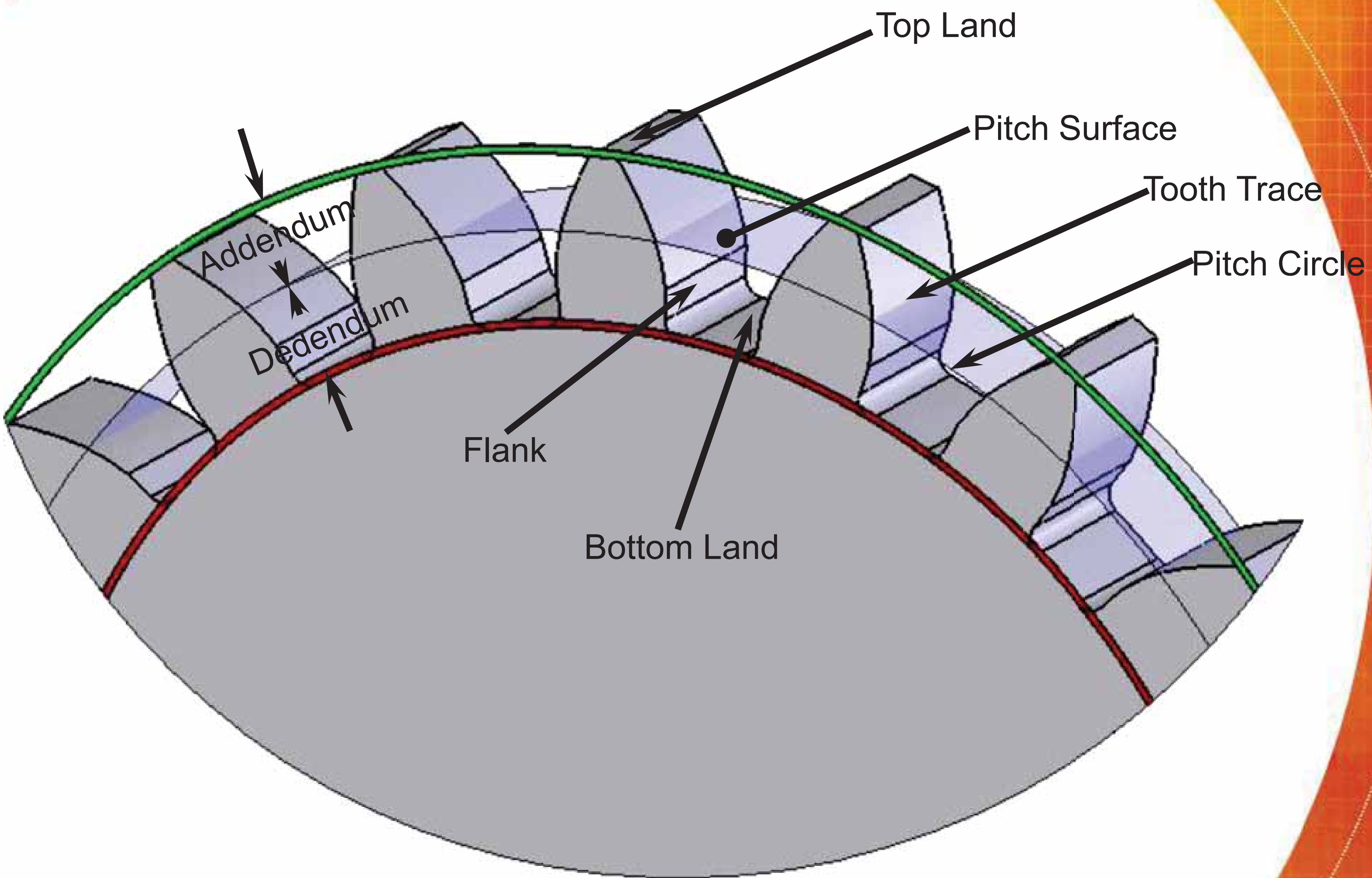


More Terminology

- **Common Tangent –** A line tangential to the two base circles along which contact between the meshing teeth takes place, also known as the **line of action**
- **Pitch Point** – Point of contact between the pitch circles of meshing gears



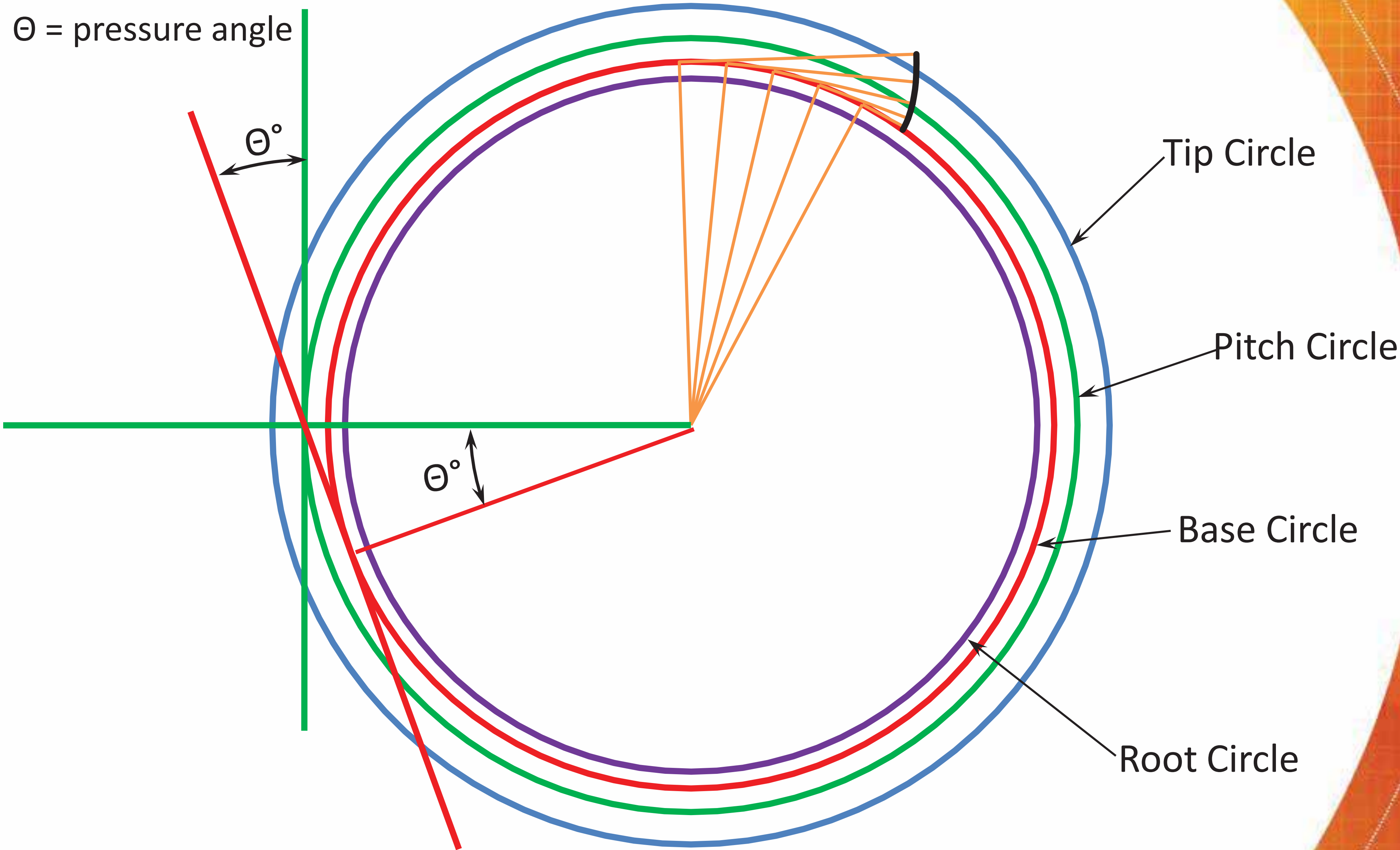
Parts of a Gear Tooth

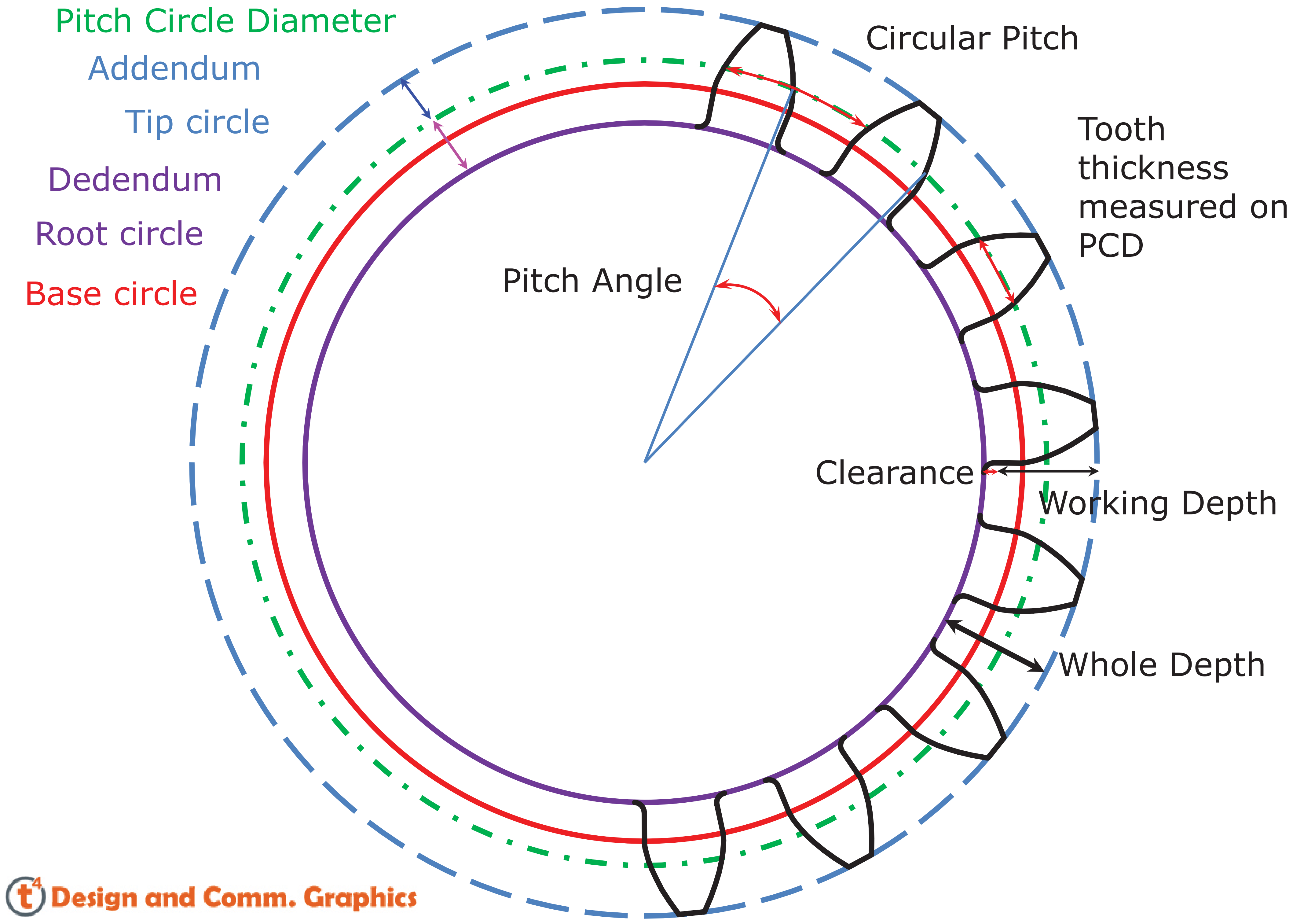


Parts of a Gear Tooth

Term	Definition
Pitch Surface	Is an imaginary cylindrical surface which contains the pitch circle of a gear
Addendum	Is the part of the tooth outside the pitch
Dedendum	Is the part of the tooth inside the pitch surface
Flank	Is the part of the tooth that comes into contact with other gears
Tip Surface	Is and imaginary surface at the top of the tooth
Root Surface	Is an imaginary surface at the bottom of the tooth
Top Land	Is the part of the tooth between opposite flanks
Bottom Land	Is the part of the root surface between opposite flanks
Tooth Trace	Is the intersection between the pitch surface and the flank of the tooth

Gear Terminology




Pitch Circle Diameter
Addendum
Tip circle
Dedendum
Root circle
Base circle
Pitch Angle
Circular Pitch
Tooth thickness measured on PCD
Clearance
Working Depth
Whole Depth
Design and Comm. Graphics

Gear Terms

Term	Symbol	Definition
Addendum	a	The part of the tooth that extends outside of the pitch circle/pitchline The addendum is always equal to the module $a=m$
Base Circle	BCD	An imaginary circle from which the tooth shape is generated The base circle = the pitch circle diameter × cos (pressure angle) $BCD=PCD \times \cos(\text{pressure angle})$ <u>This circle can be constructed graphically</u>
Circular Pitch	p	Is the distance from the point on one tooth to the corresponding point on the next tooth Measured around the pitch circle $p= \pi m$

Gear Terms

Term	Symbol	Definition
Circular Tooth Thickness		The thickness of a tooth measured along the pitch circle Circular tooth thickness = p/2
Clearance	c	Is the space underneath the tooth when it is in mesh Clearance = ¼ of the addendum c =d - a =0.25a =0.25m
Dedendum	d	Is the part of the tooth which is inside the pitch circle or the pitch line =1.25 × addendum d=1.25 a
Line of Action		Contact between the teeth of meshing gears takes place along a line tangential to the two base circles This line passes through the pitch point

Gear Terms

Term	Symbol	Definition
Module	m	Is the pitch circle diameter divided by the number of teeth The module for gears in mesh must be the same or they will vibrate and wear badly m= PCD/t
Pitch Circle Diameter	PCD	
Pinion		When two gears are in mesh the smaller gear is called the pinion
Pitch Angle		360°÷ number of teeth
Pitch Circle	PC	Is the circle representing the original cylinder which transmitted motion by friction
Pitch Point		When two gears are in mesh their pitch circles will be tangiental to each other. The pitch point is the point of contact betweeen the two circles

Gear Terms

Term	Symbol	Definition
Pressure Angle	Θ	The angle between the line of action and the common tangent to the pitch circles at the pitch point The pressure angle is normally 20° but may be 14.5°
Tip Circle		A circle through the tips of the teeth
Wheel		When two gears are in mesh the larger one is called the wheel
Whole depth		Is the depth of the tooth from tip to root Whole depth = addendum + dedendum
Working Depth		The whole depth – the clearance

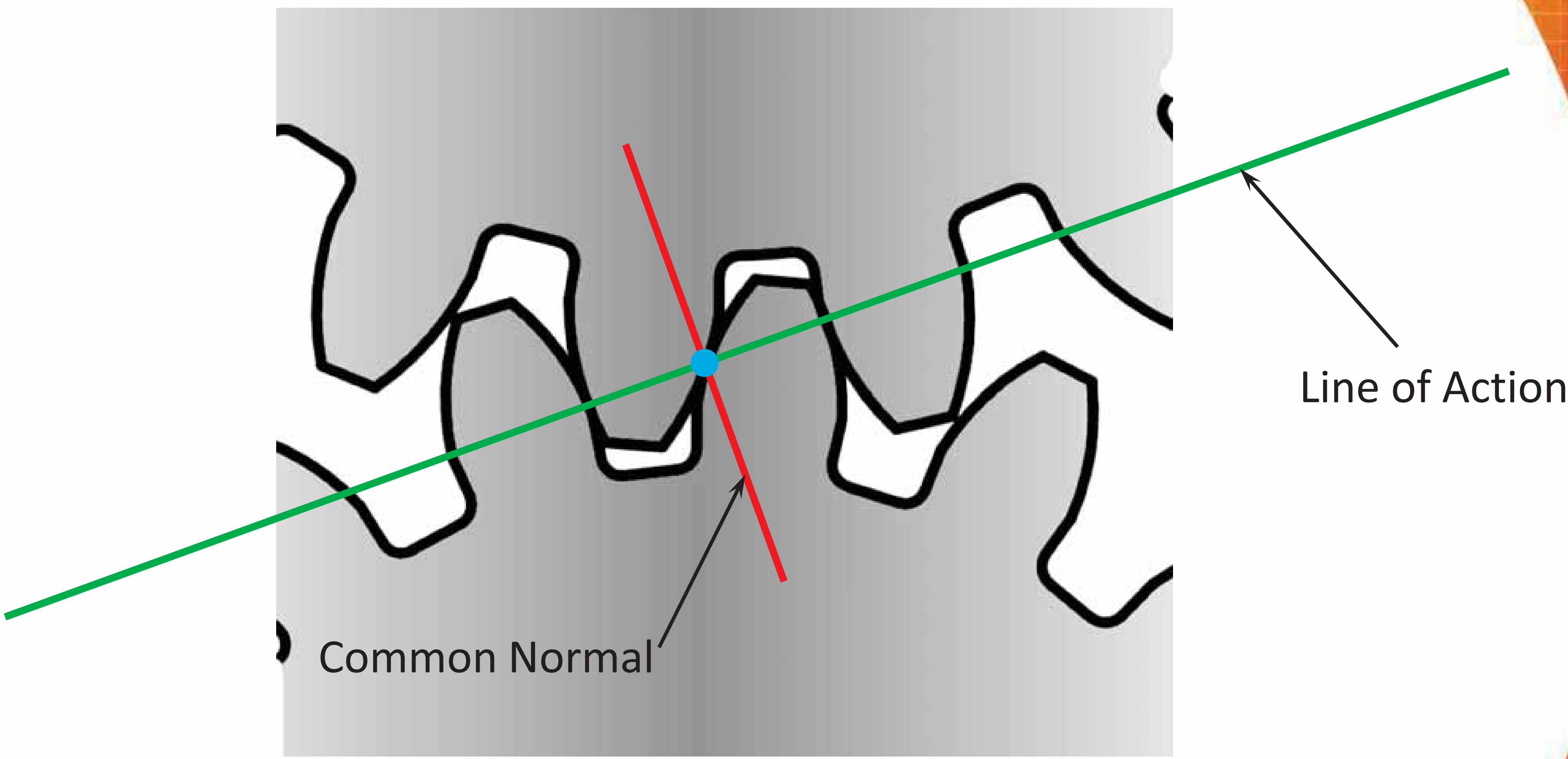
Line of action

- To ensure the gear motion is smooth, quiet and free from vibration, a direct line of transmission must act between the gear teeth
- This line of action, or common normal determines the pressure angle of the teeth and passes through the pitch point.
  – i.e. Gears in mesh meet at only one point which is the intersection of their Pitch Circle Diameters

Other line of action animations

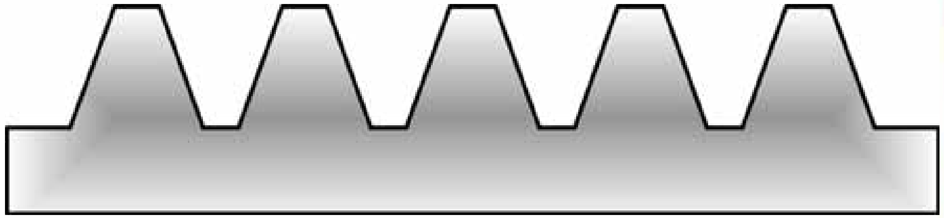
- http://science.howstuffworks.com/gear8.htm

Line of Action

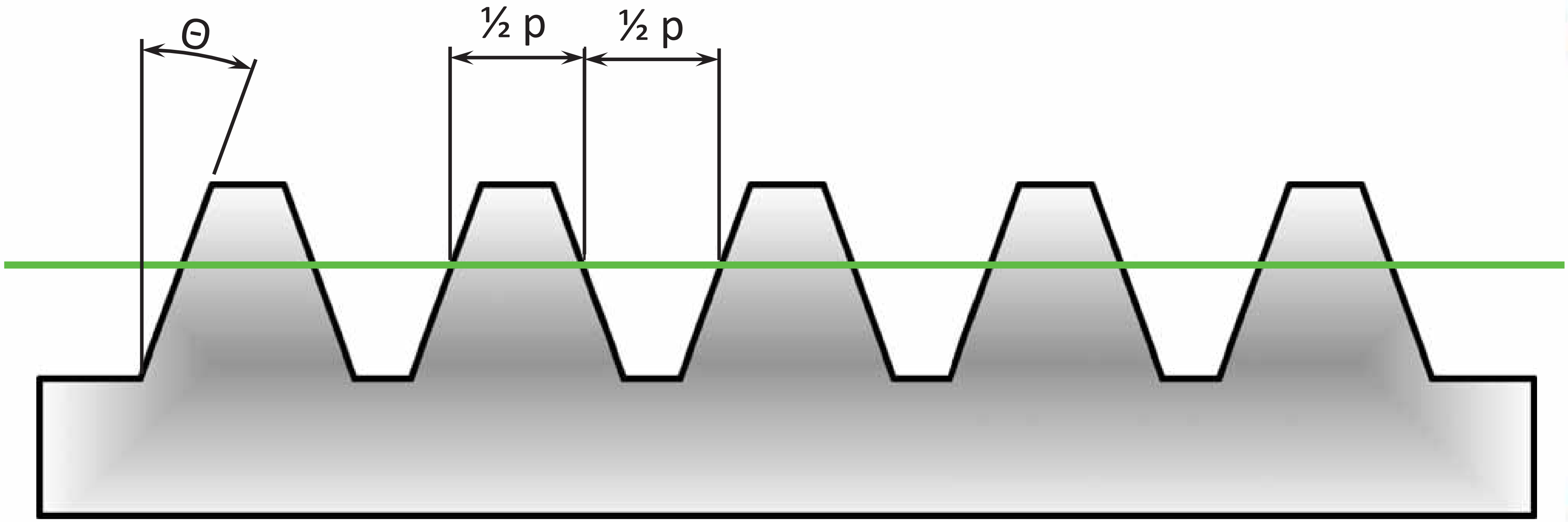


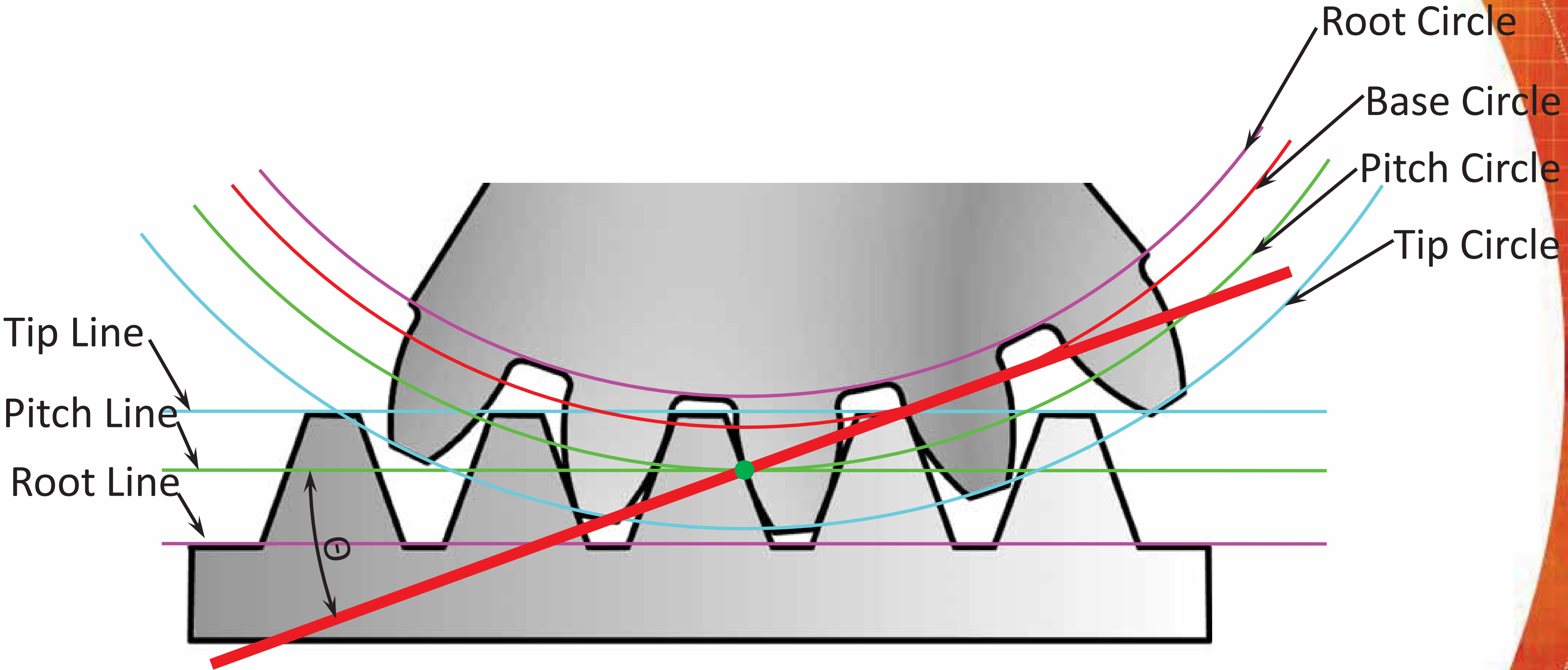
Rack

- A rack is a straight toothed bar
- Technically it is a spur gear whose radius is at infinity
- Because of this all principles of circular spur gears hold true



Rack




Root Circle
Base Circle
Pitch Circle
Tip Circle
Tip Line
Pitch Line
Root Line
Θ

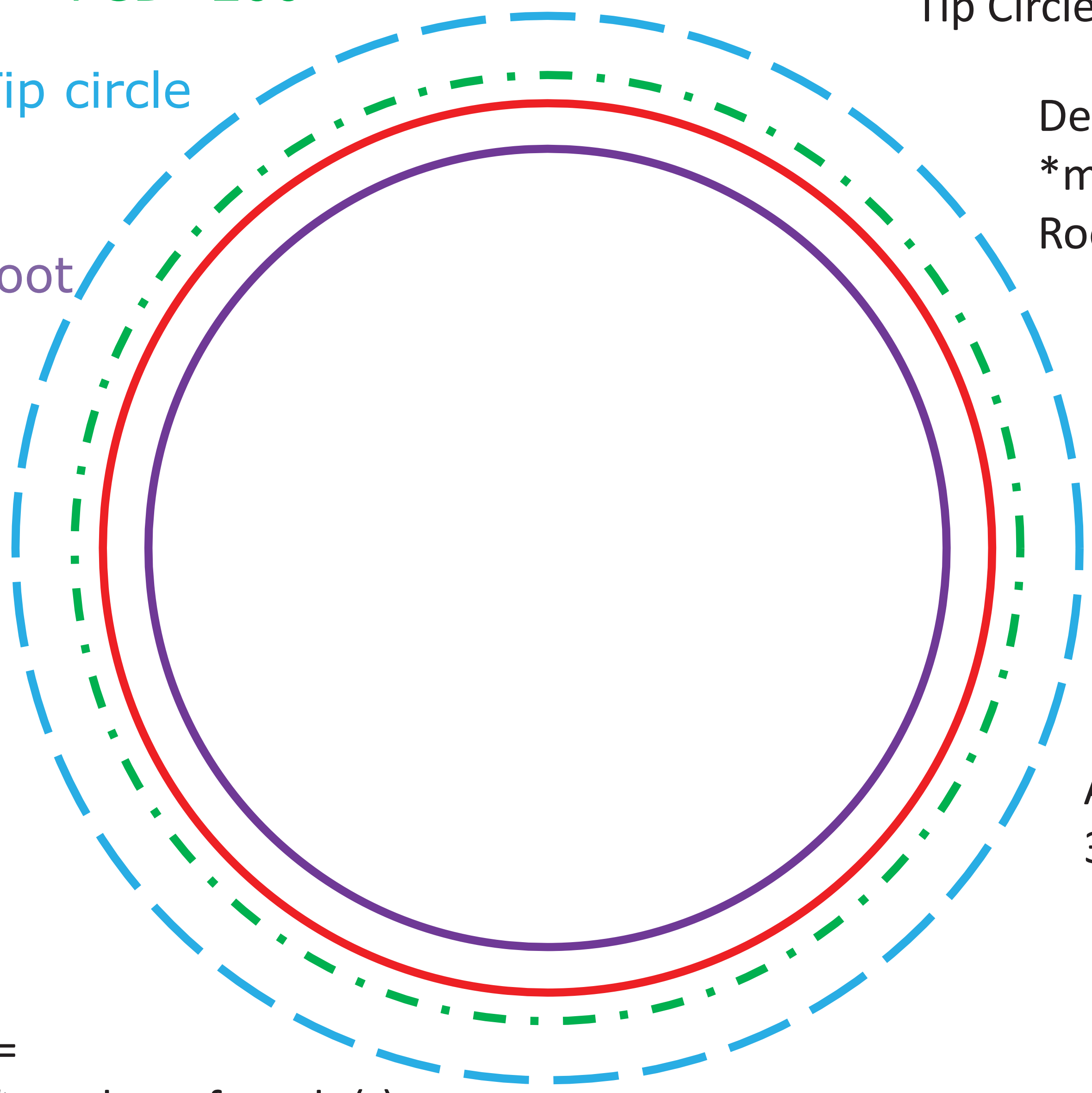
Line of action for a rack and pinion animation

- http://www.brockeng.com/mechanism/RackNPinion.htm

Gears

- Given a pitch circle diameter of 200mm and a module of 10, and a pressure angle of 20° construct the spur gear.
- Show at least four teeth on the gear
- Teeth to be constructed by the involute method.

Gear	Calculations	Results
Module (m)		10
No. of teeth (t)		20
Pressure angle (Θ)		20°
Pitch circle diameter (PCD)	m × t	200mm
Base circle diameter	PCD × cosΘ	187.9mm
Addendum (a)	a = m	10mm
Dedendum (d)	1.25 × m	12.5mm
Clearance	d - a	2.5mm
Tip circle diameter	PCD + 2a	220mm
Root circle diameter	PCD - 2d	175mm
Circular pitch (p)	π × m	31.4mm
Tooth thickness	p ÷ 2	15.7mm
Pitch angle	360° ÷ t	18°



Use graphical construction

Addendum=module
Tip Circle=Ø220

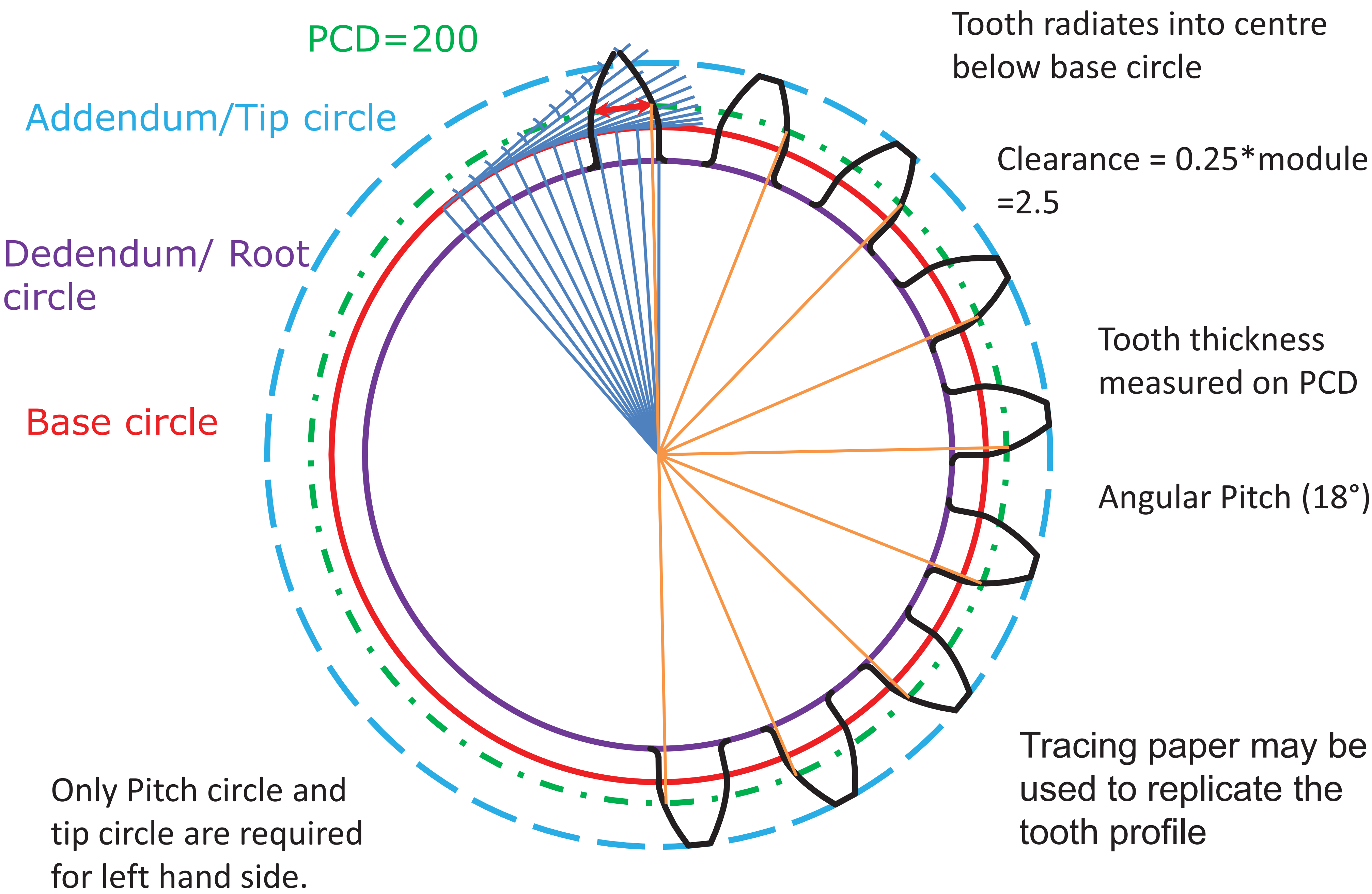
Dedendum=1.25 *module
Root circle=Ø175

Circular pitch distance=
p=πm
p=31.4

Tooth thickness=
31.4/2
=15.7

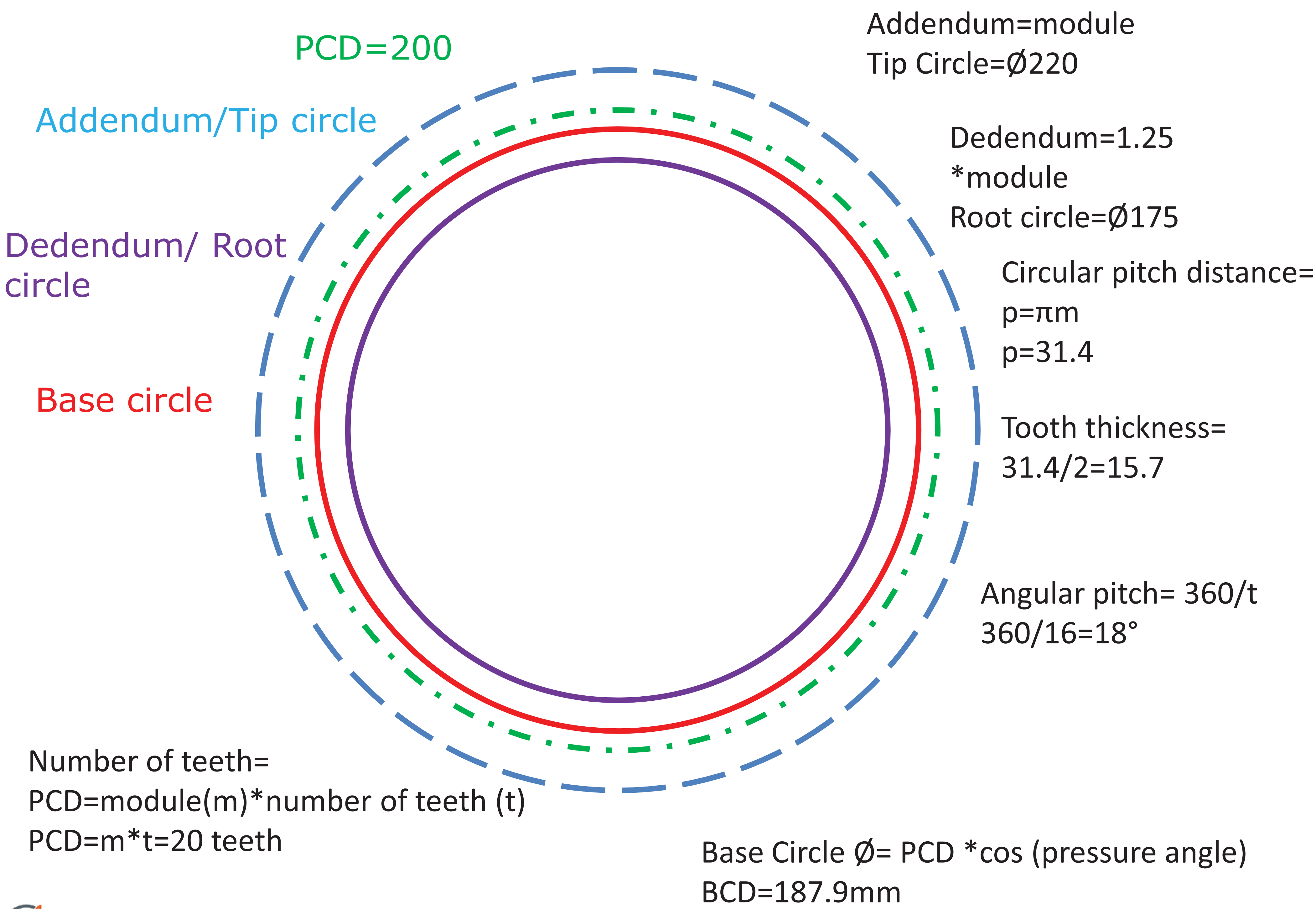
Angular pitch= 360/t
360/16=18°

Number of teeth=
PCD=module(m)*number of teeth (t)
PCD=m*t=20 teeth

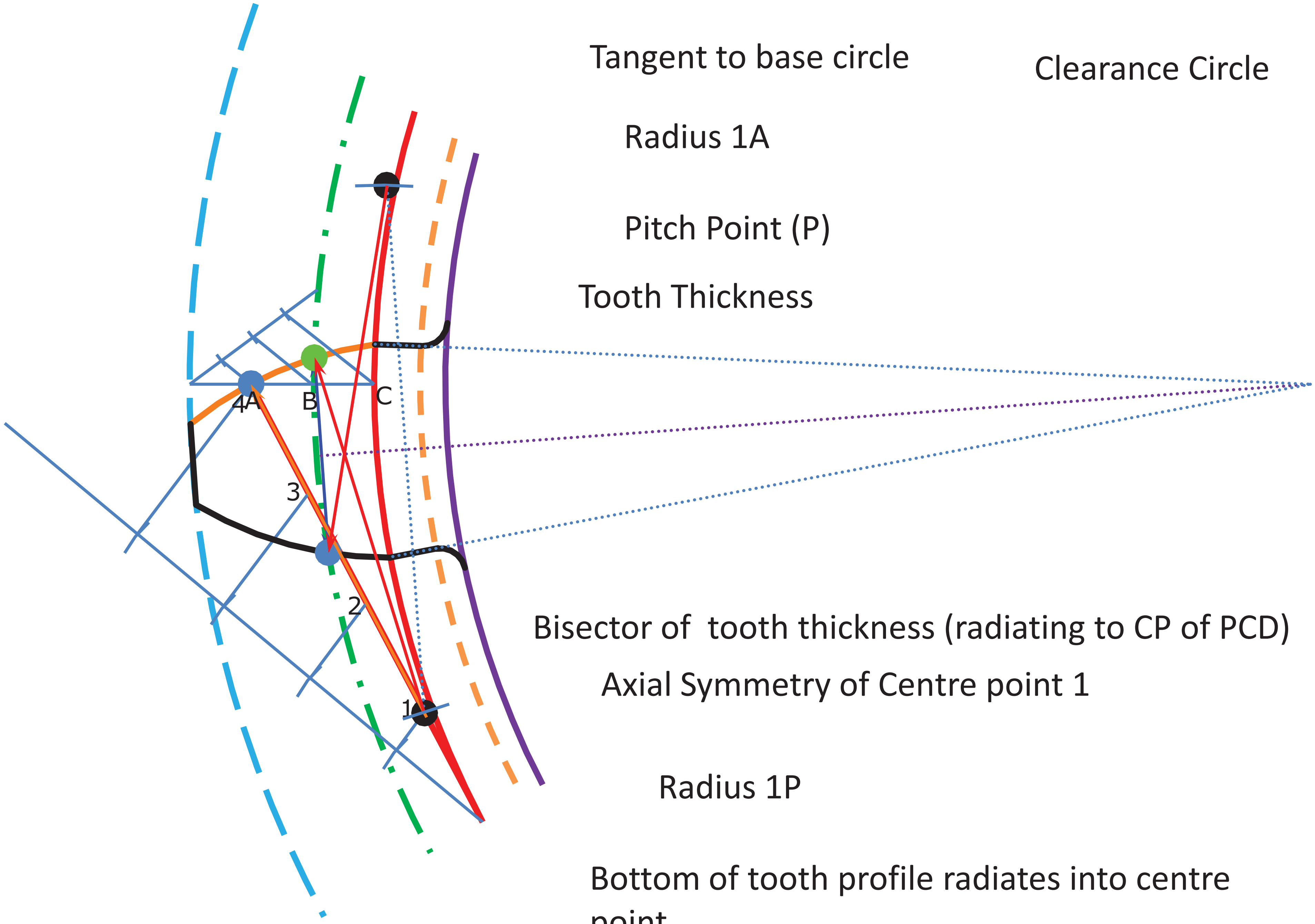

PCD=200
Tooth radiates into centre below base circle
Addendum/Tip circle
Clearance = 0.25*module =2.5
Dedendum/ Root circle
Tooth thickness measured on PCD
Base circle
Angular Pitch (18°)
Tracing paper may be used to replicate the tooth profile
Only Pitch circle and tip circle are required for left hand side.

TECHNOLOGY SUBJECTS SUPPORT SERVICE

SOLUTION USING UNWINS METHOD



Design and Comm. Graphics



Design and Comm. Graphics

Clearance Circle

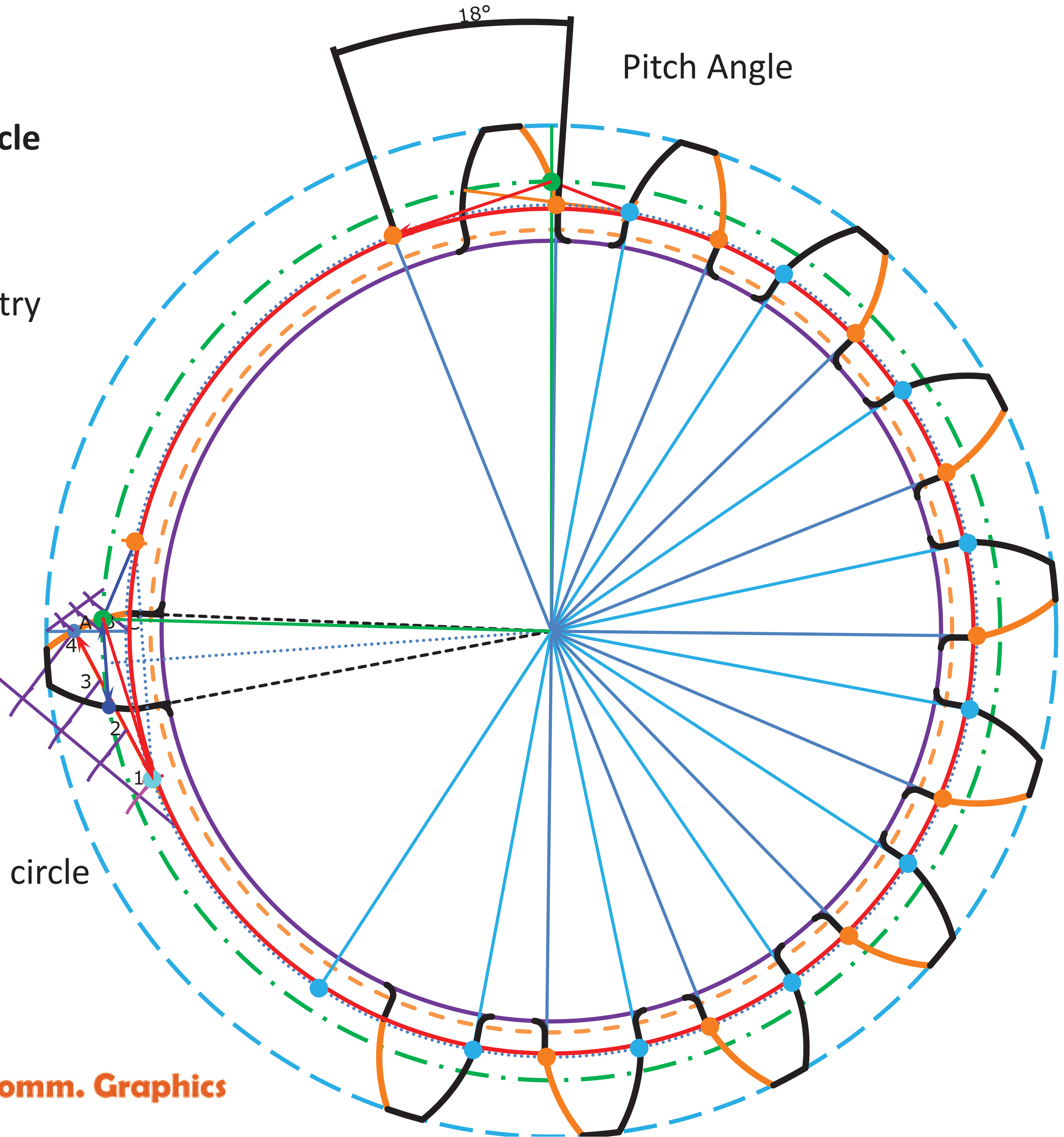
Axial symmetry

Pitch Point

Tooth thickness

Length 1 to A

Tangent to base circle



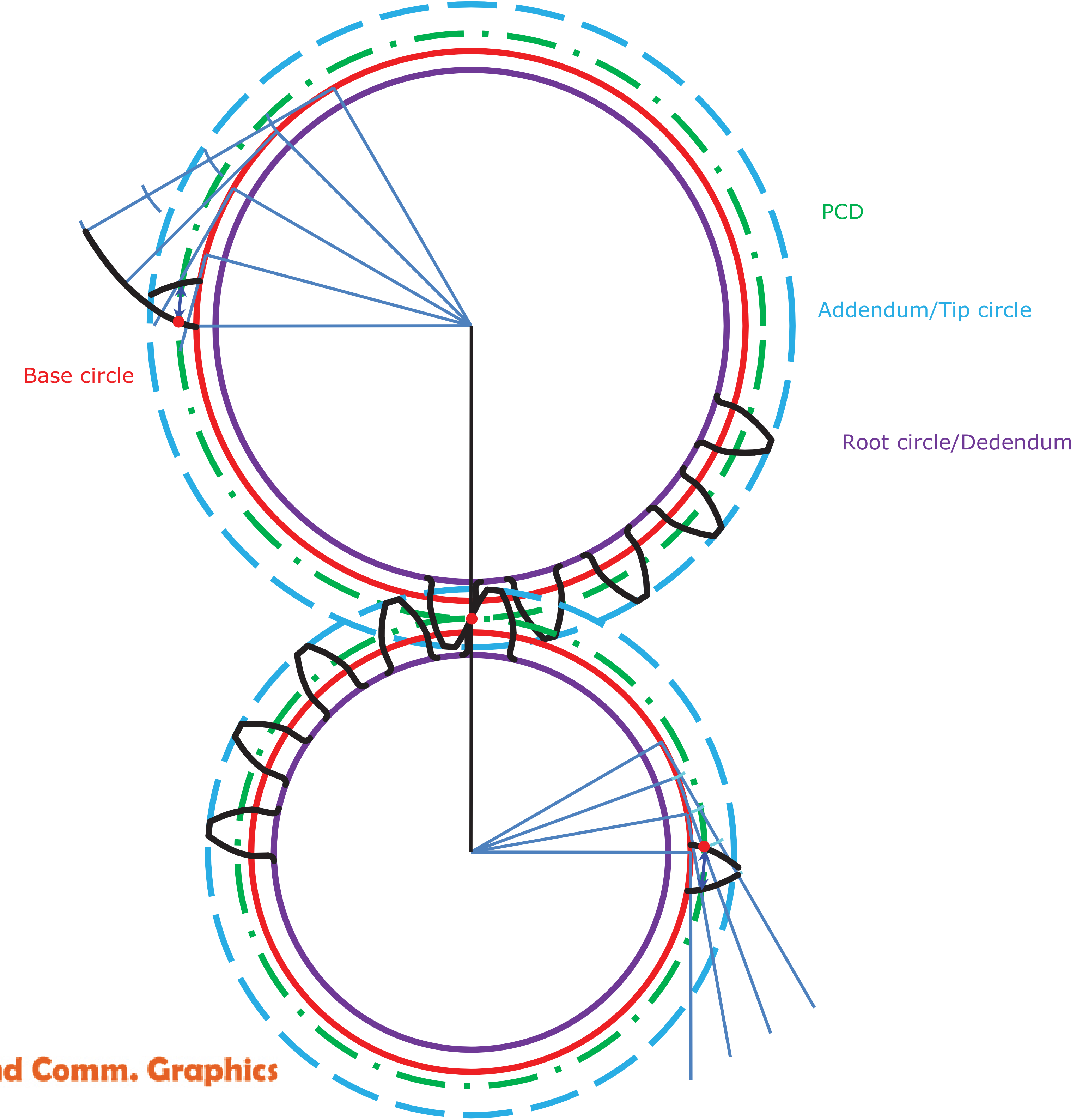
Design and Comm. Graphics

Gears

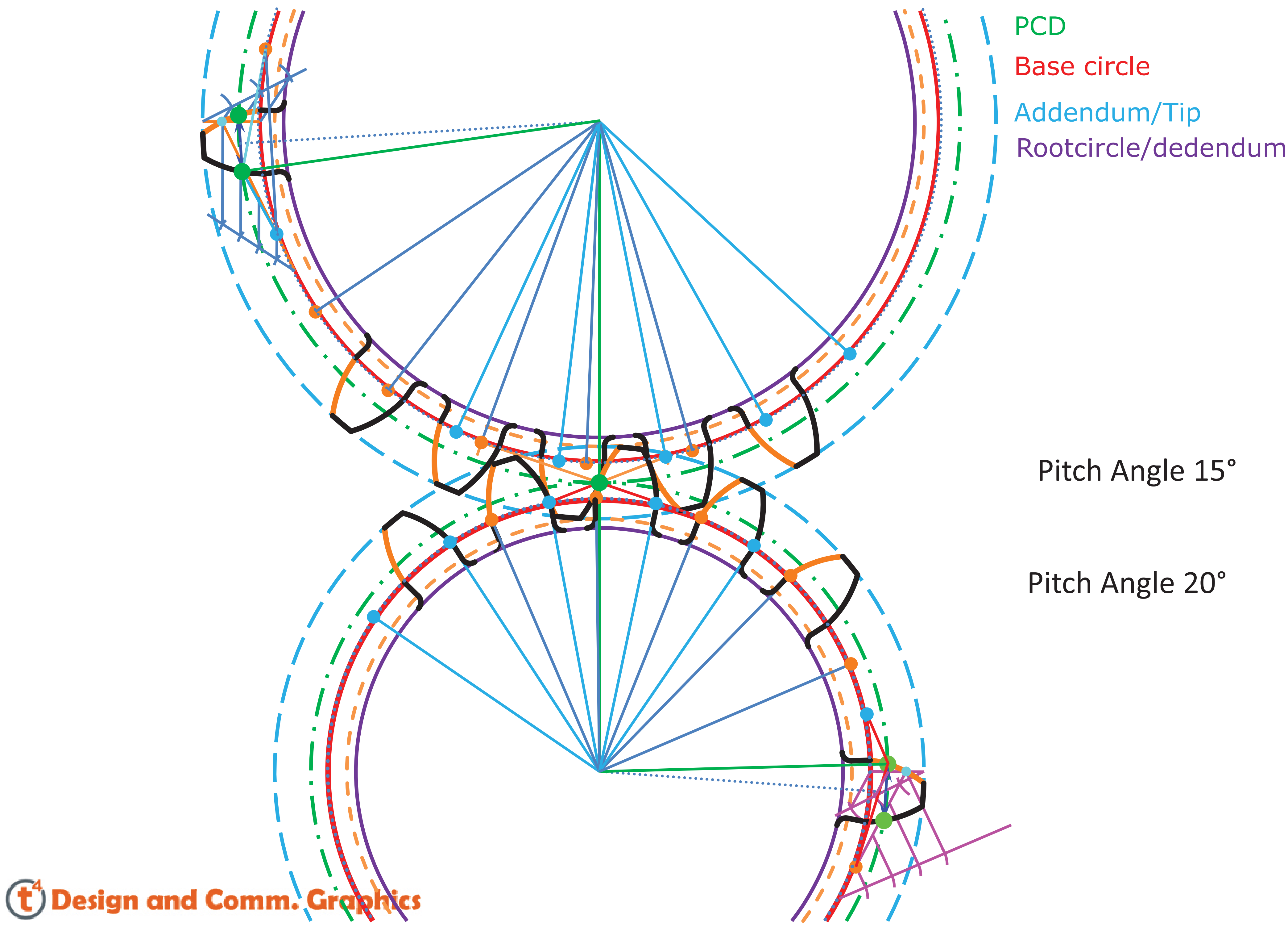
- Draw two involute spur gears in mesh and show five teeth on each gear.
- The gear ratio is 4:3.
- Driver gear details: Module=8, teeth=24, pressure angle=20°

Driver Gear	Calculations	Results
Module (m)		8
No. of teeth (t)		24
Pressure angle (Θ)		20
Pitch circle diameter (PCD)	$m \times t$	192
Base circle diameter	$PCD \times \cos\Theta$	180.4
Addendum (a)	$a = m$	8
Dedendum (d)	$1.25 \times m$	10
Clearance	$d - a$	2
Tip circle diameter	$PCD + 2a$	208
Root circle diameter	$PCD - 2d$	172
Circular pitch (p)	$\pi \times m$	25.13274123
Tooth thickness	$p \div 2$	12.56637061
Pitch angle	$360° \div t$	15

Driven Gear	Calculations	Results
Module (m)		8
No. of teeth (t)	4:3=24:18	18
Pressure angle (Θ)		20
Pitch circle diameter (PCD)	m × t	144
Base circle diameter	PCD × cosΘ	135.3
Addendum (a)	a = m	8
Dedendum (d)	1.25 × m	10
Clearance	d - a	2
Tip circle diameter	PCD + 2a	160
Root circle diameter	PCD - 2d	124
Circular pitch (p)	π × m	25.13274123
Tooth thickness	p ÷ 2	12.56637061
Pitch angle	360° ÷ t	20



Design and Comm. Graphics



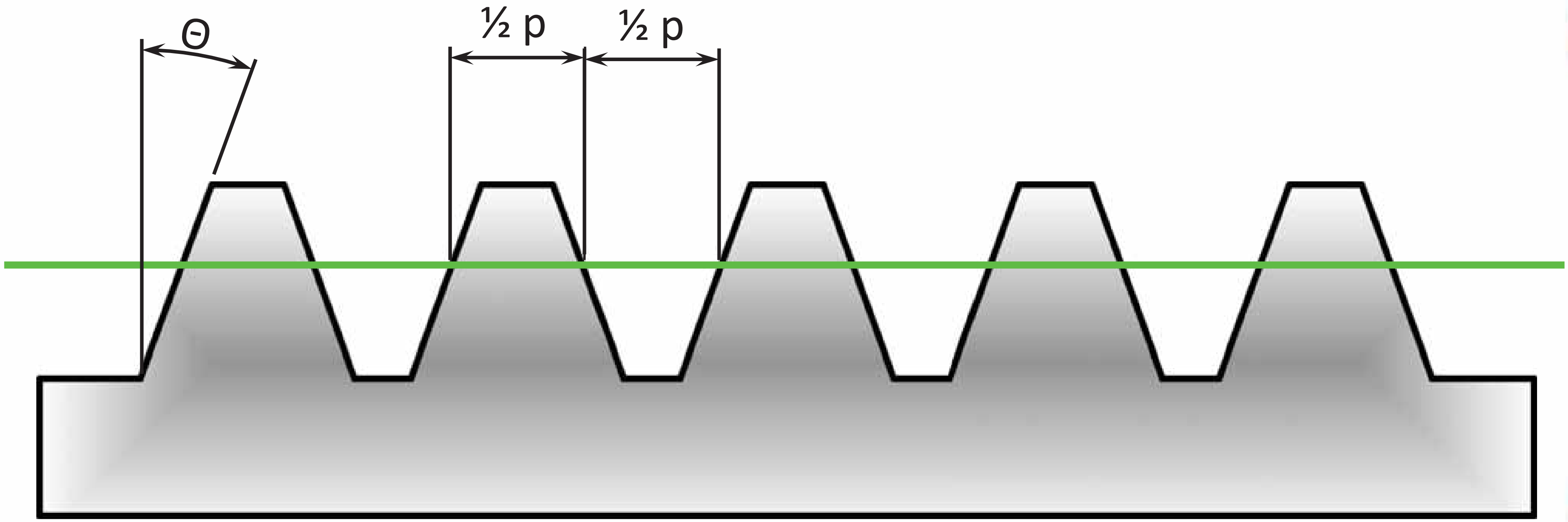
PCD
Base circle
Addendum/Tip
Rootcircle/dedendum
Pitch Angle 15°
Pitch Angle 20°

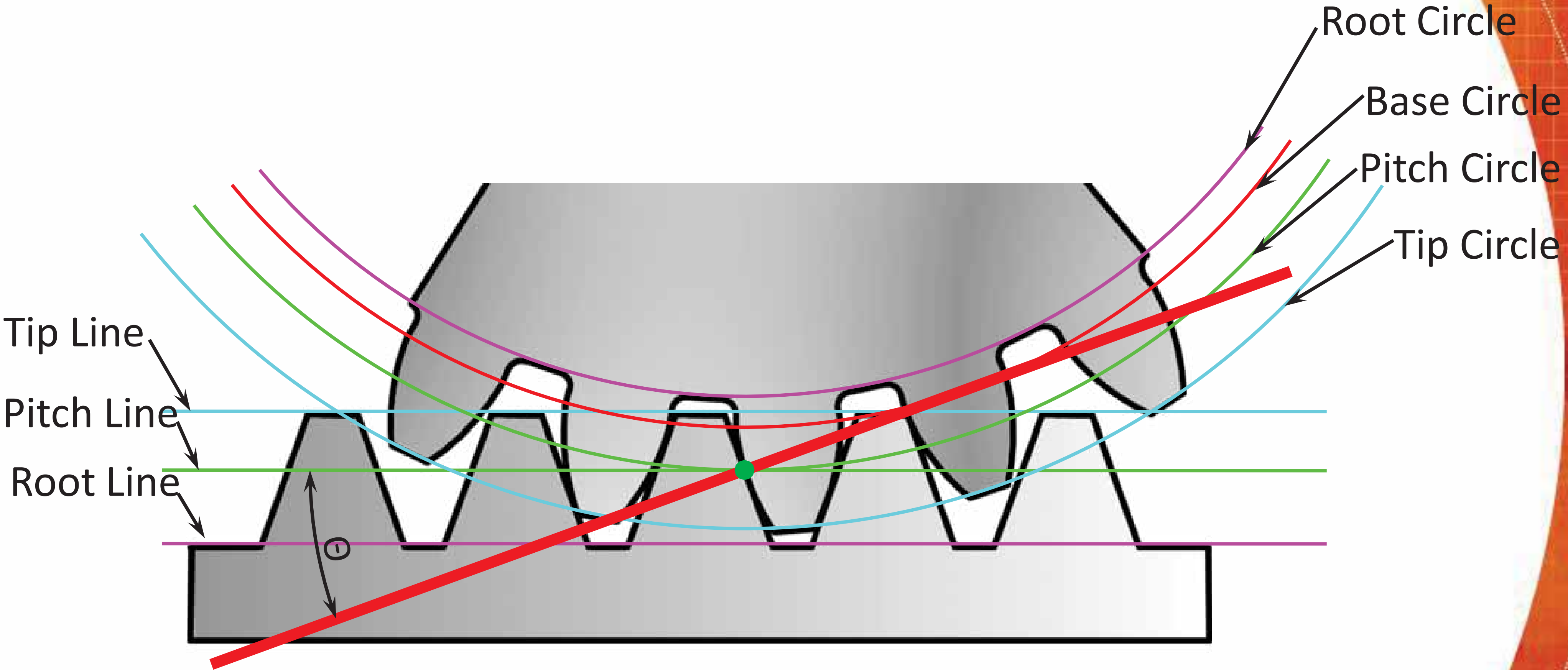
Gears

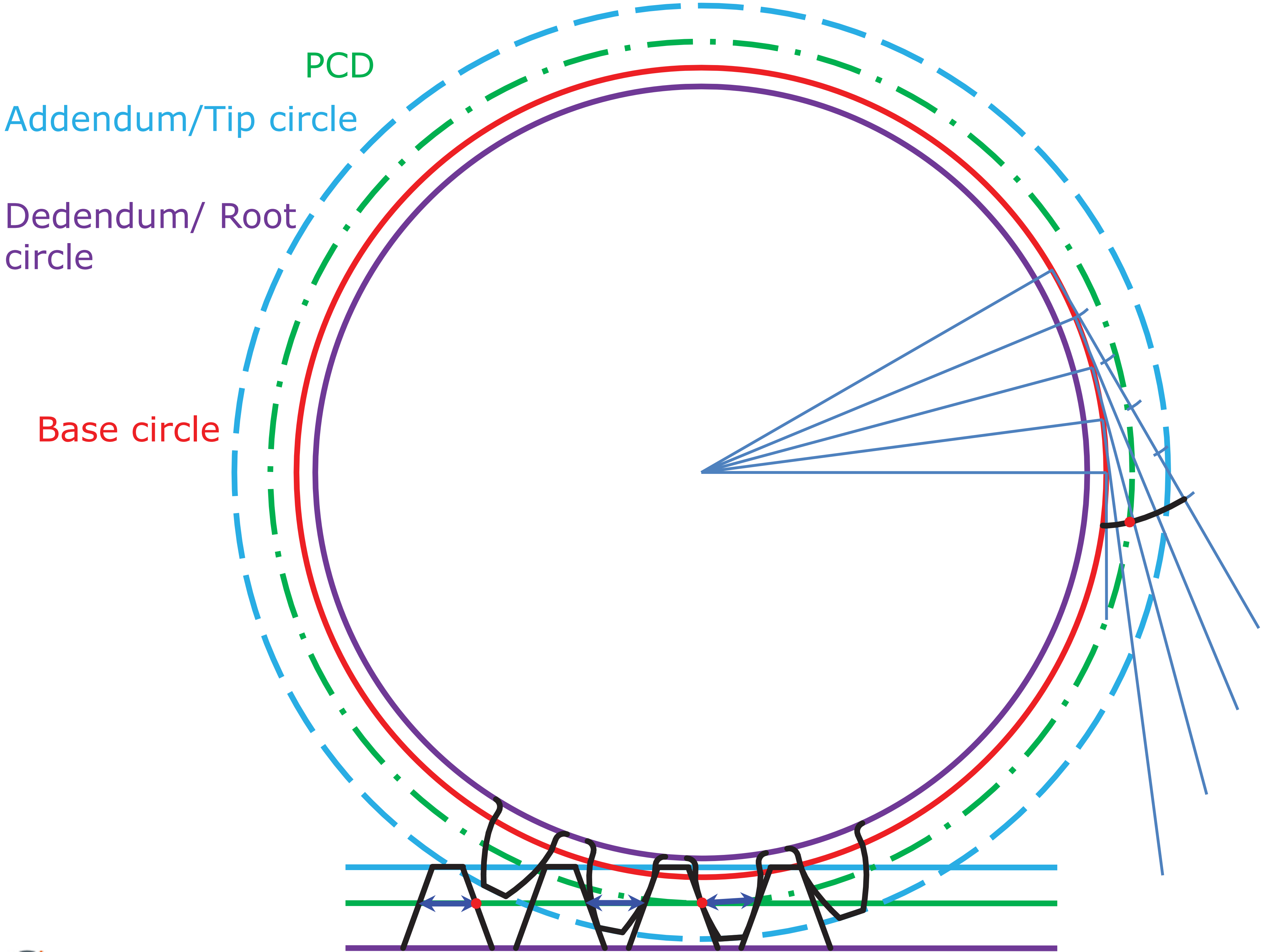
- An involute gear is in mesh with a rack.
- The involute gear has 20 teeth, a pressure angle of 20° and module of 10.
- Draw the gear and rack in mesh, showing four teeth on the gear and an equivalent on the rack.

Gear Wheel	Calculations	Result
Module (m)		10
No. of teeth (t)		20
Pressure angle (Θ)		20
Pitch circle diameter (PCD)	$m \times t$	200
Base circle diameter	$PCD \times \cos\Theta$	187.9
Addendum (a)	$a = m$	10
Dedendum (d)	$1.25 \times m$	12.5
Clearance	$d - a$	2.5
Tip circle diameter	$PCD + 2a$	220
Root circle diameter	$PCD - 2d$	175
Circular pitch (p)	$\pi \times m$	31.4
Tooth thickness	$p \div 2$	15.7
Pitch angle	$360° \div t$	18

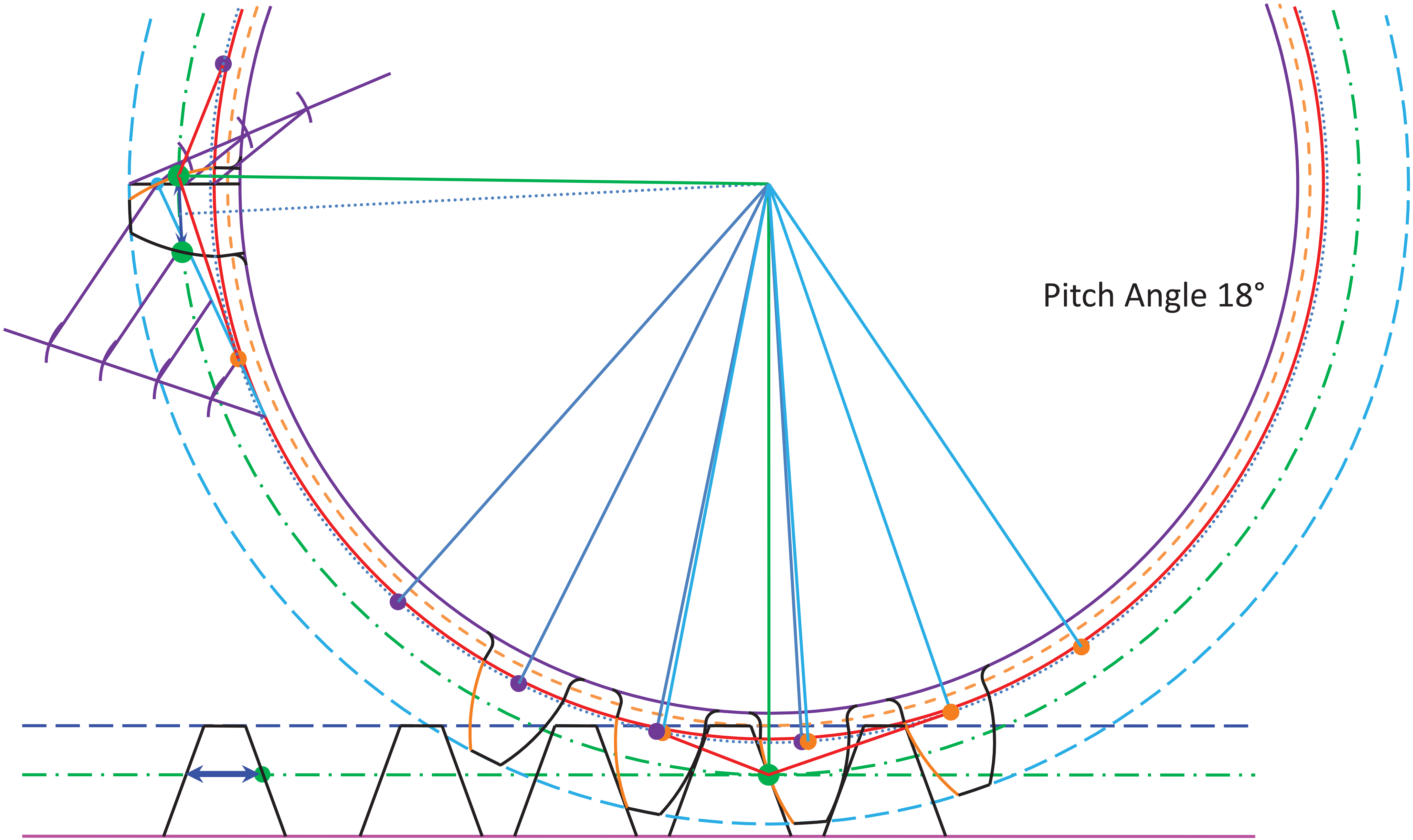
Rack	Result
Module	10
Pressure angle	20°
Addendum	10mm
Dedendum	12.5mm
Clearance	2.5mm
Pitch	31.4mm
Tooth thickness	15.7mm


Θ
½ p
½ p


Root Circle
Base Circle
Pitch Circle
Tip Circle
Tip Line
Pitch Line
Root Line
Θ



Design and Comm. Graphics


Pitch Angle 18°

Links to Dynamic Mechanisms

- http://www.ul.ie/~nolk/maincontents.htm