Dynamic Mechanisms
BASICS OF LOCI
What is a Locus?

• A **locus** is the movement of a point as it follows certain conditions.

• For example, a line is the locus of a point as it moves in a straight path.
A circle is the locus of a point which moves so it remains a constant distance from a fixed point “p”.

The fixed point is called the **centre** and the distance the **radius**.
Constructions that use Loci

• We use loci to find the midpoint of a line
• The locus of a point which moves so that it is equidistant from two fixed points is called the perpendicular bisector of the line
• The intersection of the perpendicular bisector and the line segment yields the midpoint
Constructions that use Loci

- We can use loci to solve simple problems such as finding the bisector of an angle.
- The bisector of an angle can be defined as:
- The locus of a point that moves so that it remains equidistant from 2 fixed lines.
INVOLUTES
• An involute is the locus of a point on a line as the line rolls along a shape.
• It can also be thought of as the locus of the end of a piece of string as the string is wound/unwound around the circumference of a plane figure.
Involutes are used to determine the length of belts used in pullies and other machines.

Involutes are also used to calculate the amount of material required to create tyres and wheels.

Applications of Involutes
Involute of a Square
Involute of a Triangle
Involute of a Hexagon
Involute of a Circle
Making Involute models

• [http://www.math.nmsu.edu/~breakingaway/Lessons/involute1/involute.html](http://www.math.nmsu.edu/~breakingaway/Lessons/involute1/involute.html)
TANGENTS TO INVOLUTES
Tangents to Involutes

- Involutes are curves, and as with all curves a tangent can be drawn to the involute
Tangent to an involute at point P
Tangent to an involute at point P
THE HELIX
The Helix

- A helix is the locus of a point as it moves on the surface of a cylinder so that it rotates at a constant rate around the surface of the cylinder, while also progressing in the direction of the axis at a constant rate.
The development of a helix appears as a straight line development.
A geodesic is the shortest distance between two points on a surface.

The geodesic of a cylinder may be:
- A circle
- A linear element
- A helix
Applications of the Helix

- The helix is used for the thread of bolts, reamers and drill bits
- Springs are derived from the helix
- Helical gears are derived from the helix
- Winding staircases are also derived from the helix
Applications of the Helix

- The helix may be the most important shape in the universe as the human gene code is structured around a helix.
Left and Right Hand Helix Rule

Place your thumb along the shape of the helix as shown, and you will notice that the shape of the thumb and the helix are similar. This is a left hand helix. The plan of the cylinder will be indexed clockwise.

Place your thumb along the shape of the helix as shown, and you will notice that the shape of the thumb and the helix are similar. This is a right hand helix. The plan of the cylinder will be indexed anti-clockwise.

Design and Comm. Graphics
A right hand helix of one revolution
A left hand helix of one revolution
A right handed helix of 1 ½ revolutions
Helical Screw Thread: Terminology

- **Internal Diameter**: Diameter of Shaft/internal helical curve
- **Outside Diameter**: Outermost diameter of the thread/helical curve
- **Lead**: amount of axial advance during one complete revolution of the helix
- **Pitch**: is the distance from a point on the helix to a corresponding point on the next revolution measured parallel to the axis
Right handed and Left handed Screw Threads
Variations of Helical Screw Threads

- A helical screw thread can consist of more than two helices
- A two start screw thread has four helices
- A three start would have six helices, etc
Helical Screw Thread

When large axial movement is required two or more threads may be cut on one screw

**Single Start Thread**

Pitch = Lead

**Two Start Thread**

Pitch = ½ Lead
Helical Screw Thread

- Draw one revolution of a single start right-handed screw thread (½ the pitch) given
- Inside Ø: 40mm
- Outside Ø: 82mm
- Lead: 60mm
- Square Thread: 30mm
Helix Problems

- Given the plan and elevation of a cylinder with two points on its surface, X and Y. Draw a helix starting from the base of the cylinder and finishing at the top of the cylinder and passing through X and Y.
Helix Problems

- Given the plan and elevation of a cylinder, having the point P on its surface draw a helix of one revolution so as it passes through P
Parallel to typical helix through the point
SPIRAL GEOMETRY
A spiral can be the locus of a point as it moves around a fixed point (pole) while steadily increasing its distance from the point.

If a line rotate about one of its end points (the pole), and at the same time a point moves continuously in one direction along the line, the locus of the moving point is a spiral.
Conical Helix/Conical Spiral

- A conical spiral is the locus of a point as it moves on the surface of a cone so that it rotates at a constant rate around the cone while also progressing in the direction of the axis at a constant rate.
- The plan of a conical spiral is an Archimedean spiral.
Applications of the Conical Spiral

- The conical spiral is used for augers and other boring devices such as screw tips.
- It is also used for the construction of Archimedean wells.
Construct a conical spiral
Conical Spiral

• Note: A conical spiral is not a geodesic, as the development of a conical spiral is a curve
<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
</tr>
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<tbody>
<tr>
<td>Pole</td>
<td>The point at the centre</td>
</tr>
<tr>
<td>Convolution</td>
<td>One complete rotation of the point around the spiral</td>
</tr>
<tr>
<td>Vector Angle</td>
<td>The angle between any two radius vectors</td>
</tr>
<tr>
<td>Radius Vector</td>
<td>Any line from pole to a point on the spiral</td>
</tr>
</tbody>
</table>

**Spiral Terminology**

- **Pole**: The point at the centre.
- **Convolution**: One complete rotation of the point around the spiral.
- **Vector Angle**: The angle between any two radius vectors.
- **Radius Vector**: Any line from pole to a point on the spiral.
There are two main types of spirals:
- Archimedean Spirals
- Logarithmic Spirals
Archimedean Spirals

- An Archimedean spiral is the locus of a point that moves around a circle at a constant speed while also moving away from/towards the pole at a constant speed.
Applications of the Archimedean Spiral

• The Archimedean spiral is functional as well as aesthetic
• Archimedean springs are used in watch making and door mechanisms
• Because of the link between the conical and Archimedean spiral many of the applications of the Archimedean spiral are exhibited through the conical spiral
• A logarithmic spiral is a spiral that increases/decreases proportionally according to a given rule.
• A logarithmic curve will never terminate at a pole.
Applications of the Logarithmic Spiral

- Logarithmic Spirals are naturally occurring spirals in nature
- The sea shell on the right contains a logarithmic spiral
- The natural occurrence of manmade design in nature or visa versa is known as bio-mimicry

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Applications of the Logarithmic Spiral

• Some universes branch out in a logarithmic spiral
• An understanding of geometrical shapes may assist scientists and engineers in furthering their research and discoveries
Spiral Types

Archimedean Spiral

Logarithmic Spiral
Archimedean Spirals

• Draw an archimedean spiral of one convolution given the longest radius vector as 60mm and the shortest as 0mm
Archimedean Spirals

- Draw an archimedean spiral of $1 \frac{3}{4}$ convolutions given the longest radius vector as 60mm and the shortest as 20mm
Archimedean Spirals

• Construct one convolution of an archimedean spiral given the shortest radius vector of 20mm and an increase in vector length of 5mm every 45°
Add 5mm onto the radius vector for every 45° increment
Logarithmic Spirals

Construction derived from similar triangles

A:B = Ratio of Spiral
Logarithmic Spirals

• Construct one convolution of an logarithmic spiral given a vector angle of 30° and the ratio of the vector lengths as 10:12 (Initial radius not specified)
Logarithmic spiral derived from similar triangles
TANGENTS TO SPIRALS
Tangent to an Archimedean spiral at a point P.
1 Radian = 57.3°
Tangent to a Logarithmic spiral at a point P
As this is an equiangular spiral it can be mathematically proven that the angle between any radius vector and a tangent will always be a constant 77°20'.
Animation of a logarithmic spiral involute

- http://mathworld.wolfram.com/LogarithmicSpiralInvolute.html
Applications of Loci

- A locus is the movement of a point as it follows certain conditions.
- A locus may be used to ensure that moving parts in machinery do not collide.
A cycloid is the locus of a point on the circumference of a circle which rolls without slipping along a straight line.

The valve on a car tyre generates a cycloid as the car moves.
Other cycloid animations

Draw a cycloid given the circle, the base line and the point on the circumference.
The cycloid is the locus of a point on the circumference of a circle which rolls without slipping along a straight line.
Triangulation Method with lines omitted for clarity
An inferior trochoid is the path of a point which lies **inside a circle** which rolls, without slipping, along a straight line.

The reflector on a bicycle generates an inferior trochoid as the bike moves along a flat surface.
Draw an inferior trochoid given the circle, the base line and the point P inside the circumference.
An inferior trochoid is the path of a point which lies inside a circle, which rolls, without slipping along a straight line.
Superior Trochoid

- A superior trochoid is the path of a point which lies outside a circle which rolls, without slipping, along a straight line.
- Timber moving against the cutter knife of a planer thicknesser generates a superior trochoid.
Draw a superior trochoid given the circle, the base line and the point P outside the circumference.
A superior trochoid is the path of a point which lies inside a circle, which rolls, without slipping around the inside of a fixed circle.
An epicycloid is the locus of a point on the circumference of a circle which rolls without slipping, around the outside of a fixed arc/circle.

The applications and principles of a cycloid apply to the epicycloid.

Various types of cycloids are evident in amusement rides.
If a circle rolls without slipping round the outside of a fixed circle then a point P on the circumference of the rolling circle will produce an epicycloid.
An epicycloid is the locus of a point on the circumference of a circle which rolls without slipping, around the outside of a fixed arc/circle.
An inferior epitrochoid is the path of a point which lies **inside a circle** which rolls, without slipping, around the outside of a fixed circle.

The applications and principles of the inferior trochoid apply to the inferior epitrochoid.
If a circle rolls without slipping round the inside of a fixed circle then a point P inside the circumference of the rolling circle will produce an inferior epitrochoid.
Segment lengths stepped off along base arc

An inferior epitrochoid is the path of a point which lies inside a circle, which rolls, without slipping around the outside of a fixed circle.
Superior Epitrochoid

- A superior epitrochoid is the path of a point which lies outside a circle which rolls, without slipping, around the outside of a fixed circle
- The applications and principles of the superior trochoid apply to the superior epitrochoid
If a circle rolls without slipping round the inside of a fixed circle then a point P outside the circumference of the rolling circle will produce a superior epitrochoid.
A superior epitrochoid is the path of a point which lies outside a circle, which rolls, without slipping around the outside of a fixed circle.
Hypocycloid

• A hypocycloid is the locus of a point on the circumference of a circle which rolls along without slipping around the inside of a fixed arc/circle.

• The applications of the cycloid apply to the hypocycloid.
If a circle rolls without slipping round the inside of a fixed circle then a point P on the circumference of the rolling circle will produce a hypocycloid.
Segment lengths stepped off along base arc

The hypocycloid is the locus of a point on the circumference of a circle which rolls along without slipping around the inside of a fixed arc/circle
Superior Hypotrochoid

- A superior hypotrochoid is the path of a point which lies outside a circle which rolls, without slipping, around the inside of a fixed circle.
- The applications and principles of the superior trochoid apply to the superior hypotrochoid.
If a circle rolls without slipping round the inside of a fixed circle then a point P outside the circumference of the rolling circle will produce a superior hypotrochoid.
A superior hypotrochoid is the path of a point which lies outside a circle, which rolls, without slipping around the inside of a fixed circle.
Inferior Hypotrochoid

• An inferior hypotrochoid is the path of a point which lies inside a circle which rolls, without slipping, around the inside of a fixed circle.

• The applications and principles of the inferior trochoid apply to the inferior hypotrochoid.
If a circle rolls without slipping round the inside of a fixed circle then a point P inside the circumference of the rolling circle will produce an inferior hypotrochoid.
Segment lengths stepped off along base arc

An inferior hypotrochoid is the path of a point which lies inside a circle, which rolls, without slipping around the inside of a fixed circle.
Loci of irregular paths

• The path the object follows can change as the object rolls
• The principle for solving these problems is similar ie. triangulation
• Treat each section of the path as a separate movement
• Any corner has two distinctive loci points
The circle C rolls along the path AB without slipping for one full revolution. Find the locus of point P.
Point X remains stationary while the circle rolls around the bend
TANGENTS TO LOCI
Tangent to a cycloid at a point P
Arc length = Radius of Circle
Tangent to an epicycloid at a point P
Arc length = Radius of Circle
Tangent to the hypocycloid at a point P
Arc length = Radius of Circle
Further Information on Loci

• [http://curvebank.calstatela.edu/cycloidmaple/cycloid.htm](http://curvebank.calstatela.edu/cycloidmaple/cycloid.htm)
COMBINED MOVEMENT
Shown is a circle C, which rolls clockwise along the line AB for one full revolution. Also shown is the initial position of a point P on the circle. During the rolling of the circle, the point P moves along the radial line PO until it reaches O. Draw the locus of P for the combined movement.
Shown is a circle C, which rolls clockwise along the line AB for three-quarters of a revolution. Also shown is the initial position of a point P on the circle. During the rolling of the circle, the point P moves along the semi-circle POA to A.

Draw the locus of P for the combined movement.
Combined Movement

The profile PCDA rolls clockwise along the line AB until the point D reaches the line AB. During the rolling of the profile, the point P moves along the lines PA and AD to D. Draw the locus of P for the combined movement.
LINKAGES
• Linkages are used to restrict the motion of objects so that they follow a regular path

• Linkages are commonly found in children's toys, windows, automotive parts, etc
Linkages

• Linkages are used to redirect kinetic energy or other forces
• A scissors jack is a common example of a linkage
Basic Linkages

- Sliding Link
- Pivot
- Crank
- Rocker
Sliding Link

- Sliding links are used to restrict the movement of a link to movement along an axis.
- Sliding links have many applications and functions.
- Sliding links restrict the opening in windows.
Pivot

- A pivot joins two links together and allows 360° of freedom about the pivot
- A pivot acts like a hinge
- Pivots are found in many household items
Crank

- Cranks are used to receive motion or to transfer rotary motion
- Cranks are often used with bevel gears or other variations of gears
- Common examples are in wheel braces, hand mixers etc
Rocker

- A rocker mechanism restricts the swing of a linkage, to a known angle
- Rockers are very prominent in children's cradles, chairs and similar objects
- The locus of the rocker must be found to ensure the chair doesn't swing too far back
Linkages

- Mechanisms like this are a common feature in machines.
- In order for such mechanisms to operate as desired it is necessary to plot the loci of the parts.
Converting Mechanisms to Line Drawings

P

C

B

A
Converting Mechanisms to Line Drawings
A ladder AB is leaning against a wall, with one end against the wall and the other on the floor. Plot the locus of the midpoint of the ladder as it slides to the floor.
The figure below shows a crank AB which rotates clockwise about point A. Link BC is restricted to slide vertically at C. Plot the locus of point P for one revolution of the crank.
The figure below shows a crank AB which rotates about the fixed point B in a clockwise direction. Point C is a trunnion. Crank BC pivots about the fixed point B and slides through the trunnion. Plot the locus of point P for one revolution of the crank AB.
The figure below shows a crank OA which rotates about the fixed point O. A and B are pin joints. Crank BC pivots about the fixed point C. Plot the locus of point P for one revolution of the crank OA.
The crank OD rotates anticlockwise about a fixed point O. AB oscillates about the fixed point A. D is fixed to a block, which slides along AB. Point C is constrained to slide horizontally.
Plot the locus of point P during one complete revolution of the crank OD.
The figure below shows a crank Ab which rotates anti-clockwise about pivot C. Another crank CD rotates clockwise about Pivot C. Link BE pivots at B and E. Link DEP pivots at D and E. Plot the locus of point P for one revolution of the cranks. (Both cranks rotates at the same rate)
The figure below shows a crank AB which pivots about A. B is a pivot and P is a sliding link. Plot the displacement diagram for point P for one complete clockwise revolution of the crank.
CAMS
Cams

• A cam is a machine part for transferring rotary motion to linear motion

• In a radial plate cam, the cam is mounted on a rotating shaft

• The motion is received by a follower

• To see a cam in operation click on the link

http://www.engr.colostate.edu/~dga/video_demos/mechanisms/IC_engine_cam_crank_animation.gif
Follower Types

- Followers can be knife edged, rollers or flat footed

Knife Edged

Roller

Flat Footed
Knife Edged Follower

- The point of the follower can follow very complicated cam profiles
- Wears Rapidly
- Must be used at low speeds
In order to determine the shape of a cam, a displacement diagram is drawn first.

The height of the diagram (A) is equal to the total displacement of the follower i.e., the difference between the highest and lowest points.

The width of the displacement diagram does not matter but it is divided into regular divisions representing angular increments (on the cam).

30° increments are generally used.
Uniform Velocity (UV)

- A cam that imparts uniform velocity (UV) has the following displacement diagram.
- The cam shown has a rise at uniform velocity, followed by a fall at uniform velocity:
  - The follower rises and falls at a constant speed.
- Shown over is the cam profile with uniform velocity rise and uniform velocity fall.
- The disadvantage of uniform velocity is abrupt changes of movement of the follower.
A dwell is a period when there is no displacement of the follower.
- Cam radius remains constant.

A cam will have a circular profile for periods of dwell.

Note the circular segment on the cam.
Simple Harmonic Motion (SHM) is the gentle acceleration and deceleration of the end view of a point as it rotates at constant speed around the diameter of a circle.

• Simple harmonic motion produces a sine curve.
• Shown over is the outline of a cam with SHM rise and SHM fall.
Uniform Acceleration and Retardation (UAR)

- A follower with Uniform Acceleration and Retardation (UAR) will accelerate and decelerate at the same rate.
- The path of UAR is parabolic and can be drawn using the rectangle method.
- Shown over is the outline of a cam with UAR rise and UAR fall.
• Draw the displacement diagram for a plate cam rotating in an anticlockwise direction imparting the following motion to the inline knife edge follower:
  – UV rise 0°-90° of 40mm
  – Dwell 90°-180°
  – SHM fall 180°-360° of 40mm
• The nearest approach of the follower to the cam shaft centre is 20mm
• The can shaft diameter is 15mm
• Draw the displacement diagram for a plate cam rotating in a clockwise direction imparting the following motion to the inline knife edge follower:
  – SHM rise 0° -90° of 35mm
  – UV rise 90° -210° of 10mm
  – UAR fall 210° -360° of 45mm
• The nearest approach of the follower to the cam shaft centre is 20mm
• The cam shaft diameter is 15mm
Nearest approach of follower 20mm
Ø15 mm shaft
Total rise 45
Plot the follower displacement diagram for an in-line knife-edge follower in contact with the cam profile shown below.
Roller Followers

- Are used because they give a smoother movement and they wear more evenly
• Draw the displacement diagram for a plate cam rotating in an anticlockwise direction imparting the following motion to the roller follower:
  – UV rise 0˚ -90˚ of 40mm
  – Dwell 90˚ -180˚
  – SHM fall 180˚ -360˚ of 40mm
• The roller follower has a diameter of 12mm
• The nearest approach of the roller centre to the cam shaft centre is 20mm
• The cam shaft diameter is 15mm
Total rise 40

Nearest approach of follower
20mm
Roller Ø12
Ø15 mm shaft
Flat Footed Follower

- Wears slower than a knife edge follower
- May bridge over hollows
Cams

• Draw the displacement diagram for a plate cam rotating in an anticlockwise direction imparting the following motion to the flat follower:
  – UV rise 0° -90° of 40mm
  – Dwell 90° -180°
  – SHM fall 180° -360° of 40mm
• The follower extends 6mm to either side
• The nearest approach of the follower to the cam shaft centre is 20mm
• The cam shaft diameter is 15mm
Nearest approach of follower 20mm

Total rise 40

UV rise  Dwell  SHM Fall

Flat footed follower extends 6mm to either side
Ø15 mm shaft
GEARS
Gears

- Gears are toothed wheels
- Gears are used to transmit motion
- Gears are also used to convert rotary to linear motion or visa versa
- Gears can be used to reduce or increase the torque on an object
- Gears are found in watches, engines and toys
Types of Gears

• There are many different types of gears, each of which are designed for their specific purpose.
• Different types of gears are used in the following machines:
  • Drills: Bevel Gears
  • Car engines: Helical Gears
  • Watches: Epicycloidal Gears
  • Power transmission: Involute Spur Gears
Gear animations

Imagine two disks in contact at their circumference (friction wheels)
These two disks meet at one point
If one disk rotates it imparts motion to the other disk
However these disks are prone to slipping
Large pressure must be exerted between the disks in order to create a sufficient frictional force between them
Friction wheels will only be used where low power is required
Introducing teeth will eliminate slipping occurring
Why gears?

Two Friction Wheels

Two Toothed Wheels
A spur gear is a toothed wheel. The shape of the teeth is derived from either an involute curve or an epicycloidal curve. The involute is the most commonly used curve.
Gear Terminology

Driver Gear

Pinion (Driven Gear)

Rack
<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
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<tbody>
<tr>
<td>Driver Gear</td>
<td>When two gears are in mesh the gear with the power (connected to the shaft) is called the driver</td>
</tr>
<tr>
<td>Pinion</td>
<td>When two gears are in mesh the smaller gear is called the pinion, and the gear which power is transmitted to is called the driven gear</td>
</tr>
<tr>
<td>Rack</td>
<td>It is a spur gear whose radius is at infinity)</td>
</tr>
</tbody>
</table>
Gear ratio

• The gear ratio is the ratio at which one gear rotates relative to the other and it is directly proportional to the diameter of the gears

• Diameter of driver gear = D

• Diameter of driven gear = d

• Gear ratio = D:d

• If the driver gear has a diameter of 200 and the driven gear has a diameter of 100 then the gear ratio will be 2:1
  – The driven gear rotates twice as fast as the driver gear
The aim of designing gear teeth is that the faces of the teeth will roll across each other, minimalising the sliding friction.

Two types of curves are commonly used:
- Involute
- Epicycloidal
The involute profile provides continuous contact between the teeth while also providing a smooth rolling motion.
Parts of a Gear Tooth

- An imaginary circle which corresponds to the outside diameter of the friction rollers from which the spur gears are derived.

- Formula =
  - Module × Number of teeth
Parts of a Gear Tooth

- Radial distance from the pitch circle to the top of the tooth
- Formula:
  - Addendum = module
Addendum
Parts of a Gear Tooth

- Radial distance from the pitch circle to the bottom of the tooth space
- Formula:
  - Dedendum = 1.25 × module
Dedendum

1.25 x m
Parts of a Gear Tooth

- Distance between the top of a tooth and the bottom of the mating space
- Formula:
  - Dedendum - Addendum
The base circle is the imaginary circle from which the involute is created.

Formula:
- Base Circle Diameter = Pitch Circle Diameter \times \cos (\text{pressure angle})
Parts of a Gear Tooth

- Distance measured along the pitch circle from a point on one tooth to a corresponding point on the next tooth. This includes one tooth and one space.
- Formula: 
  \[- \frac{\pi d(\text{circumference})}{n}\]
Circular Pitch

Circular Pitch

Tooth Space

Tooth Thickness
Parts of a Gear Tooth

- Circular thickness: Thickness of one tooth measured along the pitch circle, equal to \( \frac{1}{2} \) the circular pitch.

Formula:
- Circular pitch \( \div 2 \)
Parts of a Gear Tooth

- Outside Diameter: is the diameter of the circle that contains the top of the teeth =
- Formula:
  - Pitch Circle Diameter + 2 addendum
Parts of a Gear Tooth

- Diameter of the root circle
- Formula:
  - Pitch Circle Diameter - 2 dedendum
Parts of a Gear Tooth

- Full height of the tooth
- Formula:
  - Addendum + Dedendum
Parts of a Gear Tooth

- Distance a tooth projects into the mating space
- Formula:
  - \( 2 \times \text{Addendum} \)
Working Depth
Parts of a Gear Tooth

- Angle created at the centre of the gear between a point on one tooth on the PCD, and the corresponding point on an adjacent tooth.
- Formula: 
  - \(360 \div \text{number of teeth}\)
Angular Pitch

Circular Pitch

Tooth Thickness

Spacewidth

Angular Pitch
More Terminology

• **Common Tangent** – A line tangential to the two base circles along which contact between the meshing teeth takes place, also known as the **line of action**

• **Pitch Point** – Point of contact between the pitch circles of meshing gears
# Parts of a Gear Tooth

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<thead>
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<tbody>
<tr>
<td>Pitch Surface</td>
<td>Is an imaginary cylindrical surface which contains the pitch circle of a gear</td>
</tr>
<tr>
<td>Addendum</td>
<td>Is the part of the tooth outside the pitch</td>
</tr>
<tr>
<td>Dedendum</td>
<td>Is the part of the tooth inside the pitch surface</td>
</tr>
<tr>
<td>Flank</td>
<td>Is the part of the tooth that comes into contact with other gears</td>
</tr>
<tr>
<td>Tip Surface</td>
<td>Is an imaginary surface at the top of the tooth</td>
</tr>
<tr>
<td>Root Surface</td>
<td>Is an imaginary surface at the bottom of the tooth</td>
</tr>
<tr>
<td>Top Land</td>
<td>Is the part of the tooth between opposite flanks</td>
</tr>
<tr>
<td>Bottom Land</td>
<td>Is the part of the root surface between opposite flanks</td>
</tr>
<tr>
<td>Tooth Trace</td>
<td>Is the intersection between the pitch surface and the flank of the tooth</td>
</tr>
</tbody>
</table>
Gear Terminology

\[ \theta = \text{pressure angle} \]
# Gear Terms

<table>
<thead>
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<th>Symbol</th>
<th>Definition</th>
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</thead>
<tbody>
<tr>
<td>Addendum</td>
<td>a</td>
<td>The part of the tooth that extends outside of the pitch circle/pitchline</td>
</tr>
<tr>
<td></td>
<td></td>
<td>The addendum is always equal to the module</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$a = m$</td>
</tr>
<tr>
<td>Base Circle</td>
<td>BCD</td>
<td>An imaginary circle from which the tooth shape is generated</td>
</tr>
<tr>
<td></td>
<td></td>
<td>The base circle = the pitch circle diameter $\times \cos$ (pressure angle)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$BCD = PCD \times \cos$ (pressure angle)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>This circle can be constructed graphically</td>
</tr>
<tr>
<td>Circular Pitch</td>
<td>p</td>
<td>Is the distance from the point on one tooth to the corresponding point on the next tooth</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Measured around the pitch circle</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$p = \pi m$</td>
</tr>
</tbody>
</table>
## Gear Terms

<table>
<thead>
<tr>
<th>Term</th>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
</table>
| Circular Tooth Thickness |        | The thickness of a tooth measured along the pitch circle
|                        |        | Circular tooth thickness = p/2                                               |
| Clearance              | c      | Is the space underneath the tooth when it is in mesh
|                        |        | Clearance = ¼ of the addendum                                               |
|                        |        | c =d - a                                                                    |
|                        |        | =0.25a                                                                      |
| Dedendum               | d      | Is the part of the tooth which is inside the pitch circle or the pitch line |
|                        |        | =1.25 × addendum                                                            |
|                        |        | d=1.25 a                                                                    |
| Line of Action         |        | Contact between the teeth of meshing gears takes place along a line tangential to the two base circles
|                        |        | This line passes through the pitch point                                    |
## Gear Terms

<table>
<thead>
<tr>
<th>Term</th>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Module</td>
<td>m</td>
<td>Is the pitch circle diameter divided by the number of teeth The module for gears in mesh must be the same or they will vibrate and wear badly</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[ m = \frac{PCD}{t} ]</td>
</tr>
<tr>
<td>Pitch Circle Diameter</td>
<td>PCD</td>
<td></td>
</tr>
<tr>
<td>Pinion</td>
<td></td>
<td>When two gears are in mesh the smaller gear is called the pinion</td>
</tr>
<tr>
<td>Pitch Angle</td>
<td></td>
<td>360° ÷ number of teeth</td>
</tr>
<tr>
<td>Pitch Circle</td>
<td>PC</td>
<td>Is the circle representing the original cylinder which transmitted motion by friction</td>
</tr>
<tr>
<td>Pitch Point</td>
<td></td>
<td>When two gears are in mesh their pitch circles will be tangential to each other. The pitch point is the point of contact between the two circles</td>
</tr>
</tbody>
</table>
## Gear Terms

<table>
<thead>
<tr>
<th>Term</th>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure Angle</td>
<td>Θ</td>
<td>The angle between the line of action and the common tangent to the pitch circles at the pitch point. The pressure angle is normally 20° but may be 14.5°</td>
</tr>
<tr>
<td>Tip Circle</td>
<td></td>
<td>A circle through the tips of the teeth</td>
</tr>
<tr>
<td>Wheel</td>
<td></td>
<td>When two gears are in mesh the larger one is called the wheel</td>
</tr>
<tr>
<td>Whole depth</td>
<td></td>
<td>Is the depth of the tooth from tip to root. Whole depth = addendum + dedendum</td>
</tr>
<tr>
<td>Working Depth</td>
<td></td>
<td>The whole depth – the clearance</td>
</tr>
</tbody>
</table>
Line of action

• To ensure the gear motion is smooth, quiet and free from vibration, a direct line of transmission must act between the gear teeth.

• This line of action, or common normal determines the pressure angle of the teeth and passes through the pitch point.
  – i.e. Gears in mesh meet at only one point which is the intersection of their Pitch Circle Diameters.
Other line of action animations

- [http://science.howstuffworks.com/gear8.htm](http://science.howstuffworks.com/gear8.htm)
Common Normal

Line of Action
• A rack is a straight toothed bar
• Technically it is a spur gear whose radius is at infinity
• Because of this all principles of circular spur gears hold true
Rack

\[ \theta \]

\[ \frac{1}{2} p \quad \frac{1}{2} p \]
Line of action for a rack and pinion animation

Gears

• Given a pitch circle diameter of 200mm and a module of 10, and a pressure angle of 20° construct the spur gear.
• Show at least four teeth on the gear
• Teeth to be constructed by the involute method.
<table>
<thead>
<tr>
<th>Gear</th>
<th>Calculations</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Module (m)</td>
<td></td>
<td>10</td>
</tr>
<tr>
<td>No. of teeth (t)</td>
<td></td>
<td>20</td>
</tr>
<tr>
<td>Pressure angle (Θ)</td>
<td></td>
<td>20°</td>
</tr>
<tr>
<td>Pitch circle diameter (PCD)</td>
<td>( m \times t )</td>
<td>200mm</td>
</tr>
<tr>
<td>Base circle diameter</td>
<td>( PCD \times \cos \Theta )</td>
<td>187.9mm</td>
</tr>
<tr>
<td>Addendum (a)</td>
<td>( a = m )</td>
<td>10mm</td>
</tr>
<tr>
<td>Dedendum (d)</td>
<td>( 1.25 \times m )</td>
<td>12.5mm</td>
</tr>
<tr>
<td>Clearance</td>
<td>( d - a )</td>
<td>2.5mm</td>
</tr>
<tr>
<td>Tip circle diameter</td>
<td>( PCD + 2a )</td>
<td>220mm</td>
</tr>
<tr>
<td>Root circle diameter</td>
<td>( PCD - 2d )</td>
<td>175mm</td>
</tr>
<tr>
<td>Circular pitch (p)</td>
<td>( \pi \times m )</td>
<td>31.4mm</td>
</tr>
<tr>
<td>Tooth thickness</td>
<td>( p \div 2 )</td>
<td>15.7mm</td>
</tr>
<tr>
<td>Pitch angle</td>
<td>( 360° \div t )</td>
<td>18°</td>
</tr>
</tbody>
</table>
Addendum Tip circle
Addendum=module
Tip Circle=Ø220

Dedendum Root circle
Dedendum=1.25*module
Root circle=Ø175

Base circle
Circular pitch distance = \( p = \pi m \)
\( p = 31.4 \)

Tooth thickness = \( 31.4/2 = 15.7 \)

Angular pitch = \( 360/t \)
\( 360/16 = 18° \)

Number of teeth =
PCD=module(m)*number of teeth (t)
PCD=m*t=20 teeth

Design and Comm. Graphics
PCD = 200

Tooth radiates into centre below base circle

Clearance = 0.25 * module = 2.5

Tooth thickness measured on PCD

Angular Pitch (18°)

Tracing paper may be used to replicate the tooth profile

Addendum/Tip circle

Dedendum/Root circle

Base circle

Only Pitch circle and tip circle are required for left hand side.
SOLUTION USING UNWINS METHOD
PCD = 200
Addendum = module
Tip Circle = Ø220
Dedendum = 1.25
* module
Root Circle = Ø175
Circular pitch distance = 
\[ p = \pi m \]
\[ p = 31.4 \]
Tooth thickness = 
\[ 31.4/2 = 15.7 \]
Angular pitch = 360/t 
\[ 360/16 = 18^\circ \]
Number of teeth = 
PCD = module (m) * number of teeth (t) 
PCD = m*t = 20 teeth
Base Circle Ø = PCD * cos (pressure angle) 
BCD = 187.9mm
Tangent to base circle

Radius 1A

Pitch Point (P)

Tooth Thickness

Bisector of tooth thickness (radiating to CP of PCD)

Axial Symmetry of Centre point 1

Radius 1P

Bottom of tooth profile radiates into centre point
• Draw two involute spur gears in mesh and show five teeth on each gear.
• The gear ratio is 4:3.
• Driver gear details: Module=8, teeth=24, pressure angle=20°
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Calculation</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Module (m)</td>
<td></td>
<td>8</td>
</tr>
<tr>
<td>No. of teeth (t)</td>
<td></td>
<td>24</td>
</tr>
<tr>
<td>Pressure angle (Θ)</td>
<td></td>
<td>20</td>
</tr>
<tr>
<td>Pitch circle diameter (PCD)</td>
<td>$m \times t$</td>
<td>192</td>
</tr>
<tr>
<td>Base circle diameter</td>
<td>$PCD \times \cos\Theta$</td>
<td>180.4</td>
</tr>
<tr>
<td>Addendum (a)</td>
<td>$a = m$</td>
<td>8</td>
</tr>
<tr>
<td>Dedendum (d)</td>
<td>$1.25 \times m$</td>
<td>10</td>
</tr>
<tr>
<td>Clearance</td>
<td>$d - a$</td>
<td>2</td>
</tr>
<tr>
<td>Tip circle diameter</td>
<td>$PCD + 2a$</td>
<td>208</td>
</tr>
<tr>
<td>Root circle diameter</td>
<td>$PCD - 2d$</td>
<td>172</td>
</tr>
<tr>
<td>Circular pitch (p)</td>
<td>$\pi \times m$</td>
<td>25.13274123</td>
</tr>
<tr>
<td>Tooth thickness</td>
<td>$p \div 2$</td>
<td>12.56637061</td>
</tr>
<tr>
<td>Pitch angle</td>
<td>$360° \div t$</td>
<td>15</td>
</tr>
<tr>
<td>Driven Gear</td>
<td>Calculations</td>
<td>Results</td>
</tr>
<tr>
<td>-----------------------------------------</td>
<td>-------------------------------</td>
<td>----------</td>
</tr>
<tr>
<td>Module (m)</td>
<td></td>
<td>8</td>
</tr>
<tr>
<td>No. of teeth (t)</td>
<td>4:3=24:18</td>
<td>18</td>
</tr>
<tr>
<td>Pressure angle (Θ)</td>
<td></td>
<td>20</td>
</tr>
<tr>
<td>Pitch circle diameter (PCD)</td>
<td>m × t</td>
<td>144</td>
</tr>
<tr>
<td>Base circle diameter</td>
<td>PCD × cosΘ</td>
<td>135.3</td>
</tr>
<tr>
<td>Addendum (a)</td>
<td>a = m</td>
<td>8</td>
</tr>
<tr>
<td>Dedendum (d)</td>
<td>1.25 × m</td>
<td>10</td>
</tr>
<tr>
<td>Clearance</td>
<td>d - a</td>
<td>2</td>
</tr>
<tr>
<td>Tip circle diameter</td>
<td>PCD + 2a</td>
<td>160</td>
</tr>
<tr>
<td>Root circle diameter</td>
<td>PCD - 2d</td>
<td>124</td>
</tr>
<tr>
<td>Circular pitch (p)</td>
<td>π × m</td>
<td>25.13274123</td>
</tr>
<tr>
<td>Tooth thickness</td>
<td>p ÷ 2</td>
<td>12.56637061</td>
</tr>
<tr>
<td>Pitch angle</td>
<td>360° ÷ t</td>
<td>20</td>
</tr>
</tbody>
</table>
PCD
Addendum/Tip circle
Root circle/Dedendum
Base circle
Gears

• An involute gear is in mesh with a rack.
• The involute gear has 20 teeth, a pressure angle of 20° and module of 10.
• Draw the gear and rack in mesh, showing four teeth on the gear and an equivalent on the rack.
<table>
<thead>
<tr>
<th>Gear Wheel</th>
<th>Calculations</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>Module (m)</td>
<td></td>
<td>10</td>
</tr>
<tr>
<td>No. of teeth (t)</td>
<td></td>
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</tr>
<tr>
<td>Pressure angle (Θ)</td>
<td></td>
<td>20</td>
</tr>
<tr>
<td>Pitch circle diameter (PCD)</td>
<td>m × t</td>
<td>200</td>
</tr>
<tr>
<td>Base circle diameter</td>
<td>PCD × cosΘ</td>
<td>187.9</td>
</tr>
<tr>
<td>Addendum (a)</td>
<td>a = m</td>
<td>10</td>
</tr>
<tr>
<td>Dedendum (d)</td>
<td>1.25 × m</td>
<td>12.5</td>
</tr>
<tr>
<td>Clearance</td>
<td>d - a</td>
<td>2.5</td>
</tr>
<tr>
<td>Tip circle diameter</td>
<td>PCD + 2a</td>
<td>220</td>
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<tr>
<td>Root circle diameter</td>
<td>PCD - 2d</td>
<td>175</td>
</tr>
<tr>
<td>Circular pitch (p)</td>
<td>π × m</td>
<td>31.4</td>
</tr>
<tr>
<td>Tooth thickness</td>
<td>p ÷ 2</td>
<td>15.7</td>
</tr>
<tr>
<td>Pitch angle</td>
<td>360° ÷ t</td>
<td>18</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Rack</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>Module</td>
<td>10</td>
</tr>
<tr>
<td>Pressure angle</td>
<td>20°</td>
</tr>
<tr>
<td>Addendum</td>
<td>10mm</td>
</tr>
<tr>
<td>Dedendum</td>
<td>12.5mm</td>
</tr>
<tr>
<td>Clearance</td>
<td>2.5mm</td>
</tr>
<tr>
<td>Pitch</td>
<td>31.4mm</td>
</tr>
<tr>
<td>Tooth thickness</td>
<td>15.7mm</td>
</tr>
</tbody>
</table>
PCD
Addendum/ Tip circle
Dedendum/ Root circle
Base circle
Links to Dynamic Mechanisms

• http://www.ul.ie/~nolk/maincontents.htm